The Stochastic KiBaM

… or how charging probably keeps batteries alive

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Alpine Verification Meeting 2015

What 3 items would you take to a deserted island?

What items up to 1 kg & 1 liter would you take?

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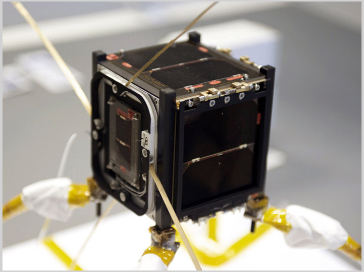
Fly Your Satellite! educational program

A Cube Satellite



Cube satellites for educational or scientific use

- **Limits:** 1 kg & 1 liter
- **Mission time:** up to 4 years

What do we have to squeeze in there?

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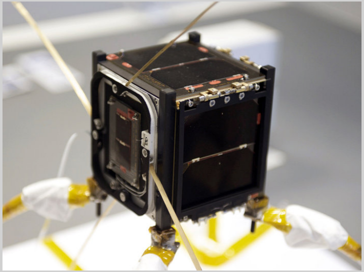

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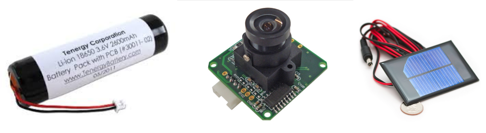
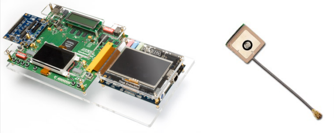
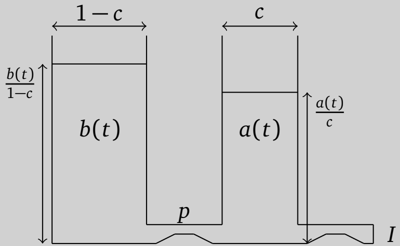




We will focus on the battery!

The kinetic battery model (KiBaM)

The two-wells illustration



Parameters

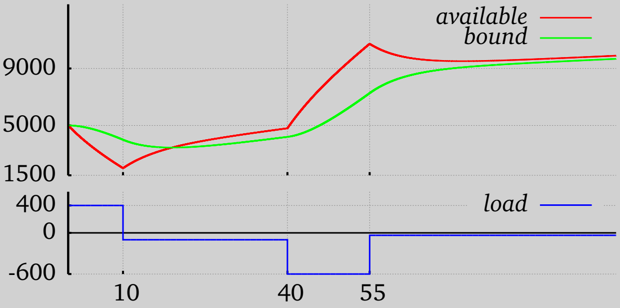
- c – Width of available charge tank
- p – Diffusion rate between tanks

KiBaM ODE System

$$\begin{aligned}\dot{a}(t) &= -I + p\left(\frac{b(t)}{1-c} - \frac{a(t)}{c}\right)\\ \dot{b}(t) &= p\left(\frac{a(t)}{c} - \frac{b(t)}{1-c}\right)\end{aligned}$$

KiBaM (ctd.)

Example (Unbounded KiBaM)



The model supports:

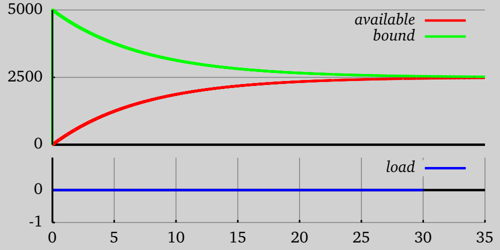
- **Discharging:**
  Load is positive ($I > 0$)
- **Charging:**
  Load is negative ($I > 0$)
- **Depletion:**
  Available charge reaches 0 ($a(t) \leq 0$)

Analysis hard if load is not piecewise constant...

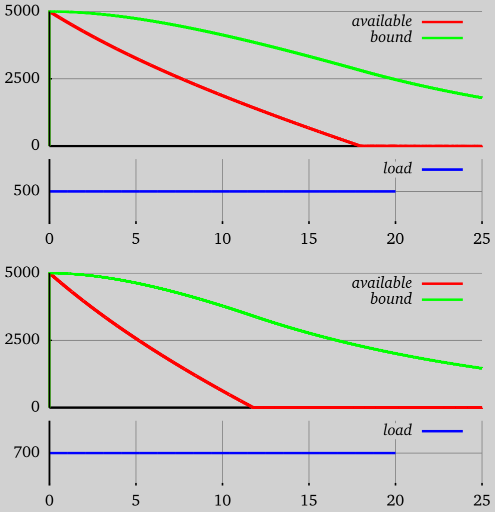
Why is the KiBaM a good model?

The KiBaM captures some realistic effects:

Recovery effect



Rate-capacity effect



Solution of KiBaM ODEs

Solution of ODE system

$$\mathsf{K}_{t,I}\begin{bmatrix} a_0 \\ b_0 \end{bmatrix} = \begin{bmatrix} \mathrm{q}_a(t) & \mathrm{r}_a(t) & \mathrm{s}_a(t) \\ \mathrm{q}_b(t) & \mathrm{r}_b(t) & \mathrm{s}_b(t) \end{bmatrix} \cdot \begin{bmatrix} a_0 \\ b_0 \\ I \end{bmatrix}$$

$\Rightarrow$ Linear in a_0, b_0 and I

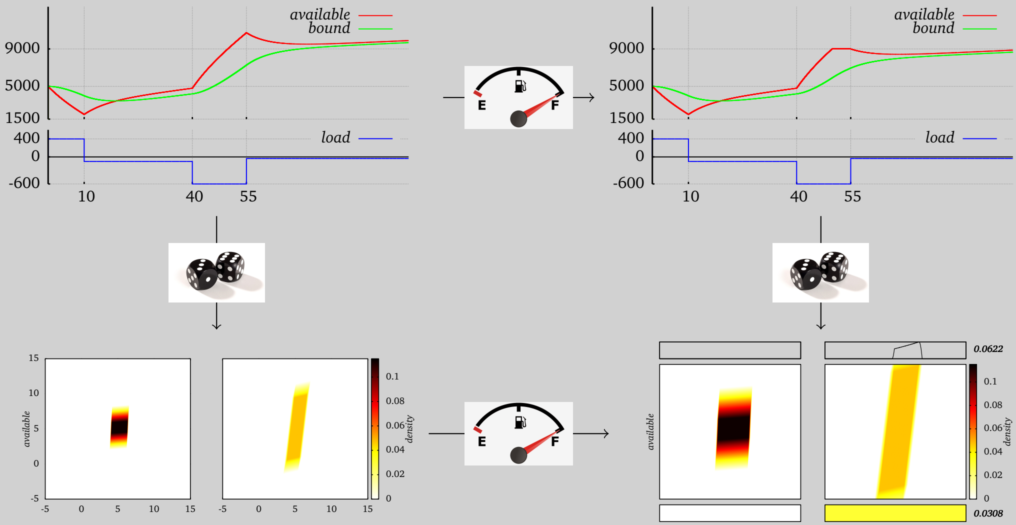
Coefficients

$$\begin{aligned} \mathrm{q}_a(t) &= (1-c)\mathrm{e}^{-kt} + c \\ \mathrm{q}_b(t) &= -(1-c)\mathrm{e}^{-kt} + (1-c) \\ \mathrm{r}_a(t) &= -c \cdot \mathrm{e}^{-kt} + c \\ \mathrm{r}_b(t) &= c \cdot \mathrm{e}^{-kt} + (1-c) \end{aligned}$$

$$\begin{aligned} \mathrm{s}_a(t) &= \frac{(1-c)(\mathrm{e}^{-kt}-1)}{k} - t \cdot c \\ \mathrm{s}_b(t) &= \frac{(1-c)(1-\mathrm{e}^{-kt})}{k} - t \cdot (1-c) \end{aligned}$$

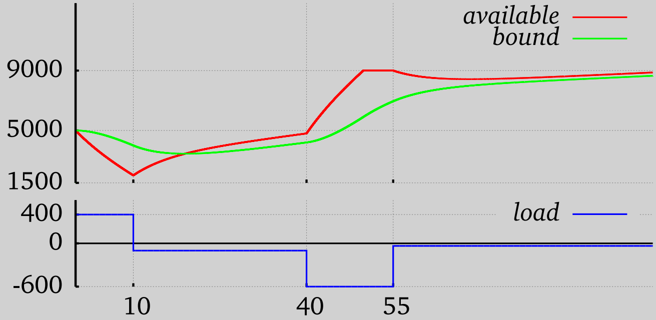
$\Rightarrow$ Not linear in t

What can be added to the KiBaM?



Capacity bounds

Example (Bounded KiBaM)



- Switching ODE systems

$$\dot{b}(t) = p\left(\frac{a_{\max}}{c} - \frac{b(t)}{1-c}\right)$$

(... *can be solved*)

- if current high enough

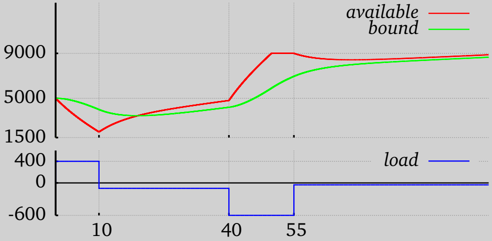
$$b_{\text{tresh}}(I) = b_{\max} + I \cdot \frac{1-c}{p}$$

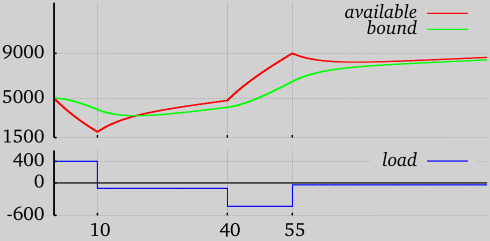
- But when?

$$t = -\mathbb{W}\left(\frac{u}{v} \cdot e^{-\frac{w}{v}}\right) - \frac{w}{v}$$

Capacity bounds

Example (Underapproximated charging current)



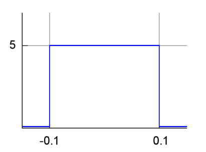


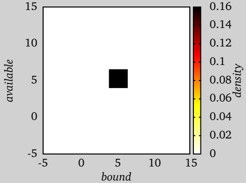
Underapproximate charging load such that capacity bound is hit when load changes

$$\bar{I}(a_0, b_0) = -\frac{q_a}{s_a} \cdot a_0 - \frac{r_a}{s_a} \cdot b_0 + \frac{a_{\max}}{s_a}.$$

Random SoC and load 

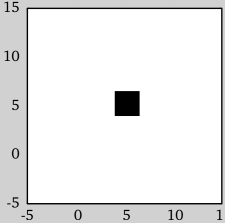
Example (Random initial SoC with random load)

Random load 

+ Random SoC 

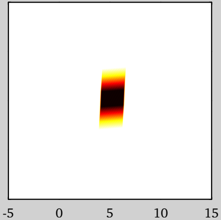
=

$t = 0$



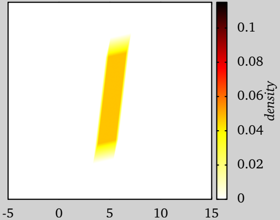
$\rightsquigarrow$

$t = 20$



$\rightsquigarrow$

$t = 60$



How do we handle  ?

Definition (Transformation Law of Random Variables)

For $f_{\mathbf{X}}$-distributed vector $\mathbf{X}$, injective and continuously differentiable function $g : \mathbb{R}^d \to \mathbb{R}^d$, express density of $\mathbf{Y} := g(\mathbf{X})$ as

$$f_{\mathbf{Y}}(y) = f_{\mathbf{X}}\left(g^{-1}(y)\right) \cdot \left|\mathbf{det}\left(J_{g^{-1}}(y)\right)\right|$$

- Transform density of SoC conditioned on $I = i$:

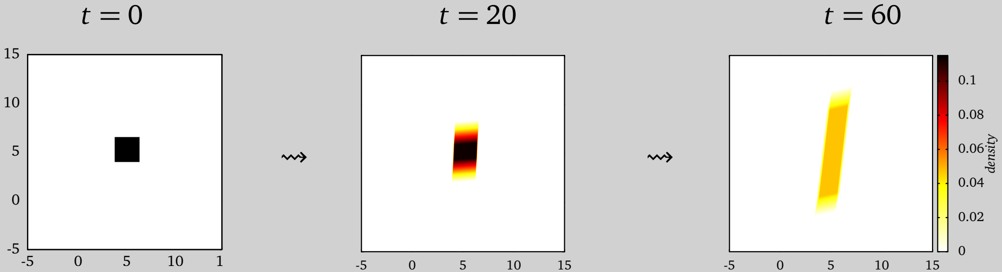
$$f_T(a, b \mid i) = f_0\left(\mathsf{K}_{T,i}^{-1}[a; b]\right) \cdot \left|\mathrm{e}^{kT}\right|$$

- Integrate information of the load afterwards

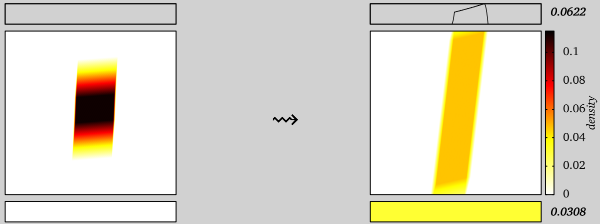
$$f_T(a, b) = \int_{-\infty}^{\infty} f_0\left(\mathsf{K}_{T,i}^{-1}[a; b]\right) \cdot \mathrm{e}^{kT} \cdot g(i)\, \mathrm{d}i.$$

Bounded random SoC and load  + 

Example (Evolution over time)

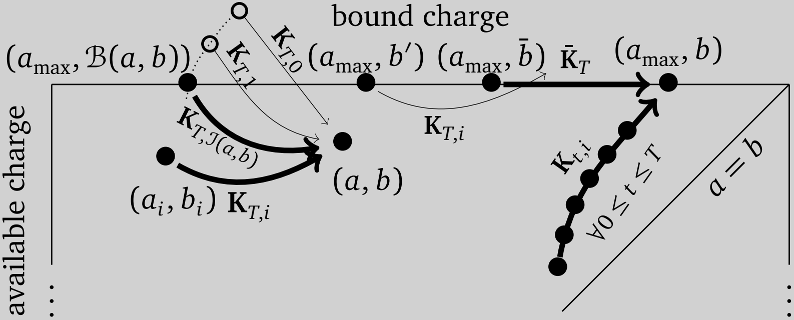


Imposing bounds 0 and 10 ⟹



What can happen at the capacity bound?

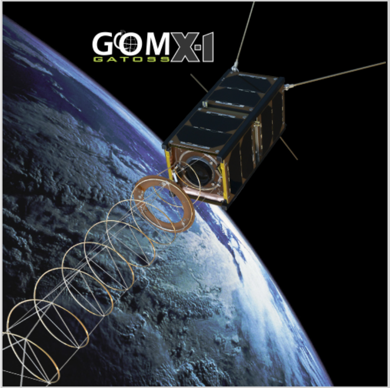
The upper bound scenario



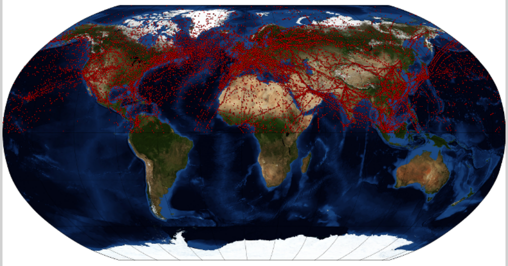
- Moving within the bounds
- Sliding along the bound
- Moving from the capacity bound back within the bounds.

$$\mathcal{J}(a,b) = (a_{\max} e^{-kt} - r_b a - q_b b) / (r_a s_b - r_b s_a),$$
$$\mathcal{B}(a,b) = -q_b a + q_a b + (q_b s_a - q_a s_b) \cdot \mathcal{J}(a,b).$$

GOMX–1 Cubesat



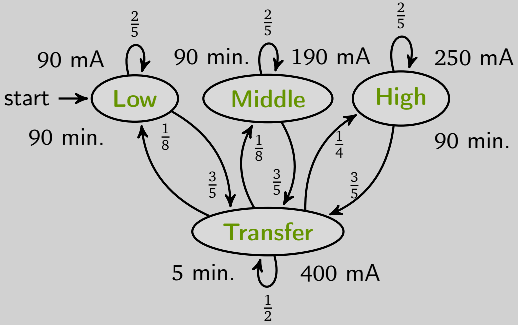
- 2-Unit Cube Satellite
- launched 21.11.2013
- tracking airplanes using their ADS-B signal



- Logging plenty of internal (battery) data

Satellite Model

Markov Task Process



- **Orbit Time:** 99 min. (1/3 in eclipse)
- **Communication:** when close to Aalborg, DK
- **Battery:** 5000 mAh, 7.2 V, Li-Ion
- **Solar charge:** 400 mA

Additional Randomness:

- SoC uniformly distributed between 70% and 90% full (battery in equilibrium)

Satellite Model

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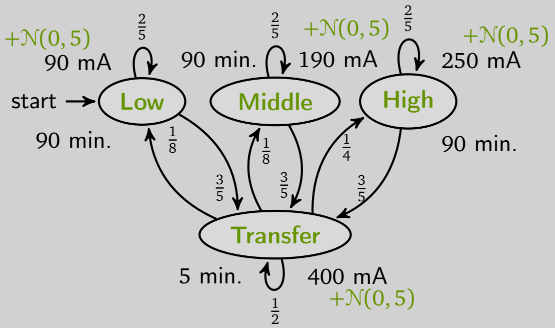

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Additional Randomness:

- SoC uniformly distributed between 70% and 90% full (battery in equilibrium)
- White noise in the workload model

Computation

- Iterative approach **stacks integrals**:

$$f_T(a,b) = \int_{-\infty}^{\infty} f_0\left(\mathsf{K}_{T,i}^{-1}[a;b]\right) \cdot \mathrm{e}^{kT} \cdot g(i)\,\mathrm{d}i.$$

- We **discretize** the battery SoC, we keep continuous time

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We tried other SHS tools

- **SiSat**
- **Faust^2**

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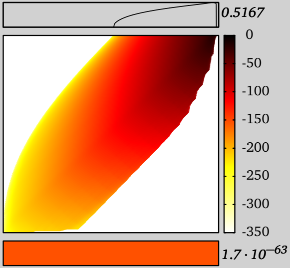
$\Longrightarrow$ Cannot handle the KiBaM system, cannot compare with our accuracy

Results – SoC Distribution after 1 year

SoC distribution for decreasing battery size

5000 mAh:



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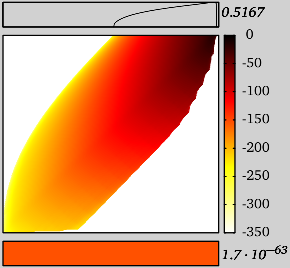
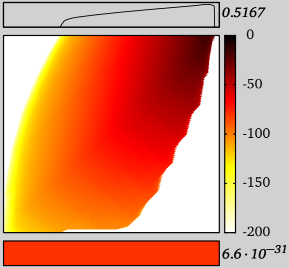


2500 mAh:



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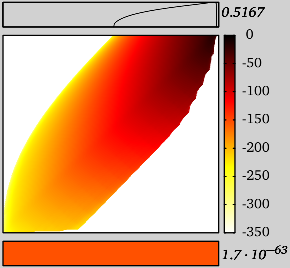

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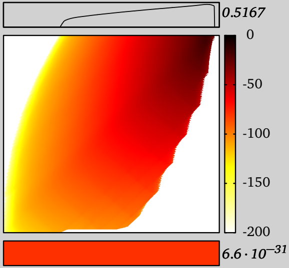
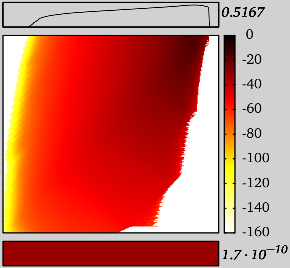


1250 mAh:



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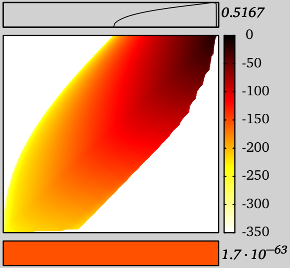

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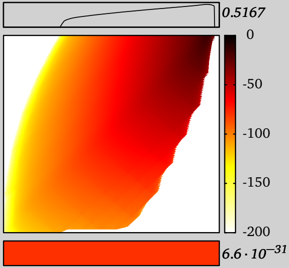

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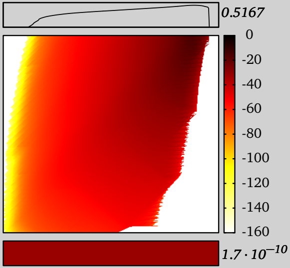
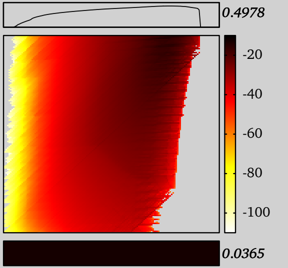


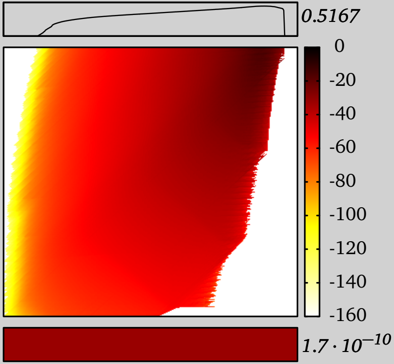
625 mAh:



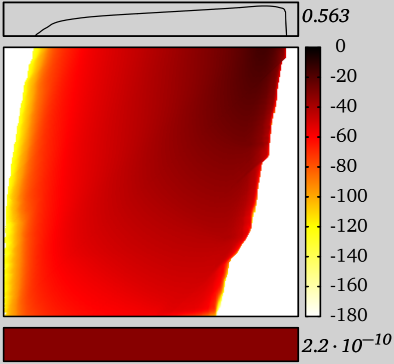
Results – SoC Distribution after 1 year

Effect of noisy loads (1250 mAh battery)

- without noise:



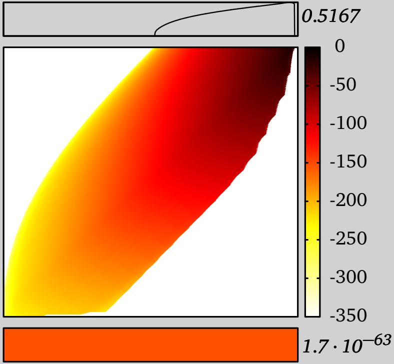
- with noise:



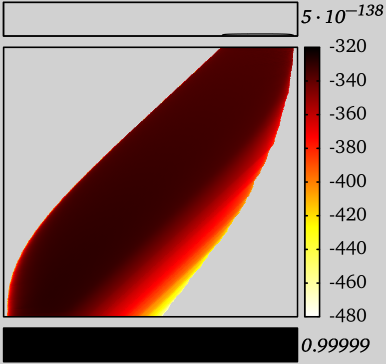
Results – SoC Distribution after 1 year

Could a 1 unit satellite survive with a 5000 mAh battery?

- 9 solar panels:

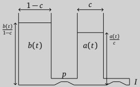


- 6 solar panels:

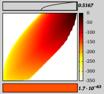


The last slide!

Summary

- We extended 

with  and 

- We get 

for  models.

The last slide!

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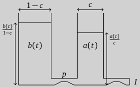
- We extended 

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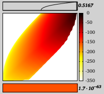
- We get 

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Future Work

- Battery wear
- Randomized capacity bounds
- Temperature dependency
- Energy optimal scheduling (GOMX–3!)

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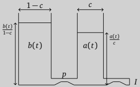
- We extended 

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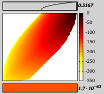
- We get 

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Questions?