A Short History of SAT Based Model Checking: From Bounded Model Checking to Interpolation

Armin Biere

Institute for Formal Models and Verification
Johannes Kepler University
Linz, Austria

Brno University of Technology
Seminar Faculty of Information Technology

Brno, Czech Republic

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BurchClarkeMcMillanDillHwang’90: Symbolic Model Checking

DavisPutnam’60: DP

CoudertMadre’89: Symbolic Reachability

McMillan’03: Interpolation

DavisLogemannLoveland’62: DPLL

Marques−SilvaSakallah’96: GRASP

Bryant’86: BDDs

BiereArthoSchuppan’01: Liveness2Safety

Pnueli’77: Temporal Logic

MoskewiczMadiganZhaoZhangMalik’01: CHAFF

McMillan’93: SMV

EenSorensson’03: MiniSAT

BiereCimattiClarkeZhu’99: Bounded Model Checking

ClarkeEmerson’82: Model Checking

EenBiere’05: SatELite

Kurshan’93: Localization

SheeranSinghStalmarck’00: k−Induction

QuielleSifakis’82: Model Checking

BallRajamani’01: SLAM

Holzmann’91: SPIN

GrafSaidi’97: Predicate Abstraction

Holzmann’81: On−The−Fly Reachability

ClarkeGrumbergJahLuVeith’03: CEGAR

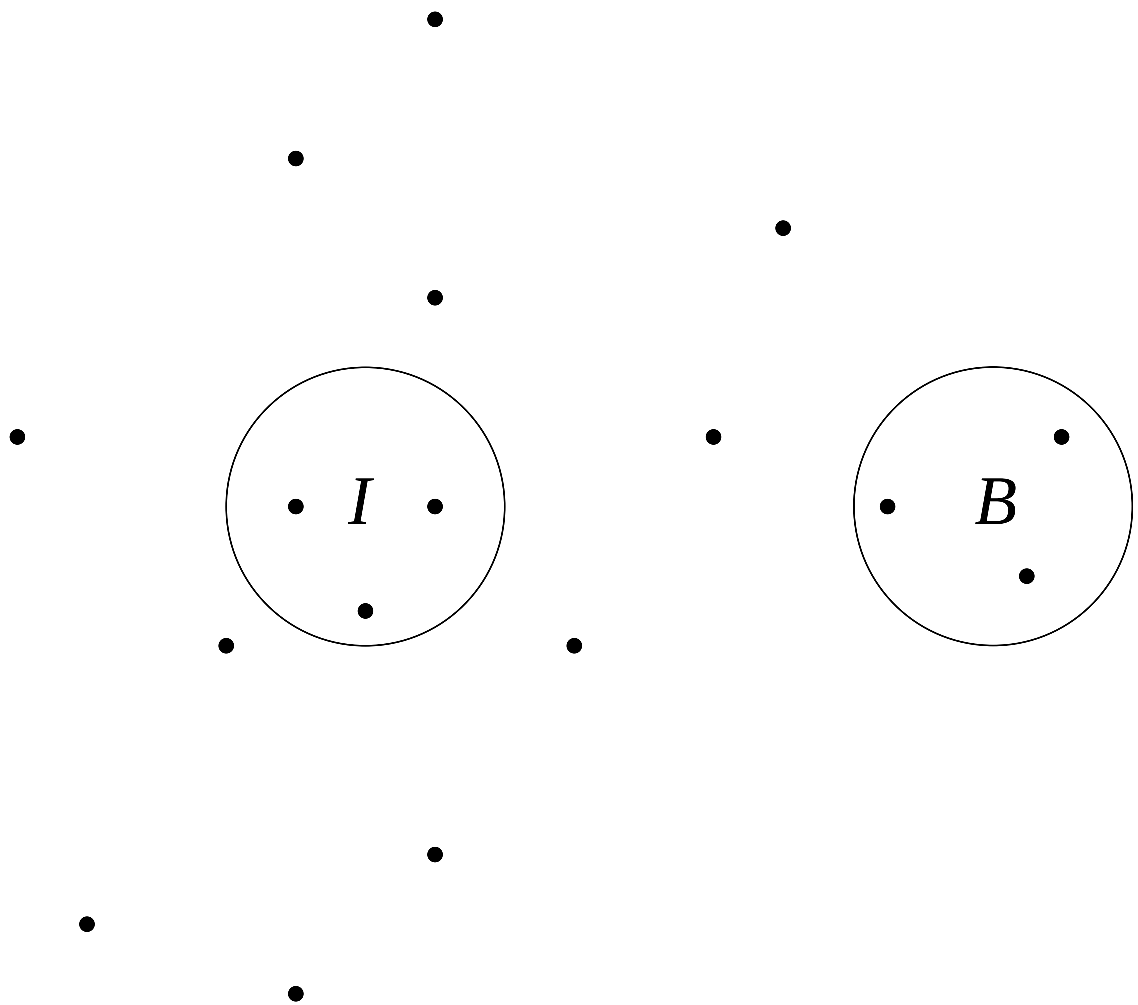
Peled’94: Partial−Order−Reduction

- set of states S, initial states I, transition relation T

- bad states B reachable from I via T?

- symbolic representation of T (ciruit, program, parallel product)
  - avoid explicit matrix representations, because of the
  - state space explosion problem, e.g. n-bit counter: $|T| = O(n)$, $|S| = O(2^n)$
  - makes reachability PSPACE complete [Savitch'70]

- on-the-fly [Holzmann'81'] for protocols
  - restrict search to reachable states
  - simulate and hash reached concrete states


I
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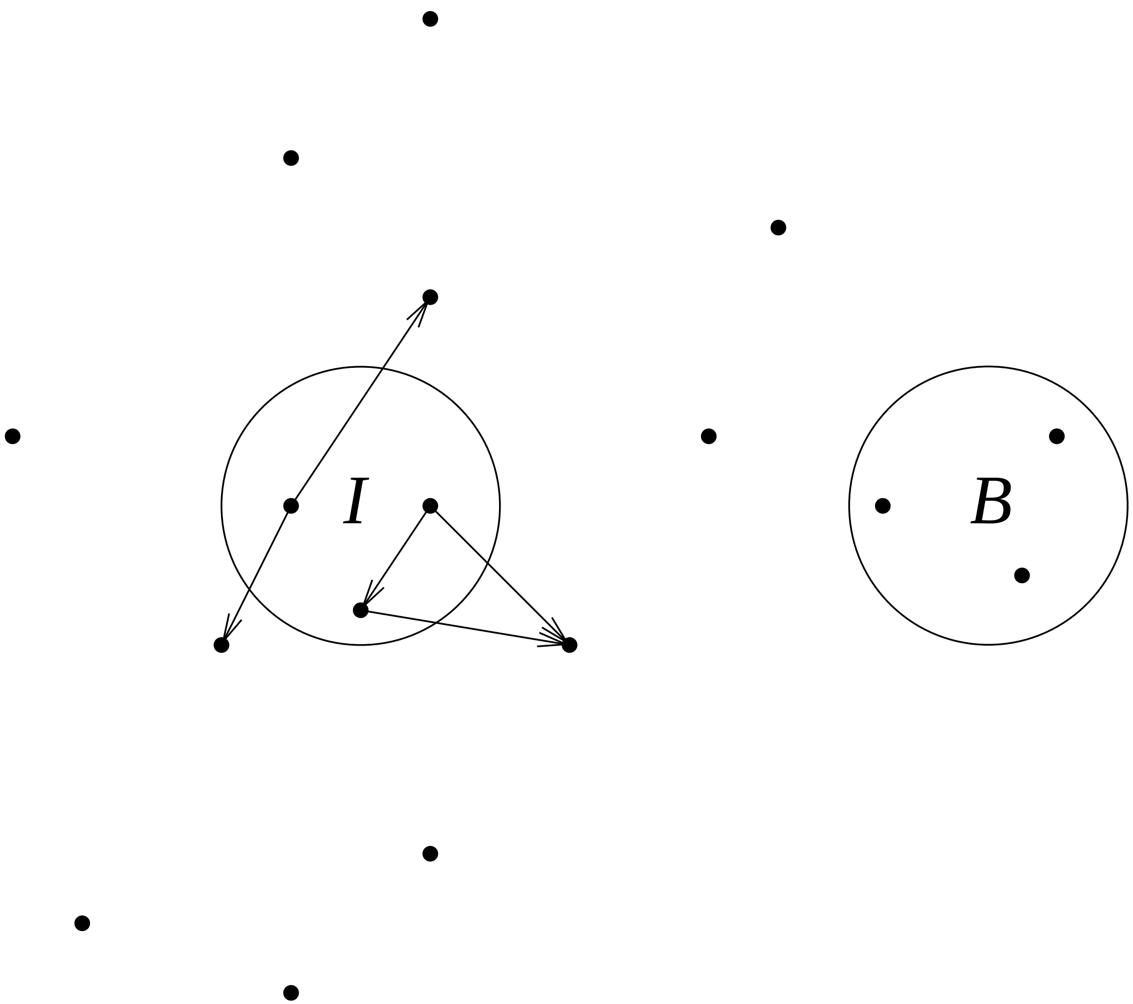
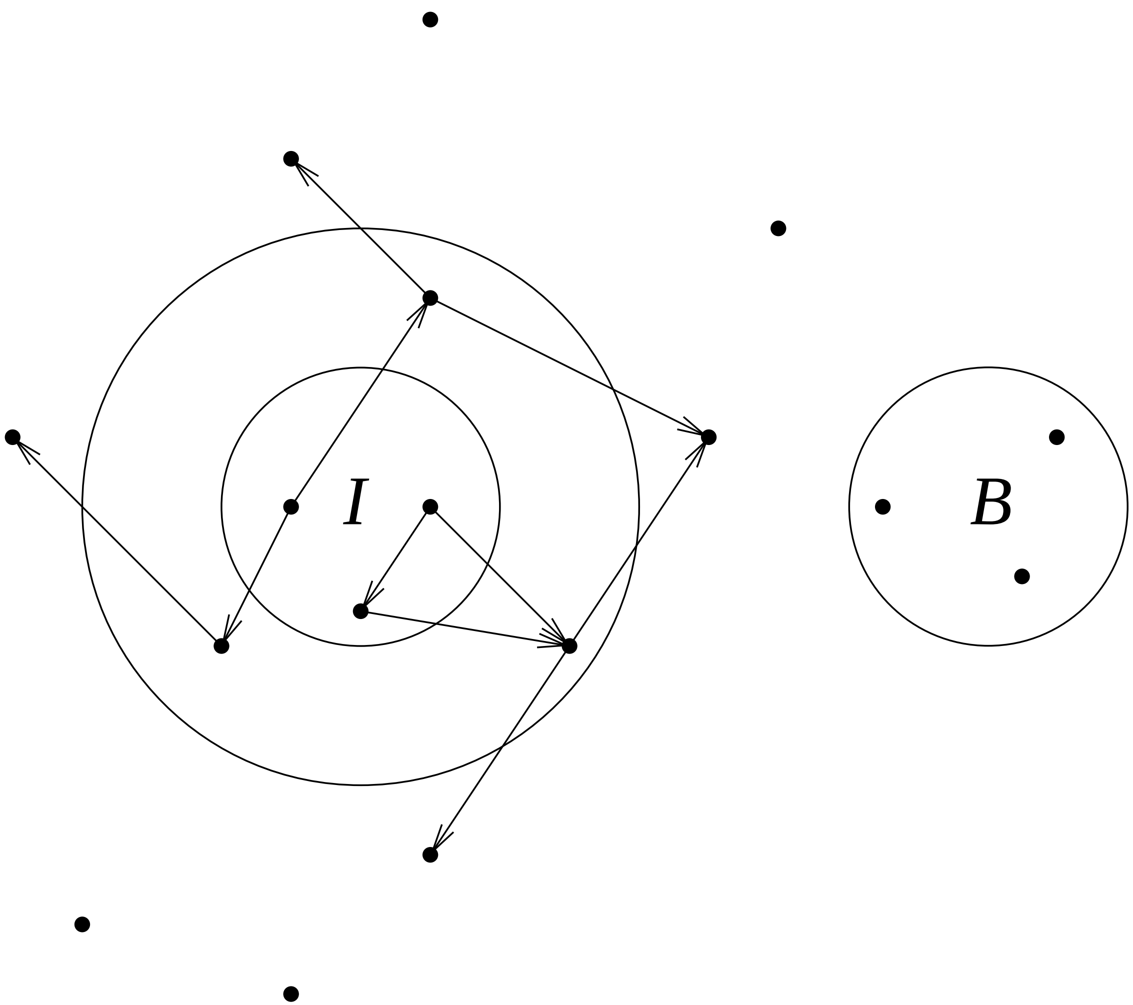
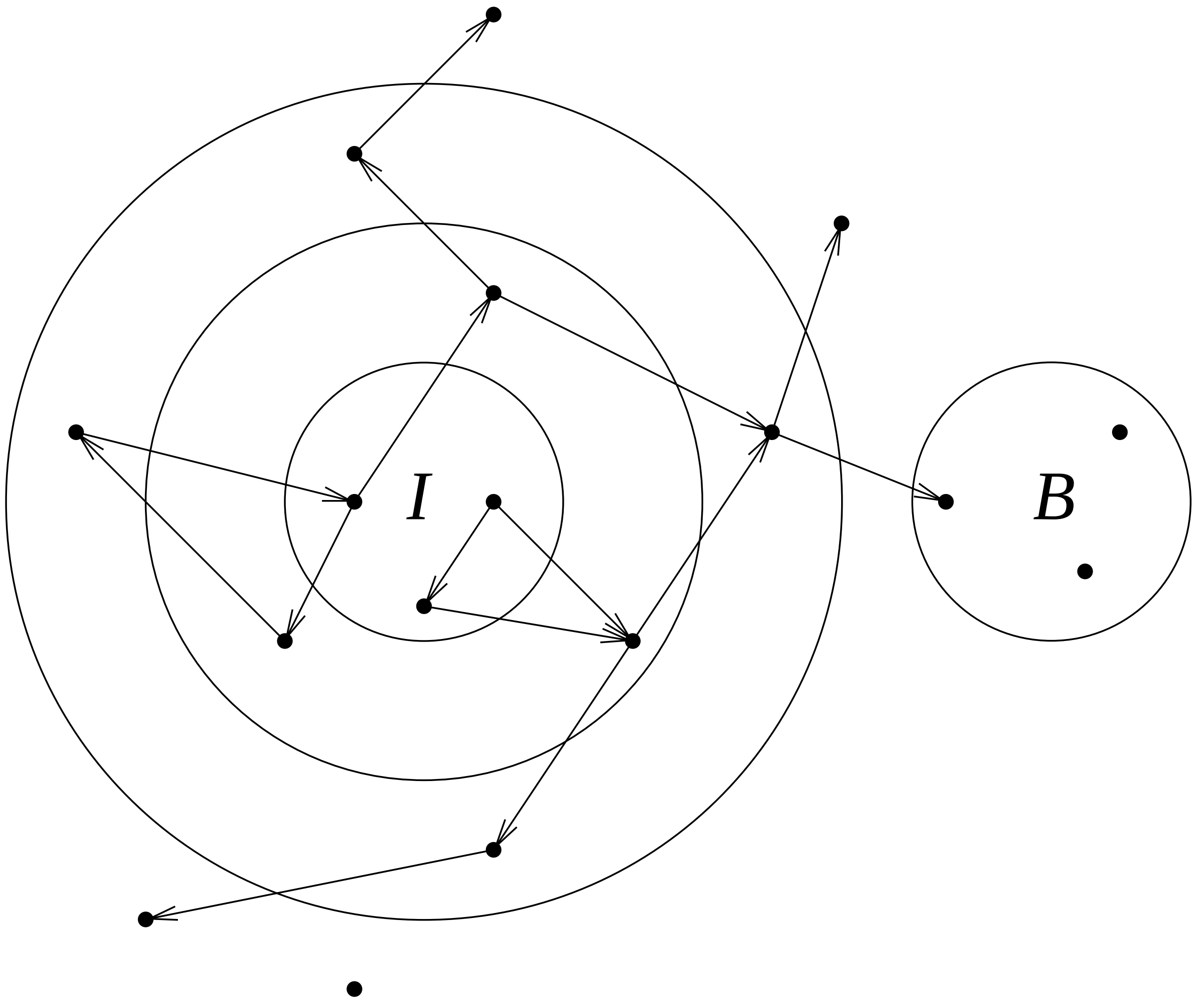
Forward Fixpoint Algorithm: Step 1



Forward Fixpoint Algorithm: Step 2

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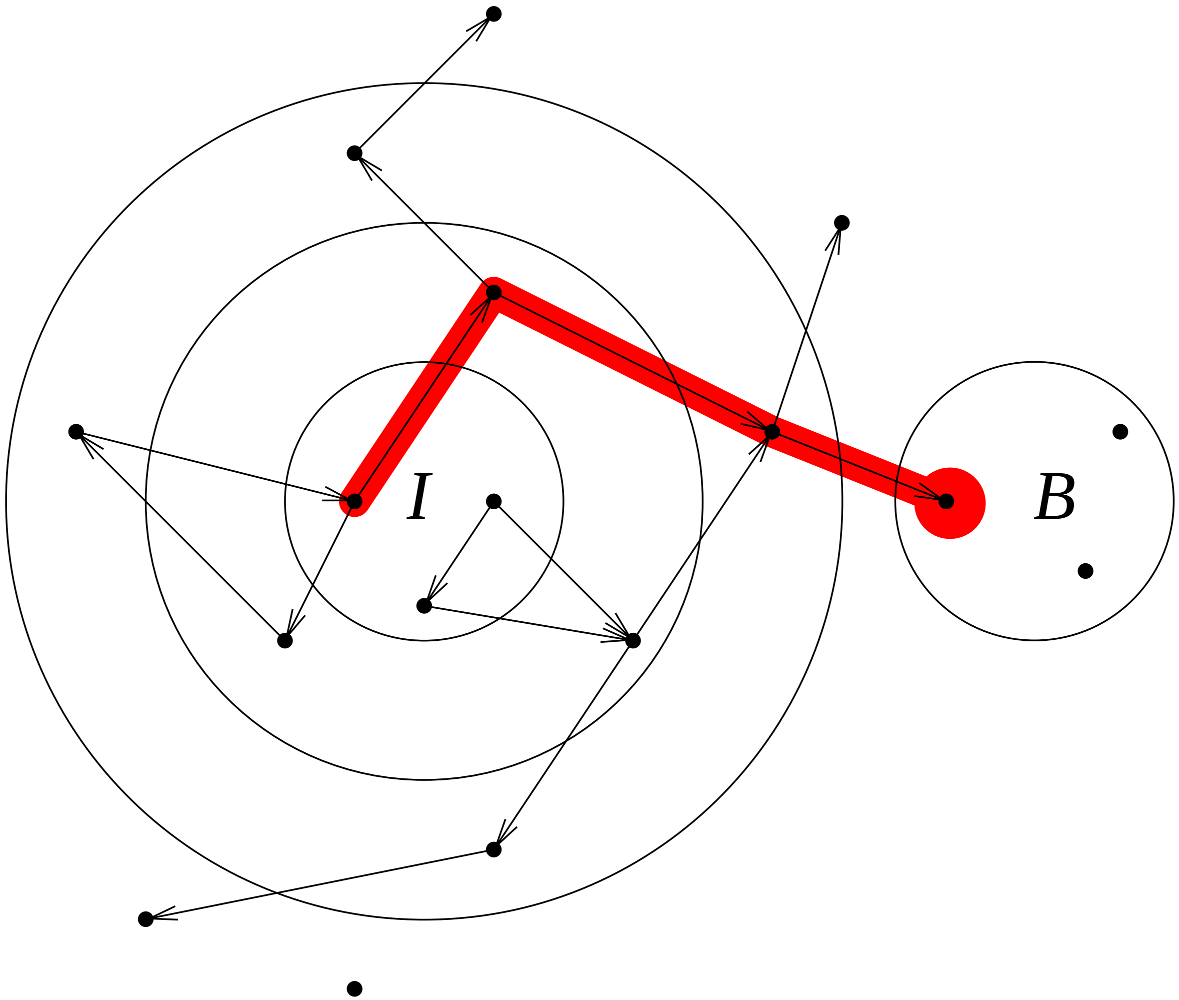
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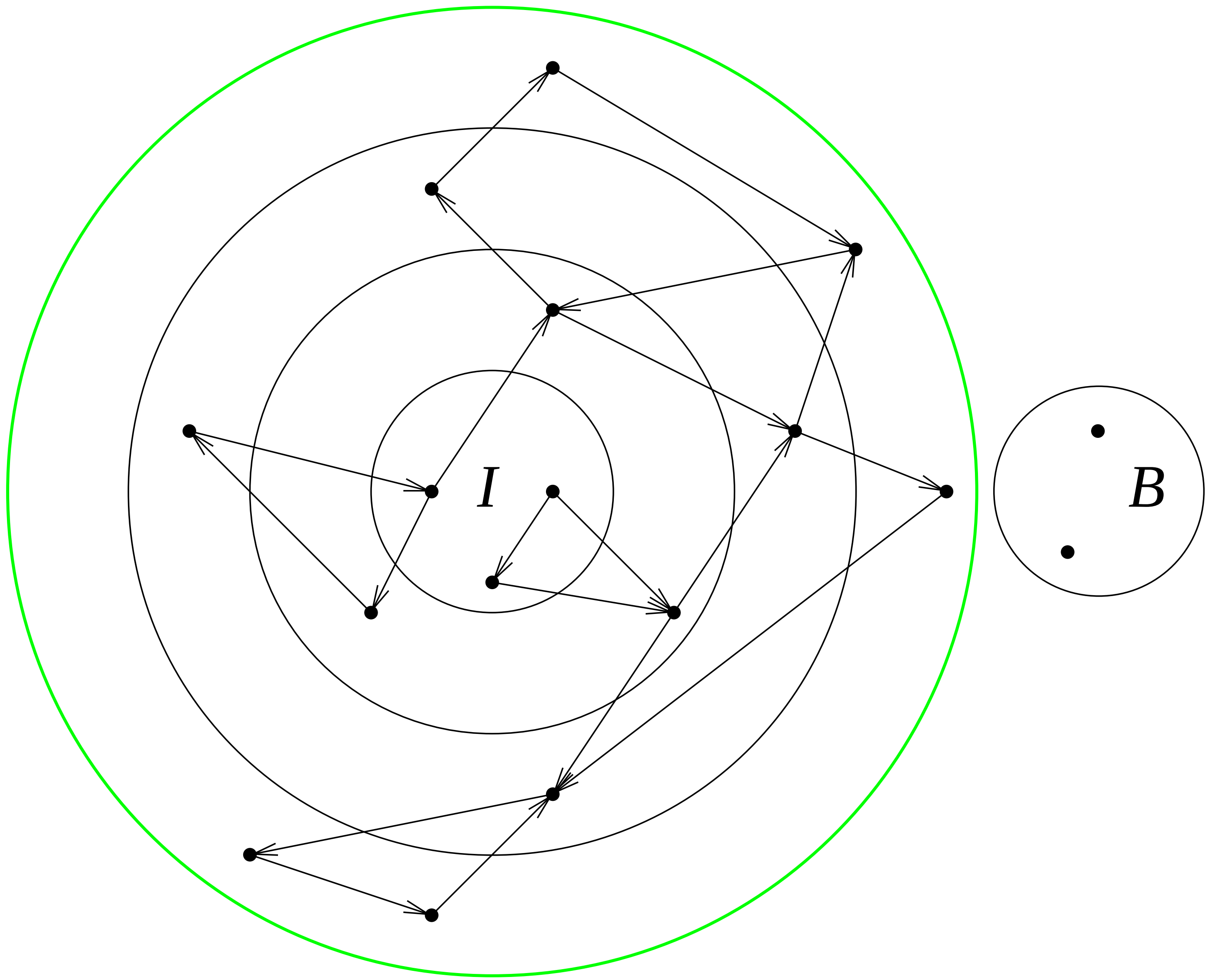
Forward Fixpoint Algorithm: Termination, No Bad State Reachable



initial states I, transition relation T, bad states B

<u>model-check</u>$^{\mu}_{\text{forward}}$ (I, T, B)

$S_C = \emptyset$; $S_N = I$;

while $S_C \neq S_N$ **do**

  if $B \cap S_N \neq \emptyset$ **then**

    return "found error trace to bad states";

  $S_C = S_N$;

  $S_N = S_C \cup Img(S_C)$;

done;

return "no bad state reachable";

- algorithms to check more general properties [ClarkeEmerson'82], [QuielleSifakis'82]
  - uses temporal logic [Pnueli'77] as property specification language
  - model checkers are usually fully automatic

    linear vs. branching time formalisms (CTL vs LTL) was hotly debated
  - either determine that property holds or ...
  - ... provide counter example for debugging purposes

- originally explicit (as in SPIN [Holzmann'91])
  - search works with concrete states,
  - bottle neck: number of states, that have to be stored
  - local (on-the-fly) and global algorithms (not on-the-fly)

- work with symbolic representations of states
  - symbolic representations are potentially exponentially more succinct
  - favors BFS: next frontier set of states in BFS is calculated symbolically
- originally “symbolic” meant model checking with BDDs [CoudertMadre’89/’90,BurchClarkeMcMillanDillHwang’90,McMillan’93]
- Binary Decision Diagrams [Bryant’86]
  - canonical representation for boolean functions
  - BDDs have fast operations (but image computation is expensive)
  - often blow up in space
  - restricted to hundreds of variables

Unrolling of Forward Least Fixpoint Algorithm

0: continue?	$S_C^0 \neq S_N^0$	$\exists s_0[I(s_0)]$
0: terminate?	$S_C^0 = S_N^0$	$\forall s_0[\neg I(s_0)]$
0: bad state?	$B \cap S_N^0 \neq \emptyset$	$\exists s_0[I(s_0) \wedge B(s_0)]$
1: continue?	$S_C^1 \neq S_N^1$	$\exists s_0, s_1[I(s_0) \wedge T(s_0, s_1) \wedge \neg I(s_1)]$
1: terminate?	$S_C^1 = S_N^1$	$\forall s_0, s_1[I(s_0) \wedge T(s_0, s_1) \rightarrow I(s_1)]$
1: bad state?	$B \cap S_N^1 \neq \emptyset$	$\exists s_0, s_1[I(s_0) \wedge T(s_0, s_1) \wedge B(s_1)]$
2: continue?	$S_C^2 \neq S_N^2$	$\exists s_0, s_1, s_2[I(s_0) \wedge T(s_0, s_1) \wedge T(s_1, s_2) \wedge \neg(I(s_2) \vee \exists t_0[I(t_0) \wedge T(t_0, s_2)])]$
2: terminate?	$S_C^2 = S_N^2$	$\forall s_0, s_1, s_2[I(s_0) \wedge T(s_0, s_1) \wedge T(s_1, s_2) \rightarrow I(s_2) \vee \exists t_0[I(t_0) \wedge T(t_0, s_2)]]$
2: bad state?	$B \cap S_N^1 \neq \emptyset$	$\exists s_0, s_1, s_2[I(s_0) \wedge T(s_0, s_1) \wedge T(s_1, s_2) \wedge B(s_2)]$

~~0: continue?~~ $S_C^0 \neq S_N^0$ $\exists s_0[I(s_0)]$

~~0: terminate?~~ $S_C^0 = S_N^0$ $\forall s_0[\neg I(s_0)]$

0: bad state? $B \cap S_N^0 \neq \emptyset$ $\boxed{\exists s_0[I(s_0) \wedge B(s_0)]}$

~~1: continue?~~ $S_C^1 \neq S_N^1$ $\exists s_0, s_1[I(s_0) \wedge T(s_0, s_1) \wedge \neg I(s_1)]$

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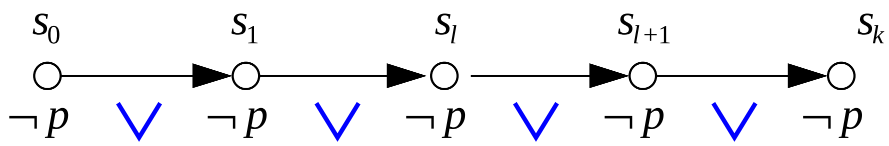
2: bad state? $B \cap S_N^1 \neq \emptyset$ $\boxed{\exists s_0, s_1, s_2[I(s_0) \wedge T(s_0, s_1) \wedge T(s_1, s_2) \wedge B(s_2)]}$

[BiereCimattiClarkeZhu'99]

- look only for counter example made of k states (the bound)



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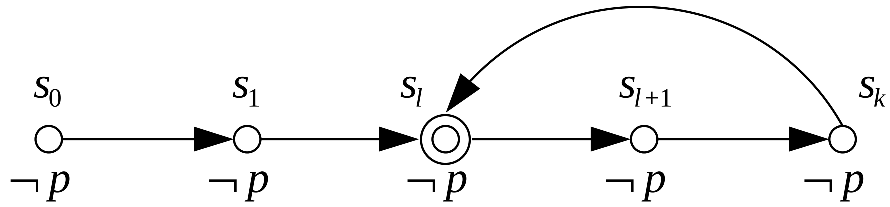


- simple for safety properties p is invariantly true (e.g. $p = \neg B$)

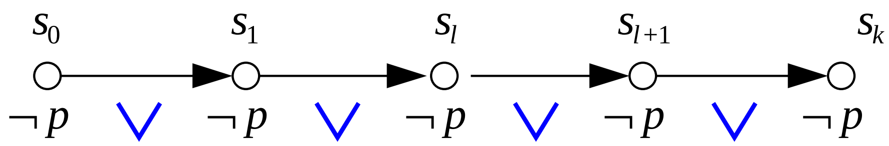
$$I(s_0) \wedge T(s_0, s_1)) \wedge \cdots \wedge T(s_{k-1}, s_k) \wedge \bigvee_{i=0}^{k} \neg p(s_i)$$

- harder for liveness properties p is eventually true

$$I(s_0) \wedge T(s_0, s_1)) \wedge \cdots \wedge T(s_{k-1}, s_k) \wedge \bigwedge_{i=0}^{k} \neg p(s_i) \wedge \exists l\; T(s_k, s_l)$$

[BiereCimattiClarkeZhu'99]

- look only for counter example made of k states (the bound)



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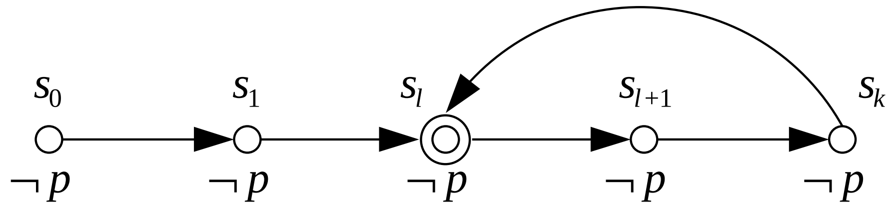

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- satisfiability checking (SAT)
  - of propositional/combinational problems (only boolean variables)
  - actually restricted to conjunctive normal form (CNF)
  - classical NP hard problem [Cook'71]
- key motivation of BMC
  - leverage capacity of SAT solvers
  - SAT solvers could handle 10000 variables in late 90'ties
  - compared to hundreds of variables with BDDs
- key insight: trade capacity for completeness

- increase in efficiency of SAT solvers [ZChaff,MiniSAT,SatELite]
- SAT more robust than BDDs in bug finding

  (shallow bugs are easily reached by explicit model checking or testing)
- better unbounded but still SAT based model checking algorithms
  - k-induction [SinghSheeranStalmarck'00]
  - interpolation [McMillan'03]
- 4th Intl. Workshop on Bounded Model Checking (BMC'06)
- other logics beside LTL, better encodings, e.g. [LatvalaBiereHeljankoJuntilla'04]
- other system models, such as hybrid automata

[SinghSheeranStalmarck'00]

- more specifically k-induction
  - does there exist k such that the following formula is *unsatisfiable*

$$\overline{B(s_0)} \wedge \cdots \wedge \overline{B(s_{k-1})} \wedge T(s_0, s_1) \wedge \cdots \wedge T(s_{k-1}, s_k) \wedge B(s_k) \wedge \bigwedge_{0 \leq i < j \leq k} s_i \neq s_j$$

  - if *unsatisfiable* and $\neg$BMC(k) then bad state unreachable

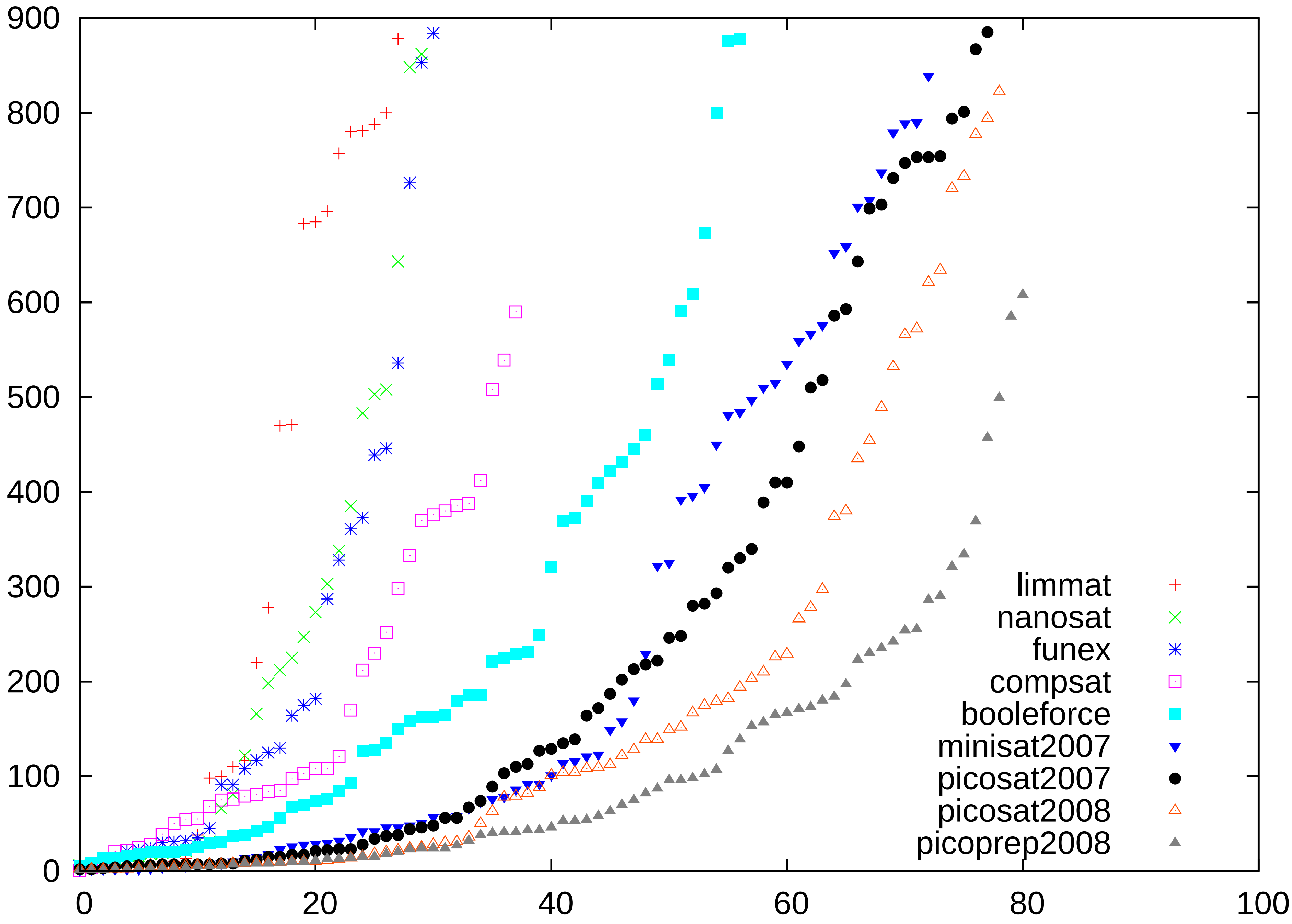
- bound on k: length of longest cycle free path

- $k = 0$ check whether $\neg B$ tautological (propositionally)

- $k = 1$ check whether $\neg B$ inductive for T

- Davis and Putnam procedure
  - DP: elimination procedure [DavisPutnam'60]
  - DPLL: splitting [DavisLogemannLoveland'62]
- modern SAT solvers are mostly based on DPLL
  - learning: GRASP [MarquesSilvaSakallah'96], RelSAT [BayardoSchrag'97]
  - watched literals, VSIDS: CHAFF [MoskewiczMadiganZhaoZhangMalik'01]
  - improved heuristics: MiniSAT [EenSorensson'03] actually Version from 2005
- preprocessing is a hot topic:
  - currently fastest solvers use SatELite style preprocessing [EenBiere'05] DP
- www.satcompetition.org since 2002

Cactus Plot for SAT'06 Race Instances

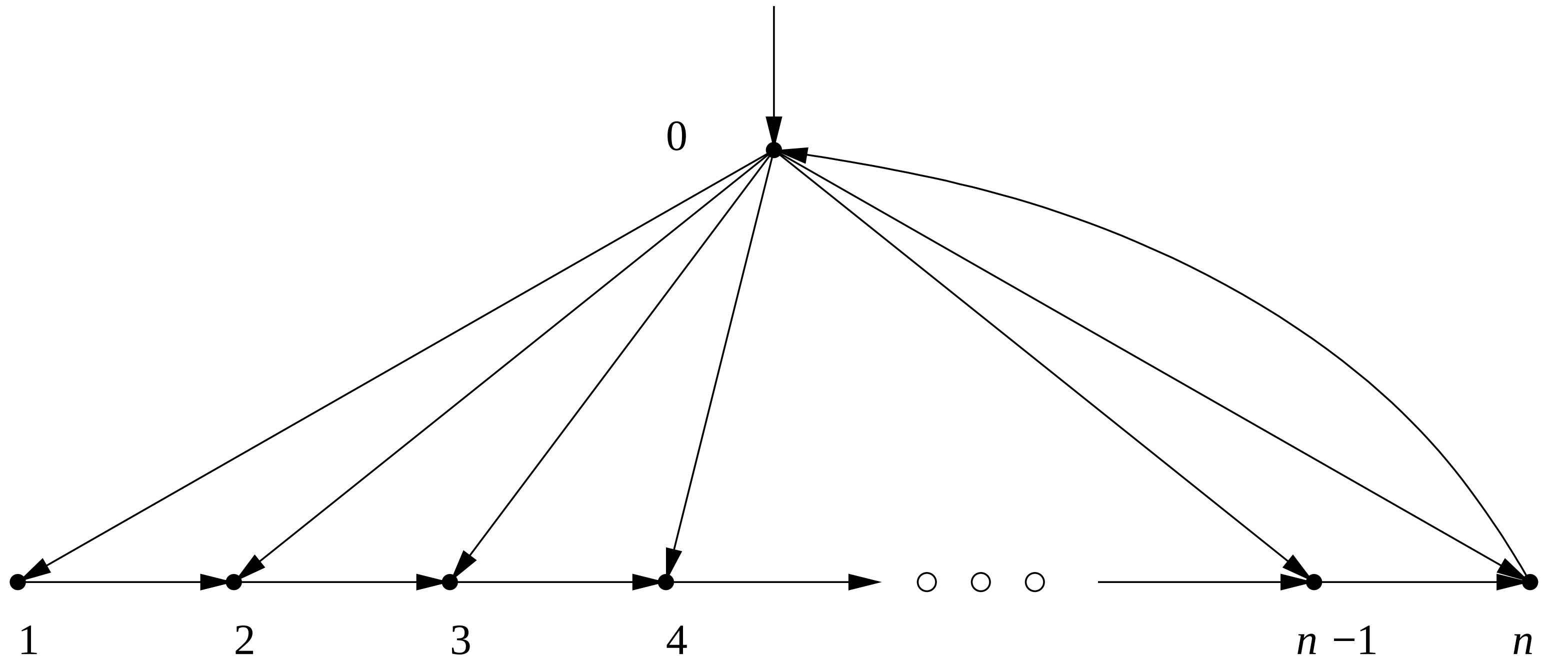


[McMillan'03]

- SAT based technique to overapproximate frontiers $Img(S_C)$
  - currently most effective technique to show that bad states are unreachable
  - better than BDDs and k-induction in most cases [HWMCC'07]
- starts from a resolution proof refutation of a BMC problem with bound $k+1$

$$S_C(s_0) \wedge T(s_0, s_1) \wedge T(s_1, s_2) \wedge \cdots \wedge T(s_k, s_{k+1}) \wedge B(s_{k+1})$$

  - result is a characteristic function $f(s_1)$ over variables of the second state s_1
  - these states do not reach the bad state s_{k+1} in k steps
  - any state reachable from S_C satisfies f: $S_C(s_0) \wedge T(s_0, s_1) \Rightarrow f(s_1)$
- k is bounded by the diameter (exponentially smaller than longest cycle free path)



length of longest shortest path $O(n)$

diameter $O(1)$

- further convergence between theorem proving and model checking
  - as pioneered by SLAM [BallRajamani'01] using
    * predicate abstraction [GrafSaidi'97] and
    * counter example guided abstraction refinement [ClarkeGrumbergJahLuVeith'03]
  - handle large software and hardware systems precisely
  - automate compositional reasoning, e.g. alias analysis
- improve Satisfiability Modulo Theory (SMT) procedures
  - What is the right way to handle bit-vectors, arrays?
  - Quantifiers, interpolation for bit-vectors and arrays?

- Satisfiability Solver (SAT) (standard NP hard problem)
  - improve heuristics, remove magic constants
  - more aggresive incremental preprocessing
  - effective incorporation of more powerful reasoning engines
- Quantified Boolean Formulas (QBF) (standard PSPACE hard problem)
  - new paradigms?
  - improve capacity and effectively apply QBF to real problems
- and do not forget testing, debugging, simulation