Multi-Core Partial-Order Reduction for LTL Model Checking

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joint work with **Anton Wijs** (Eindhoven University of Technology)

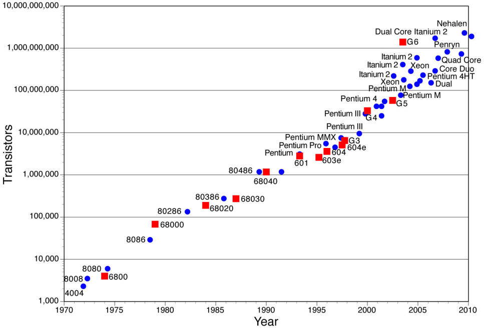
Formal Methods in Systems Engineering
Vienna University of Technology

May 5, 2015

Goals

Combine:

- Parallel model checking (exponential gains)
- Partial-Order Reduction (POR) (exponential gains)



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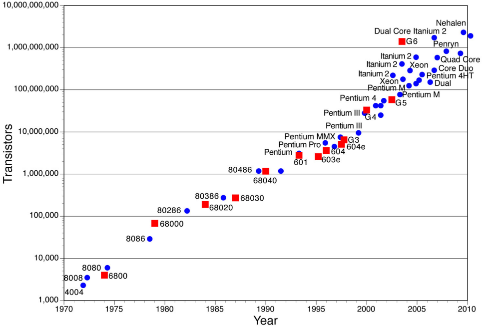
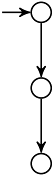
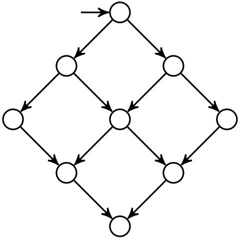


P_i



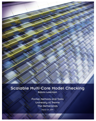
$P_1 \| P_2$



Scalable Multi-Core Model Checking

Research questions

- Can model checking scale on modern multi-cores?
- Retain compatibility with different optimizations?



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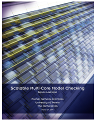

1. On-the-fly
2. Partial-order reduction
3. State compression
4. OR Symbolic with BDDs
   [van Dijk, L, van de Pol, 2013]

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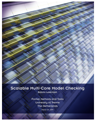

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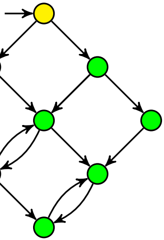
Formalism	Property	Explicit state	+ Compression	+ On-the-fly	+ POR	Symbolic
Plain	Reachability	✓	✓	✓	✓	✓
	Liveness	✓	✓	✓	?	✓
Timed	Reachability	✓	✓	✓	✓	✓
	Liveness	✓	✓	✓	?	✓

1. Shared hash table approach (as opposed to distributed algorithms)
2. Lockless data structures
3. Parallel algorithms (Multi-Core Nested-DFS)

Partial-Order Reduction for LTL

State-space graph: $\mathcal{G} = (\mathcal{S}, T, s_0, AP)$
On-the-fly exploration: $en : \mathcal{S} \rightarrow \mathcal{S}$



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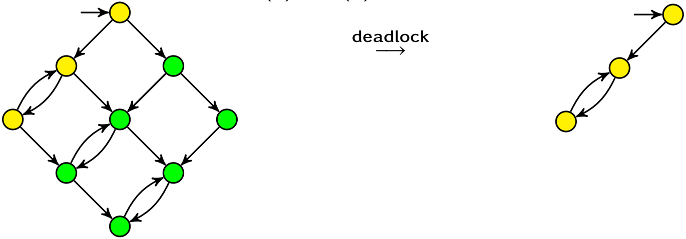
Reduce successor function: $por(s) \subseteq en(s)$.



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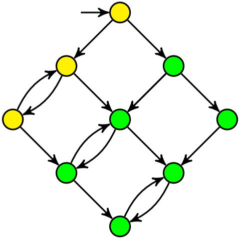
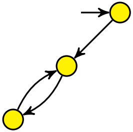
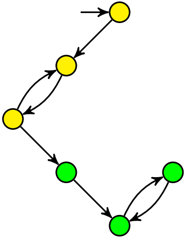


deadlock $\longrightarrow$



$\downarrow$ +ignoring



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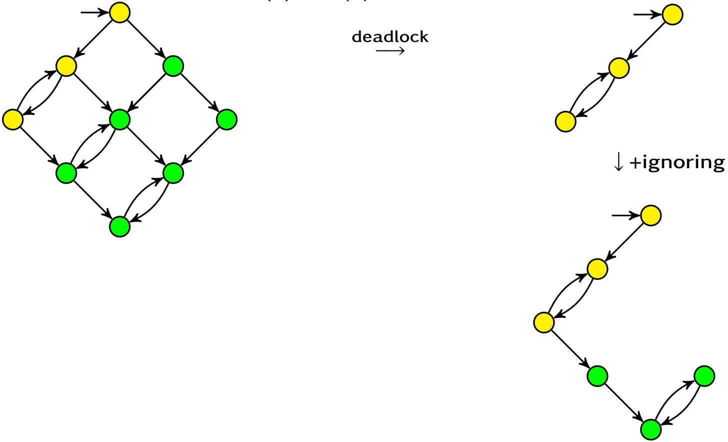

Smaller reduced set $por()$ leads to smaller state space.

DFS Stack Proviso

```
procedure DFS(s)
    for all s′ in por(s) do
        if s′ is not on stack and not visited then
            DFS(s′)
    if successor of s is on the stack then
        explore s fully ( por(s) := en(s) )
    mark s visited
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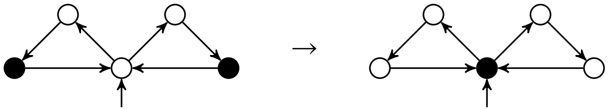

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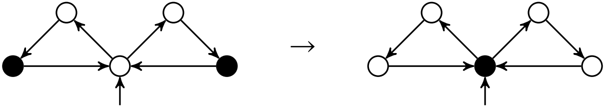

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- (Minimal) feedback vertex set (FVS) → NP-complete
- Stack proviso is the best we can do on-the-fly and in linear time

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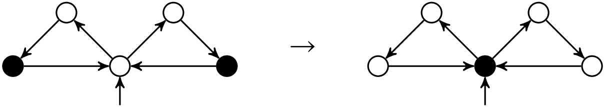

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DFS is P-complete ⇒ inherently sequential (assuming P ≠ NC)

Related Work (Parallel LTL + POR)

Algorithm/Proviso	Reduction	Scalability
NDFS/Stack	++	
TwoPhase [Gopalakrishnan et al.]	+-	??
Topological sort [Barnat et al.]	+-	+
Sticky transitions [Peled et al]	-	+

Related Work (Parallel LTL + POR)

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Topological sort [Barnat et al.]	+-	+
Sticky transitions [Peled et al]	-	+
MC-NDFS/n/a	n/a	++

Challenge: do as good as DFS stack proviso in the parallel setting

Nested Depth-First Search for LTL

[Courcoubetis'93]

Büchi graph: $\mathcal{G} = (\mathcal{S}, \mathcal{F}, T, s_0, AP)$
On-the-fly exploration: $en : \mathcal{S} \rightarrow \mathcal{S}$
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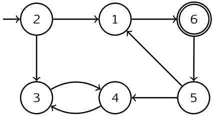
Accepting cycle detection in Büchi automaton ($6 \in \mathcal{F}$):



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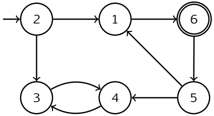

accepting-cycles($\mathcal{G}$) $\subseteq$ cycles($\mathcal{G}$)

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  s.cyan := true
  for all s′ **in** en(s) **do**
    if ¬s′.blue∧¬s′.cyan **then**
      DFSblue(s′)
  if $s \in \mathcal{F}$ **then**
    DFSred(s)
  s.blue := true
  s.cyan := false
procedure DFSred(s)
  s.red := true
  for all s′∈en(s) **do**
    if s′.cyan **then** ExitCycle
    if ¬s′.red **then** DFSred(s′)

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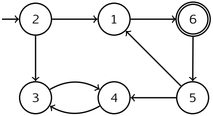

accepting-cycles($\mathcal{G}$) ⊆ cycles($\mathcal{G}$)

Nested DFS (NDFS)

- Linear time

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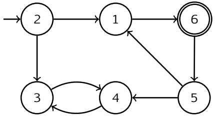

accepting-cycles($\mathcal{G}$) ⊆ cycles($\mathcal{G}$)

Nested DFS (NDFS)

- Linear time
- DFS itself is likely not parallelizable
  - DFS order is P-complete

Swarm Nested Depth-First Search

[Holzmann, 2010]

```
code for worker p:
procedure DFSblue(s, p)
    s.cyan[p] := true
    for all s′ in shuffle(en(s)) do
        if ¬s′.blue[p]∧¬t.cyan[p] then
            DFSblue(s′, p)
    if s ∈ ℱ then
        DFSred(s, p)
    s.blue[p] := true
    s.cyan[p] := false
procedure DFSred(s, p)
    s.red[p] := true
    for all s′ ∈ shuffle(en(s)) do
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Multi-core Nested Depth-First Search

[ATVA11], [PDMC11], [ATVA12]

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    if s ∈ F then
        R := ∅
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    R := R ∪ {s}
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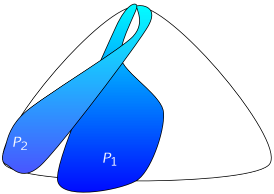

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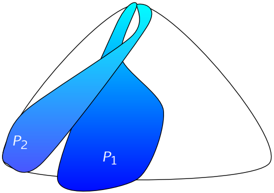

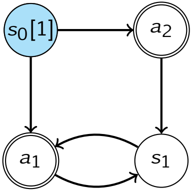

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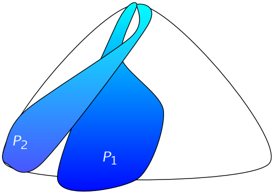

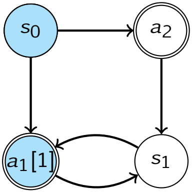

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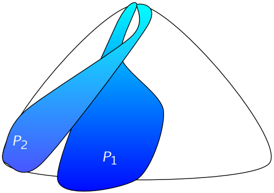

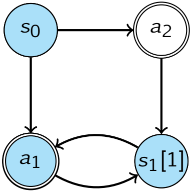

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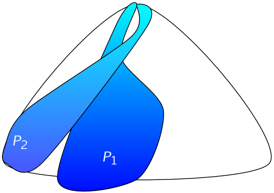

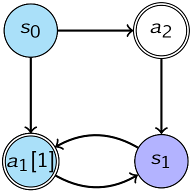

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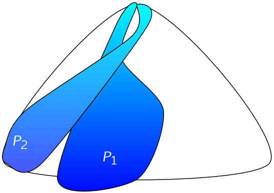

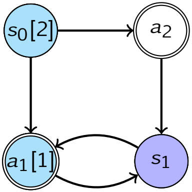

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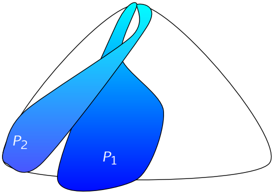

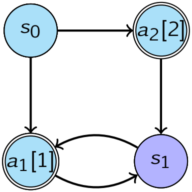

Multi-core Nested Depth-First Search

[ATVA11], [PDMC11], [ATVA12]

code for worker p:
procedure DFSblue(s, p)
  s.cyan[p] := true
  for all s′ **in** shuffle(en(s)) **do**
    if ¬s′.blue∧¬t.cyan[p] **then**
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  if $s \in \mathcal{F}$ **then**
    $R := \emptyset$
    DFSred(s, p)

    mark all in R red
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  s.cyan[p] := false
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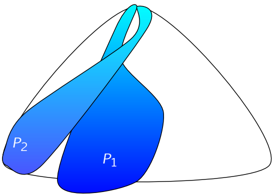
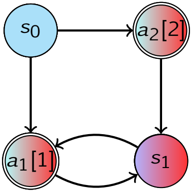
    if $s' \notin R \wedge \neg s.red$ **then** DFSred(s′, p)





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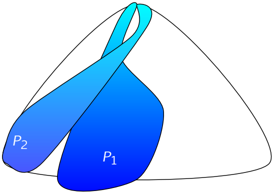

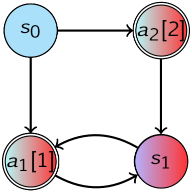

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code for worker p:

procedure DFSblue(s, p)

  s.cyan[p] := true

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    if ¬ ...

  if $s \in$ ...

    R := ...

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    await all accepting in $R \setminus \{s\}$ **are red**

    mark all in R red

  s.blue := true

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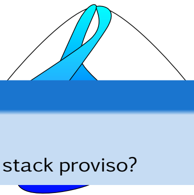
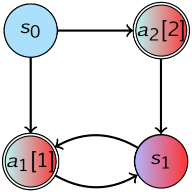


Conclusions

- MC-NDFS scales in practice and uses DFS
- Does it preserve enough order to implement stack proviso?



Stack Proviso in Parallel

```
procedure DFSblue(s, p)
    s.cyan[p] := true
    for all s′ in shuffle(por(s)) do
        if ¬s′.blue ∧ ¬t.cyan[p] then
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    if s ∈ 𝓕 then
        R := ∅
        DFSred(s, p)
        await all accepting in R \ {s} are red
        mark all in R red
    if ∃s′ ∈ por(s) : s′.cyan then
        explore s fully with DFSblue
    s.blue := true
    s.cyan[p] := false
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```
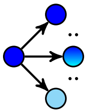

Soundness trivial
Completeness

$Blue \subseteq \Box(Blue \cup \bigcup_p Cyan_p)$



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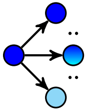

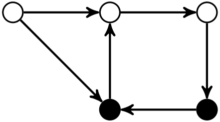

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    if s′.cyan[p] **then** ExitCycle
    if $s' \notin R \wedge \neg s.red$ **then**DFSred(s′, p)
  if successor of s is on DFSred_p **stack then**
    explore s fully with DFSred

Soundness trivial
Completeness

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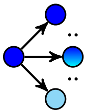

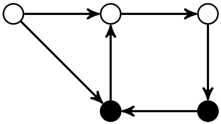

Re-visiting problem
[Holzmann et al., 1996 –
On nested depth-first search]

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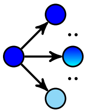

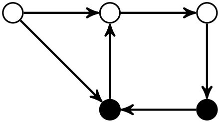

Re-visiting problem
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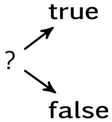
No termination!

The Parallel Cycle Proviso

Add a state proviso flag:




```
procedure dfsBlue(s, p)
    ...
    prov := successor of s is on the local DFSblue stack
    compare_and_swap (s.proviso, ?, prov)
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    ...
```
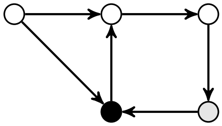

The Parallel Cycle Proviso

Add a state proviso flag: ? → **true** / **false**

Performance



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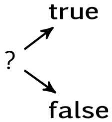
Add a state proviso flag: 

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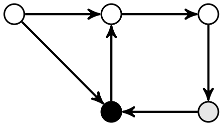

Correctness
Backtracked states (blue and red):
$F = \{s \mid s.proviso \neq ?\}$
$V = \{s \mid s.proviso = \textbf{false}\}$
Lemma 6. Backtracked states have a proviso set: $(B \cup R) \subseteq F$.
Lemma 8. Successors of V states are backtracked: $V \subseteq \Box(B \cup R)$.

Conclusions

Parallel cycle proviso (% reduction)

Model	Threads 1	Threads 64
leader_filters.7	2.35	2.35
elevator.3	94.20	94.96
leader_election.4	3.02	3.02
leader_election.6	0.70	0.70
anderson.6	48.43	51.71
garp	18.69	20.79
peterson4	15.52	15.67
iprotocol-2	34.80	37.91
pacemaker_distributed	47.81	48.26
pacemaker_concurrent	45.90	46.00

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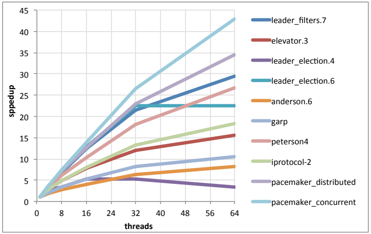

- DFS-proviso's reduction power is preserved
- Speedups maintained

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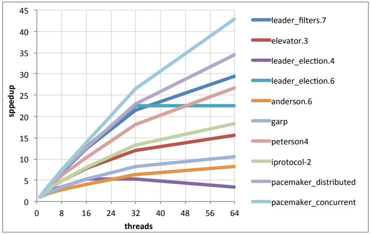

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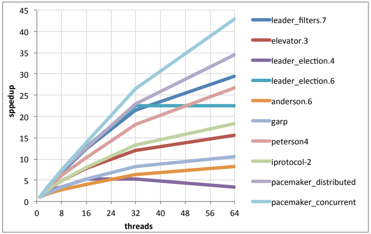

- DFS-proviso's reduction power is preserved
- Speedups maintained
- Demonstrates the strength of parallel DFS-based algortihms
- How much of the DFS order is preserved?
- On which type of graphs does CNDFS scale?