Deterministic ω-Automata for LTL: A safraless, compositional, and mechanically verified construction

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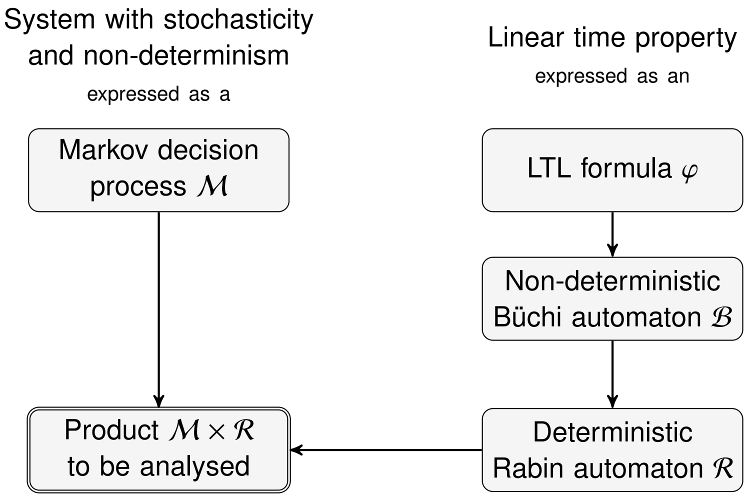
System with stochasticity and non-determinism
expressed as a
Markov decision process $\mathcal{M}$
Linear time property
expressed as an
LTL formula φ
Non-deterministic Büchi automaton $\mathcal{B}$
Deterministic Rabin automaton $\mathcal{R}$
Product $\mathcal{M} \times \mathcal{R}$ to be analysed

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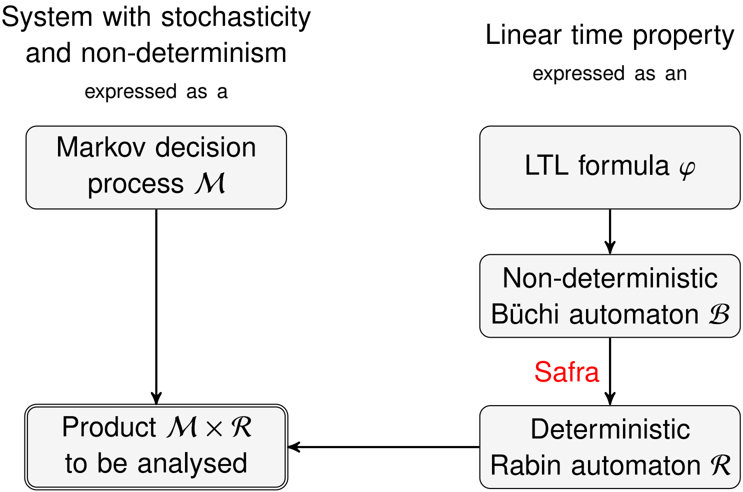

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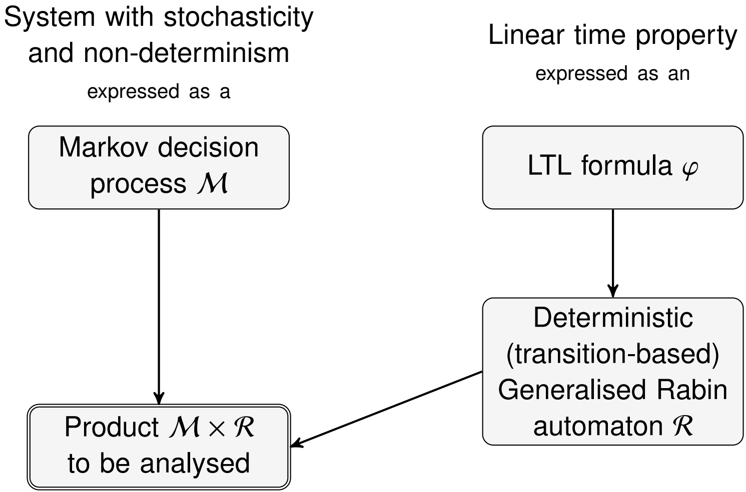

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Experimental Data

$\bigwedge_{i\in\{1,\ldots,n\}} \mathbf{GF}a_i \Rightarrow \mathbf{GF}b_i$	NBA LTL2BA	DRA ltl2dstar	DTGRA Rabinizer 3
$n = 1$	4		
$n = 2$	14		
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ω-Words and LTL

An ω-word is an infinite sequence: $w = a_0 a_1 a_2 a_3 \ldots$.

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Definition (LTL Semantics, Negation-Normal-Form)

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\square & \models & \square & :: & \alpha\ \mathit{set\ word} \to \alpha\ \mathit{ltl} \to \mathbb{B} \\
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Unfolding Modal Operators

$$\begin{aligned} \mathbf{F}\varphi &\equiv \mathbf{X}\mathbf{F}\varphi \vee \varphi \\ \mathbf{G}\varphi &\equiv \mathbf{X}\mathbf{G}\varphi \wedge \varphi \\ \psi\mathbf{U}\varphi &\equiv \varphi \vee (\psi \wedge \mathbf{X}(\psi\mathbf{U}\varphi)) \end{aligned}$$

Co-Büchi Automata for **G**-free φ

$$\varphi = a \vee (b \mathbf{U} c)$$

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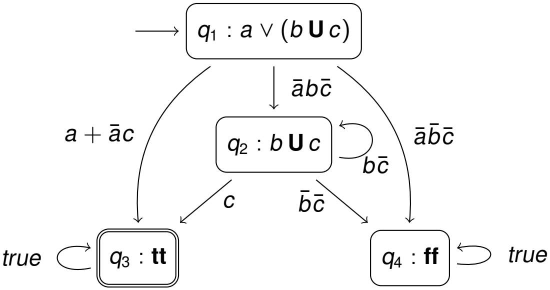
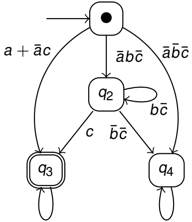


Tackling the **G**-Operator

- Relaxed case: $\mathbf{FG}\varphi$
  - $w \models \mathbf{FG}\varphi$ *iff* $w_{i\infty} \models \varphi$ for almost all i
- Reason: **G**-subformulae may be nested inside $\mathbf{X}, \mathbf{F}, \mathbf{U}$.

Automata for **FG**φ where φ is **G**-free

$$w = \ \ldots$$



Automata for $\mathbf{FG}\varphi$ where φ is $\mathbf{G}$-free

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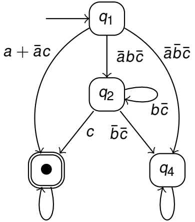

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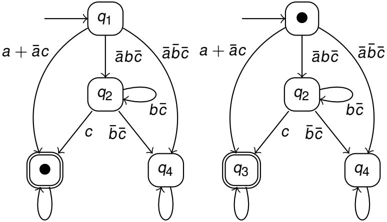

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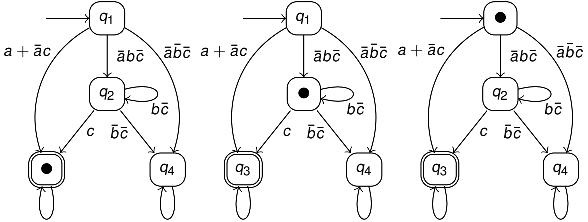

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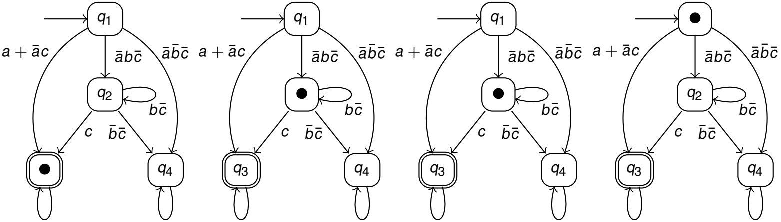

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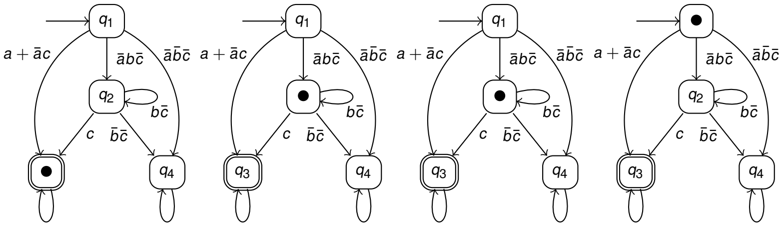
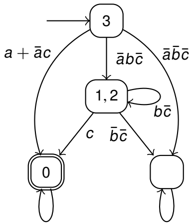
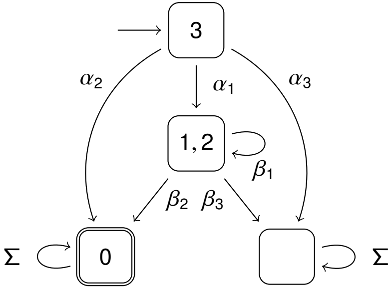
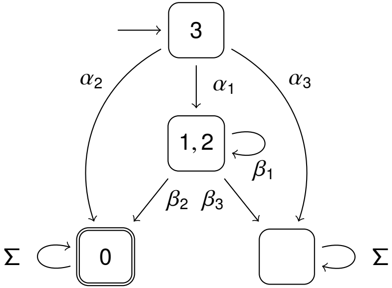




Mojmir Automata





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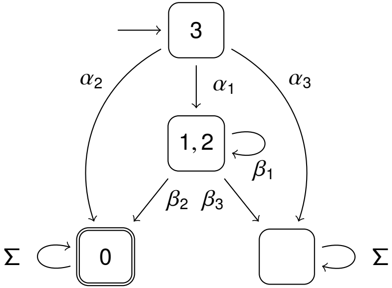

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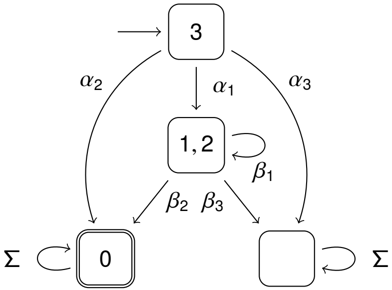

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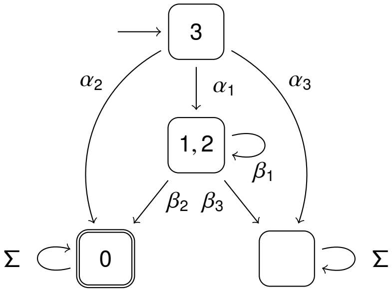

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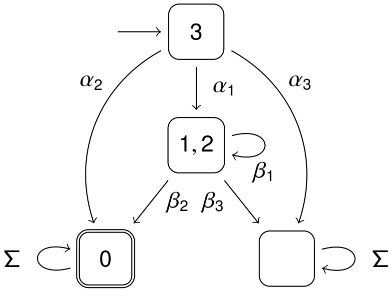

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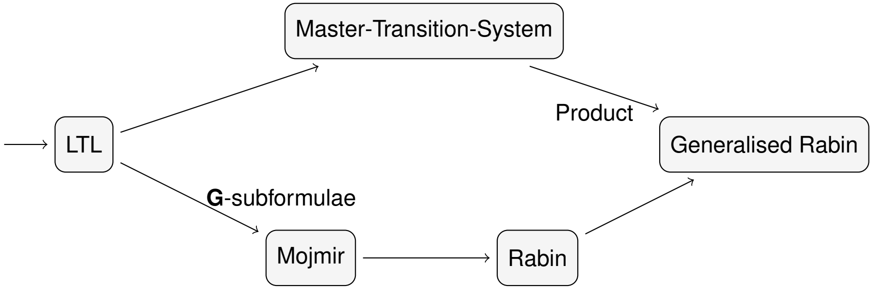
- Mojmir Automata for $\mathbf{FG}\varphi$ for arbitrary φ
  - Divide-and-conquer approach
  - Construct for every **G**-subformula a separate automaton
  - Instead of expanding **G**'s rely on the other automata
  - Intersection and Union of several Mojmir Automata

Overview of the Construction



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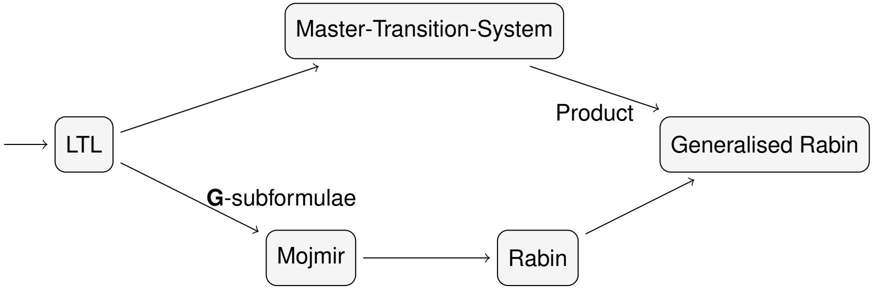

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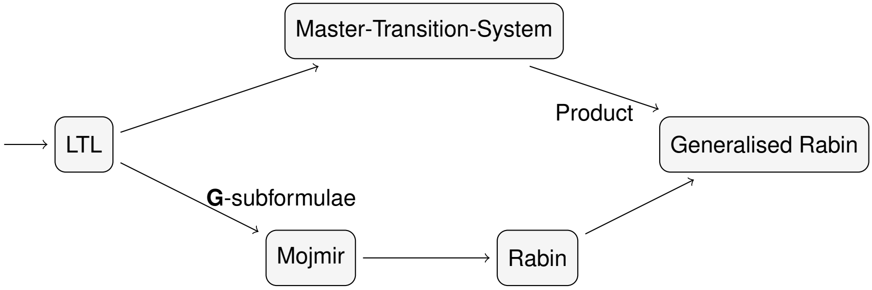

- The Master-Transition-System tracks a finite prefix of the ω-word.
- Acceptance:
  1. Guess the set of eventually true **G**-subformulae
  2. Verify this guess using the Mojmir automata
  3. Accept *iff* almost all the time this guess entails the current state of the master-transition-system

Conclusion and Future Work

The presented translation . . .

- preservers the logical structure of the formula
- is compositional
  - Aggressive optimization can lead to huge space savings
  - Some optimizations are already verified
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Open Problems:

- Explore and formalize further optimizations
- Adapt construction to support:
  - Alternation-free linear-time μ-calculus (contains LTL)
  - Parity automata

Getting More Information

- Javier Esparza, Jan Kretínský: From LTL to Deterministic Automata: A Safraless Compositional Approach. CAV 2014: pages 192–208

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