Program Analysis with Local Policy Iteration

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Outline

Motivation

- Program verification

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- Program verification
- Finding inductive invariants

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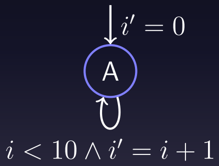
- Program verification
- Finding inductive invariants
- LPI
  - Scalable algorithm for policy iteration
  - Sent to FMCAD'15

Program Modeling

- Control Flow Automaton (CFA)

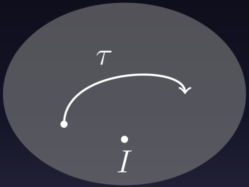
```
int i=0;
while (i<10) {
    i++;
}
```



Inductive Invariant

Motivation

- Task: verify program properties
- Prove: by induction
- Aim: find inductive invariant
  - Includes initial state
  - Closed under transition



Inductive Invariant

Abstract Interpretation Limitations

- Usual tool: abstract interpretation
- Relies on widenings/narrowings to enforce convergence
- Can be very brittle

Policy Iteration

Historical Perspective

- Game-theoretique technique
- Solving markov processes

Policy Iteration

Historical Perspective

- Game-theoretique technique
- Solving markov processes
- Used for poker AI

Policy Iteration

Introduction - 2

- Finds least inductive invariant in the given abstract domain
- Considers the program as a set of equations
- Game-theoretic algorithm adapted to find inductive invariant
- Requires abstract semantics to be monotone & concave

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Guarantees

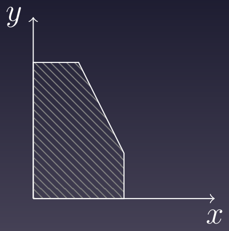
Least inductive invariant, not least invariant in general!

Outline

Template Constraints Domain

Domain used in our work

- Choose linear inequalities to be tracked before the analysis
- E.g. $x, y, x + y$ (templates)
- We want to find inductive invariant $x \leq d_1 \wedge y \leq d_2 \wedge x + y \leq d_3$ for all control states
- An element of the domain above is a vector $(3, 2, 4)$ which corresponds to $x \leq 3 \wedge y \leq 2 \wedge x + y \leq 4$



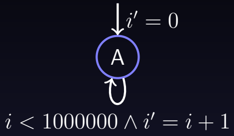
Template Constraints Domain

Abstract Semantics

- Abstract Semantics: transition relation in the abstract domain
- Convex optimization:
  - Template x, transition $x' = x + 1$, previous element $x \leq 5$
  - New element given by $\max x'$ s. t. $x' = x + 1 \land x \leq 5$

Policy Iteration

Simple Example



- Template constraints domain $\{i\}$
- Aim: find smallest d, s.t. $i \leq d$ is an inductive invariant
- Use semantical equations for d

Policy Iteration

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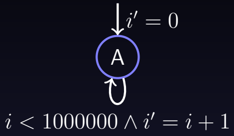

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- Aim: find smallest d, s.t. $i \leq d$ is an inductive invariant
- Use semantical equations for d
- Necessary and sufficient condition:
- $d = \sup i'$ s.t.
  $$i' = i + 1 \land i < 1000000 \land i \leq d \bigvee i' = 0 \bigvee \perp$$
  - Disjunctions come from multiple edges

Policy Iteration

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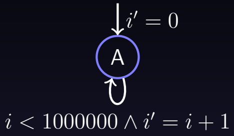

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  - Disjunctions come from multiple edges
  - $\perp$ represents unreachable state
  - We take supremum as the answer can be ∞ (unbounded) or $-\infty$ (unreachable)

Policy Iteration

Explanation By Example

- We have a min-max equation:
  $d = \min\left(\sup i' \text{ s.t. } i' = i + 1 \land i < 1000000 \land i \leq d \bigvee i' = 0 \bigvee \bot\right)$

Policy Iteration

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  $d = \min\left(\sup i' \text{ s.t. } i' = i + 1 \wedge i < 1000000 \wedge i \leq d \bigvee i' = 0 \bigvee \perp\right)$
- We consider separate cases for disjunctions
- Replacing each disjunction with one argument
  - $d = \sup i' \text{ s.t. } i' = 0$
  - Referred to as a policy

Policy Iteration

Explanation By Example - 2

- $d = \sup i'$ s.t. $i' = 0$
- Simplified system (with no disjunctions):
  - Monotone and concave
  - ≤ 2 fixpoints
  - Can be solved using LP

Policy Iteration Example

Algorithm Run

- $d = \sup i'$ s.t.
  $$i' = i + 1 \land i < 1000000 \land i \leq d \bigvee i' = 0 \bigvee \perp$$

Policy Iteration Example

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Policy Iteration Example

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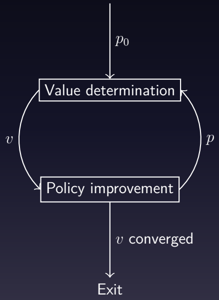
5. Increase to 1000000 using
   $d = \sup i'$ s.t. $i' = i + 1 \wedge i < 1000000 \wedge i \leq d$
6. Substitute, holds!
   $1000000 = \sup i'$ s.t. $i' = i+1 \wedge i < 1000000 \wedge i \leq d \bigvee i' = 0 \bigvee \perp$

Policy Iteration

Algorithm Overview



Policy $p \leftarrow p_0$
repeat
  $v \leftarrow$ value determination based on p
  $p \leftarrow$ policy based on v
until v converges

Policy Iteration

Algorithm Continued

- Policy Improvement: SMT call
  - Policy which can improve current value?

- Value Determination: LP call
  - Maximum value for current policy?

Path Focusing

Similar to Large Block Encoding

- Unknown per node per template

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- Over-approximates invariant in the abstract domain
- Loss of precision

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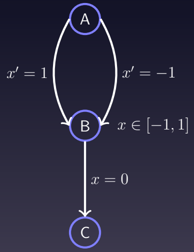
```
if (unknown()) {
    x = -1;
} else {
    x = 1;
}
assert(x != 0);
```



Path Focusing

Adding disjunctions

- Solution: remove nodes!
- CFG Compaction:
  - Edges (A, τ_1, B), (B, τ_2, C), with no other incoming to B
    - Converted to $(A, \tau_1 \wedge \tau_2, C)$, B removed
  - Edges (A, τ_1, B), (A, τ_2, B), no other incoming to B
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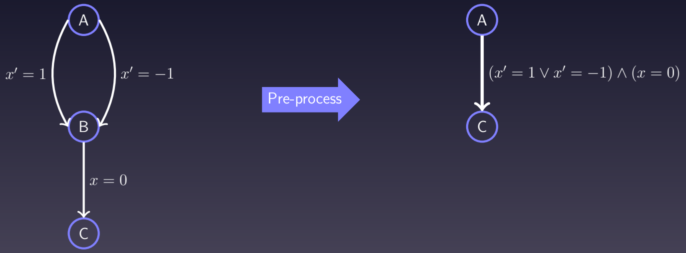

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- For a well-structured graph: only loop-heads remain

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Path Focusing

Properties

- For a well-structured graph: only loop-heads remain
- Disjunctions create new policies
- Possible improvement: cut-set
  - Set of nodes which cut all the cycles in the graph
- Disadvantage: requires pre-processing

Outline

Problems of Policy Iteration

Motivation for our work

- Problems with the approach above:
  - Scalability: at each step, we update each policy, and at each step, we solve the global equation system (of the size of the entire program)
  - Cooperability: policy iterations don't fit into any existing framework, pre-processing makes it worse

Problems of Policy Iteration

Our Contribution

- Our work: LPI (Local Policy Iteration)
  - Exploits the locality to avoid redundant computation
  - Avoids solving the global equation at each point
  - Unifies policy iteration with other approaches using CPA (Configurable Program Analysis Framework)
  - No pre-processing is involved

LPI as a Configurable Program Analysis

$$x \leq 4 \xrightarrow{x' = x + 1} ?$$

- Transfer Relation:
  - Similar to abstract interpretation
  - Record the bound along with the policy

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  - Record the bound along with the policy
- Global map M : location $\rightarrow$ abstract state

- When two states for the same node, merge
  - When merge closes the loop, perform value determination
  - Follow backpointers to re-create global problem

Abstract Domain

- Two lattices:
  - Abstracted State (element of template constraints domain)
  - Intermediate State (formula)
- Idea: avoid pre-processing

Abstract Domain

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  - Abstracted State (element of template constraints domain)
  - Intermediate State (formula)
- Idea: avoid pre-processing
- Propagate intermediate states, convert to abstracted at loop-heads

LPI Abstract Domain

Abstracted States

Abstracted State

Set of tuples (bound, policy, backpointer) for each template.
Current bound, policy and the previous value used to derive that bound.

- Abstracted state example: $\{i : (0, i' = 0, A)\}$

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- Abstracted state example: $\{i : (0, i' = 0, A)\}$
- Partial order given by component-wise comparison on bounds
- On merge:
  - Pick the upper bound for each template
  - Keep the corresponding policy and backpointer

Intermediate State

Formula $\phi(X')$ representing set of reachable states

Ω meta-variables instead of backpointers

- Intermediate State example:
    - $x' = 1 \land \Omega = A \lor x' = 0 \land \Omega = B$

LPI Abstract Domain

Intermediate States

Intermediate State

Formula $\phi(X')$ representing set of reachable states

Ω meta-variables instead of backpointers

- Intermediate State example:
  - $x' = 1 \land \Omega = A \lor x' = 0 \land \Omega = B$
- Propagation: symbolic execution
- Can be converted to abstracted state using abstraction
  - Maximizing for every template
  - Recording policy and backpointer

LPI

Propagation Overview

- Start with abstracted state at node A: $\{x : (0, x' = 0, I)\}$

LPI

Propagation Overview

- Start with abstracted state at node A: $\{x : (0, x' = 0, I)\}$
- Successor under edge $x' = x + 5$

LPI

Propagation Overview

- Start with abstracted state at node A: $\{x : (0, x' = 0, I)\}$
- Successor under edge $x' = x + 5$
- Intermediate state: $x' = x + 5 \wedge x \leq 0 \wedge \Omega = A$

LPI

Propagation Overview

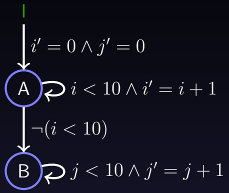
- Start with abstracted state at node A: $\{x : (0, x' = 0, I)\}$
- Successor under edge $x' = x + 5$
- Intermediate state: $x' = x + 5 \land x \leq 0 \land \Omega = A$
- If we need to perform abstraction, we get $\{x : (5, x' = x + 5, A)\}$

Local Value Determination

- On closing the loop (at abstracted state):
  - Follow backpointers, keep adding constraints
  - Create value determination problem
    - Potentially size of the largest loop

Local Policy Iteration

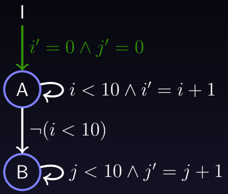
Algorithm Example



1. Start with abstracted state $\top$

Local Policy Iteration

Algorithm Example



1. Start with abstracted state $\top$
2. Intermediate state $i' = 0 \wedge j' = 0 \wedge \Omega = I$

Local Policy Iteration

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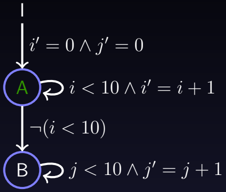

1. Start with abstracted state $\top$
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3. Abstracted to $\{i : (0, I), j : (0, I)\}$

Local Policy Iteration

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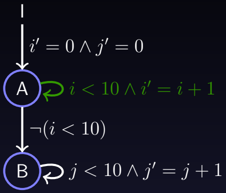

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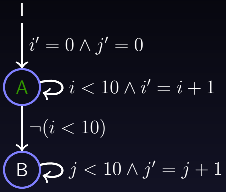

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Local Policy Iteration

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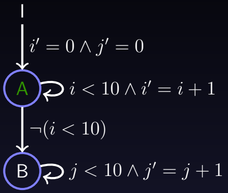

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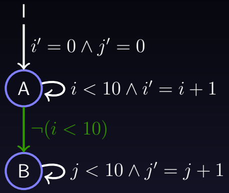
6. Merge A, val. det.: $\{i : (10, A), j : (0, I)\}$

Local Policy Iteration

Algorithm Example



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Local Policy Iteration

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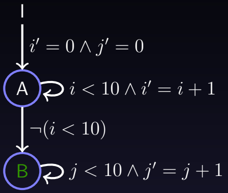

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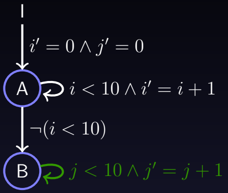

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8. Abstracted: $\{i : (10, A), j : (0, A)\}$
9. Intermediate:
   $i = 10 \wedge j \leq 0 \wedge j' = j + 1 \wedge \Omega = B$

Local Policy Iteration

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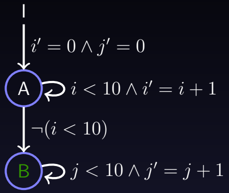

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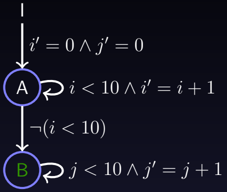

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   $i = 10 \land j \leq 0 \land j' = j + 1 \land \Omega = B$
10. Abstracted: $\{i : (10, B), j : (1, B)\}$
11. Merge B, val. det.: $\{i : (10, B), j : (10, A)\}$

Reachability of Bad States

- Whether we are safe:
  - $\phi \wedge E$ is unsat
  - Example: $(x \leq 10) \wedge (x = 11)$

Reachability of Bad States

- Whether we are safe:
  - $\phi \land E$ is unsat
  - Example: $(x \leq 10) \land (x = 11)$
- Whether we are unsafe:
  - $\phi \land \neg E$ is unsat
  - Example: $(x = 0) \land (x = 0)$

Algorithm Properties

- Soundness
  - Only terminate when inductive

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Algorithm Properties

- Soundness
  - Only terminate when inductive
- Termination
  - Bounds can only grow
  - Each bound corresponds to some policy
  - Finite number of policies
- Least invariant property
  - Only select feasible policies

LPI Configurations

- Configurations
  - Intervals ($\pm x$)
  - Octagons (above and $\pm x, \pm x \pm y$)
  - Rich Templates (above and $\pm 2x \pm y, \pm x \pm y \pm z, \pm 2x \pm y \pm z$)
  - Unrolling
  - Simple Congruence Analysis

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  - Unrolling
  - Simple Congruence Analysis
- Refinement: progressively switch to more expensive config

Contrast with Classical Policy Iteration

- Only update policies which need updating
- Run value determination on a reduced program section
- Stated in CPA framework
- (Unguided) refinement of template precision
- Local value determination optimizations
  Not in the presentation

Outline

Local Policy Iteration

Code analysis tool

Tool

Included in CPAchecker trunk.

https://github.com/dbeyer/cpachecker

- Configurations:
  - `-policy-intervals`
  - `-policy`
  - `-policy-ensemble`
  - `-policy-counterexample-checking`

LPI Evaluation

- Evaluated on SV-Comp "Loops" category
- Compared with
  - BLAST(2014)
  - CPAchecker-SVcomp15
  - PAGAI
- Across true benchmarks

Results

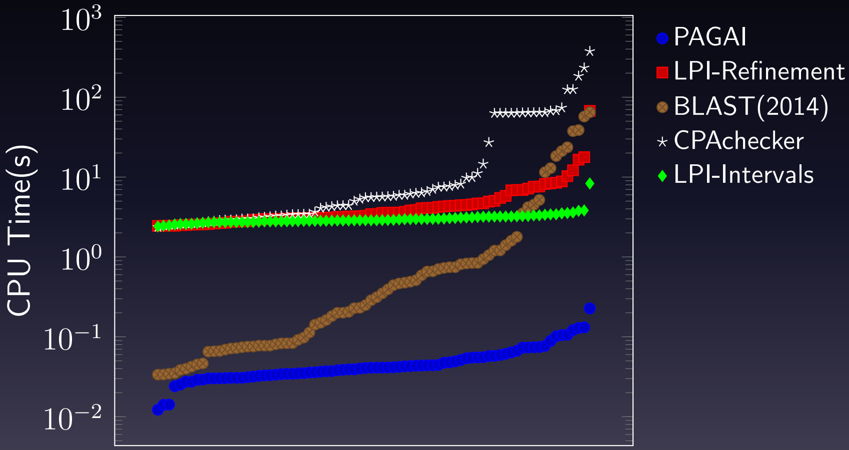
Comparison of Approaches

	vs. LPI	PAGAI	BLAST	CPAchecker	Unique	**Verified**	**Incorrect**
LPI		13	21	22	8	60	1
PAGAI	5		14	15	0	52	1
BLAST	4	5		7	0	43	1
CPAchecker	19	20	21		12	57	2

- Difference between approaches
- Reads: A vs. B

Results

Timing Results



Contributions

Re-iterating

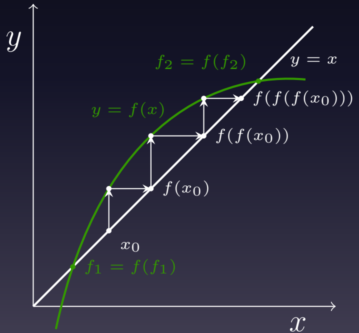
- New scalable algorithm for policy iteration
- Tool for program analysis (using CPAchecker framework)
  - The only policy-iteration based tool capable of dealing with C

Questions?

Policy Iteration

Fixpoints and Concavity

- Concavity and monotonicity limits the number of fixpoints
- Can solve for $x = f(x)$



Policy Iteration

Concavity of Abstract Semantics

- Linear Semantics: $x' = Tx \wedge G(x)$
- Let $t' = t(Tx)$