A Short History of SAT Based Model Checking: From Bounded Model Checking to Interpolation

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A Personal History of Model Checking

BurchClarkeMcMillanDillHwang’90: Symbolic Model Checking

DavisPutnam’60: DP

CoudertMadre’89: Symbolic Reachability

McMillan’03: Interpolation

DavisLogemannLoveland’62: DPLL

Bryant’86: BDDs

Marques–SilvaSakallah’96: GRASP

Pnueli’77: Temporal Logic

BiereArthoSchuppan’01: Liveness2Safety

McMillan’93: SMV

MoskewiczMadiganZhaoZhangMalik’01: CHAFF

ClarkeEmerson’82: Model Checking

EenSorensson’03: MiniSAT

Kurshan’93: Localization

BiereCimattiClarkeZhu’99: Bounded Model Checking

QuielleSifakis’82: Model Checking

EenBiere’05: SatELite

Kurshan’91: SPIN

SheeranSinghStalmarck’00: $k$–Induction

Holzmann’81: On–The–Fly Reachability

BallRajamani’01: SLAM

Holzmann’91: SPIN

GrafSaidi’97: Predicate Abstraction

Peled’94: Partial–Order–Reduction

ClarkeGrumbergJahLuVeith’03: CEGAR
Reachability

- set of states $S$, initial states $I$, transition relation $T$
- bad states $B$ reachable from $I$ via $T$?
- symbolic representation of $T$ (circuit, program, parallel product)
  - avoid explicit matrix representations, because of the
  - state space explosion problem, e.g. $n$-bit counter: $|T| = O(n)$, $|S| = O(2^n)$
  - makes reachability PSPACE complete [Savitch’70]
- on-the-fly [Holzmann’81] for protocols
  - restrict search to reachable states
  - simulate and hash reached concrete states
Forward Fixpoint Algorithm: Initial and Bad States
Forward Fixpoint Algorithm: Step 2

\[ I \quad B \]
Forward Fixpoint Algorithm: Step 3
Forward Fixpoint Algorithm: Bad State Reached
Forward Fixpoint Algorithm: Termination, No Bad State Reachable
initial states $I$, transition relation $T$, bad states $B$

\[
\text{model-check}^\mu_{\text{forward}} (I, T, B) \\
S_C = \emptyset; \ S_N = I; \\
\text{while } S_C \neq S_N \text{ do} \\
\quad \text{if } B \cap S_N \neq \emptyset \text{ then} \\
\quad \quad \text{return} \text{ “found error trace to bad states”;} \\
\quad S_C = S_N; \\
\quad S_N = S_C \cup \text{Img}(S_C); \\
\text{done;} \\
\text{return} \text{ “no bad state reachable”;}
\]
Model Checking

- algorithms to check more general properties [ClarkeEmerson’82], [QuielleSifakis’82]
  - uses temporal logic [Pnueli’77] as property specification language
  - model checkers are usually fully automatic
    linear vs. branching time formalisms (CTL vs LTL) was hotly debated
    - either determine that property holds or …
    - … provide counter example for debugging purposes

- originally explicit (as in SPIN [Holzmann’91])
  - search works with concrete states,
  - bottle neck: number of states, that have to be stored
  - local (on-the-fly) and global algorithms (not on-the-fly)
Symbolic Model Checking

- work with symbolic representations of states
  - symbolic representations are potentially exponentially more succinct
  - favors BFS: next frontier set of states in BFS is calculated symbolically

- originally “symbolic” meant model checking with BDDs
  [CoudertMadre’89/’90,BurchClarkeMcMillanDillHwang’90,McMillan’93]

- Binary Decision Diagrams  [Bryant’86]
  - canonical representation for boolean functions
  - BDDs have fast operations  (but image computation is expensive)
  - often blow up in space
  - restricted to hundreds of variables
Unrolling of Forward Least Fixpoint Algorithm

0: continue? \(S_0^C \neq S_0^N\) \(\exists s_0[I(s_0)]\)
0: terminate? \(S_0^C = S_0^N\) \(\forall s_0[\neg I(s_0)]\)
0: bad state? \(B \cap S_0^N \neq \emptyset\) \(\exists s_0[I(s_0)] \land B(s_0)]\)

1: continue? \(S_1^C \neq S_1^N\) \(\exists s_0,s_1[I(s_0) \land T(s_0,s_1) \land \neg I(s_1)]\)
1: terminate? \(S_1^C = S_1^N\) \(\forall s_0,s_1[I(s_0) \land T(s_0,s_1) \rightarrow I(s_1)]\)
1: bad state? \(B \cap S_1^N \neq \emptyset\) \(\exists s_0,s_1[I(s_0) \land T(s_0,s_1)] \land B(s_1)]\)

2: continue? \(S_2^C \neq S_2^N\) \(\exists s_0,s_1,s_2[I(s_0) \land T(s_0,s_1) \land T(s_1,s_2) \land \neg (I(s_2) \lor \exists t_0[I(t_0) \land T(t_0,s_2)])\]
2: terminate? \(S_2^C = S_2^N\) \(\forall s_0,s_1,s_2[I(s_0) \land T(s_0,s_1) \land T(s_1,s_2) \rightarrow I(s_2) \lor \exists t_0[I(t_0) \land T(t_0,s_2)]\]
2: bad state? \(B \cap S_2^N \neq \emptyset\) \(\exists s_0,s_1,s_2[I(s_0) \land T(s_0,s_1) \land T(s_1,s_2) \land B(s_2)]\)
Falsification Part of Fixpoint Algorithm

0: continue? \[ S_0^C \neq S_0^N \quad \exists s_0[I(s_0)] \]

0: terminate? \[ S_0^C = S_0^N \quad \forall s_0[\neg I(s_0)] \]

0: bad state? \[ B \cap S_0^N \neq 0 \quad \exists s_0[I(s_0) \land B(s_0)] \]

1: continue? \[ S_1^C \neq S_1^N \quad \exists s_0, s_1[I(s_0) \land T(s_0, s_1) \land \neg I(s_1)] \]

1: terminate? \[ S_1^C = S_1^N \quad \forall s_0, s_1[I(s_0) \land T(s_0, s_1) \rightarrow I(s_1)] \]

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2: continue? \[ S_2^C \neq S_2^N \quad \exists s_0, s_1, s_2[I(s_0) \land T(s_0, s_1) \land T(s_1, s_2) \land \neg (I(s_2) \lor \exists t_0[I(t_0) \land T(t_0, s_2)])] \]

2: terminate? \[ S_2^C = S_2^N \quad \forall s_0, s_1, s_2[I(s_0) \land T(s_0, s_1) \land T(s_1, s_2) \rightarrow I(s_2) \lor \exists t_0[I(t_0) \land T(t_0, s_2)]] \]

2: bad state? \[ B \cap S_1^N \neq 0 \quad \exists s_0, s_1, s_2[I(s_0) \land T(s_0, s_1) \land T(s_1, s_2) \land B(s_2)] \]
Bounded Model Checking (BMC)

[BiereCimattiClarkeZhu’99]

- look only for counter example made of $k$ states  (the bound)

$$I(s_0) \land T(s_0,s_1) \land \cdots \land T(s_{k-1},s_k) \land \bigvee_{i=0}^{k} \neg p(s_i)$$

- simple for safety properties  $p$ is invariantly true  (e.g. $p = \neg B$)

- harder for liveness properties  $p$ is eventually true

$$I(s_0) \land T(s_0,s_1) \land \cdots \land T(s_{k-1},s_k) \land \bigwedge_{i=0}^{k} \neg p(s_i) \land \exists l \ T(s_k,s_l)$$
Bounded Model Checking (BMC)

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\[
I(s_0) \land T(s_0, s_1)) \land \cdots \land T(s_{k-1}, s_k) \land \bigwedge_{i=0}^{k} \neg p(s_i) \land \bigvee_{l=0}^{k} T(s_k, s_l)
\]
• satisfiability checking (SAT)
  – of propositional/combinational problems (only boolean variables)
  – actually restricted to conjunctive normal form (CNF)
  – classical NP hard problem [Cook’71]

• key motivation of BMC
  – leverage capacity of SAT solvers
  – SAT solvers could handle 10000 variables in late 90’ties
  – compared to hundreds of variables with BDDs

• key insight: trade capacity for completeness
Bounded Model Checking State-of-the-Art

- Increase in efficiency of SAT solvers \([\text{ZChaff, MiniSAT, SatELite}]\)

- SAT more robust than BDDs in bug finding
  
  (shallow bugs are easily reached by explicit model checking or testing)

- Better \textit{unbounded} but still SAT based model checking algorithms
  
  - \(k\)-induction \([\text{SinghSheeranStalmarck’00}]\)
  
  - Interpolation \([\text{McMillan’03}]\)

- 4th Intl. Workshop on Bounded Model Checking (BMC’06)

- Other logics beside LTL, better encodings, e.g. \([\text{LatvalaBiereHeljankoJuntilla’04}]\)

- Other system models, such as hybrid automata
Induction with SAT

[SinghSheeranStalmarck’00]

- more specifically \( k \)-induction
  
  - does there exist \( k \) such that the following formula is \textit{unsatisfiable}
    \[
    \overline{B(s_0)} \land \cdots \land \overline{B(s_{k-1})} \land T(s_0, s_1) \land \cdots \land T(s_{k-1}, s_k) \land B(s_k) \land \bigwedge_{0 \leq i < j \leq k} s_i \neq s_j
    \]
  
  - if \textit{unsatisfiable} and \( \lnot \text{BMC}(k) \) then \textit{bad state unreachable}

- bound on \( k \): length of \textit{longest cycle free path}

- \( k = 0 \) check whether \( \lnot B \) tautological (propositionally)

- \( k = 1 \) check whether \( \lnot B \) inductive for \( T \)
A Short SAT Solver History

- Davis and Putnam procedure
  - DP: elimination procedure [DavisPutnam’60]
  - DPLL: splitting [DavisLogemannLoveland’62]

- modern SAT solvers are mostly based on DPLL
  - learning: GRASP [MarquesSilvaSakallah’96], RelSAT [BayardoSchrag’97]
  - watched literals, VSIDS: CHAFF [MoskewiczMadiganZhaoZhangMalik’01]
  - improved heuristics: MiniSAT [EenSorensson’03] actually Version from 2005

- preprocessing is a hot topic:
  - currently fastest solvers use SatELite style preprocessing [EenBiere’05] DP

- www.satcompetition.org since 2002
Interpolation

[McMillan’03]

- SAT based technique to overapproximate frontiers $\text{Img}(S_C)$
  - currently most effective technique to show that bad states are unreachable
  - better than BDDs and $k$-induction in most cases [HWMCC’07]

- starts from a resolution proof refutation of a BMC problem with bound $k + 1$
  \[
  S_C(s_0) \land T(s_0, s_1) \land T(s_1, s_2) \land \cdots \land T(s_k, s_{k+1}) \land B(s_{k+1})
  \]
  - result is a characteristic function $f(s_1)$ over variables of the second state $s_1$
  - these states do not reach the bad state $s_{k+1}$ in $k$ steps
  - any state reachable from $S_C$ satisfies $f$: $S_C(s_0) \land T(s_0, s_1) \Rightarrow f(s_1)$

- $k$ is bounded by the diameter (exponentially smaller than longest cycle free path)
length of longest shortest path $O(n)$

diameter $O(1)$
Challenges I

• further convergence between theorem proving and model checking
  
  – as pioneered by SLAM [BallRajamani’01] using
    
    * predicate abstraction [GrafSaidi’97] and
    
    * counter example guided abstraction refinement [ClarkeGrumbergJahLuVeith’03]
  
  – handle large software and hardware systems precisely
  
  – automate compositional reasoning, e.g. alias analysis

• improve Satisfiability Modulo Theory (SMT) procedures
  
  – What is the right way to handle bit-vectors, arrays?
  
  – Quantifiers, interpolation for bit-vectors and arrays?
Challenges II

• Satisfiability Solver (SAT) (standard NP hard problem)
  – improve heuristics, remove magic constants
  – more aggressive incremental preprocessing
  – effective incorporation of more powerful reasoning engines

• Quantified Boolean Formulas (QBF) (standard PSPACE hard problem)
  – new paradigms?
  – improve capacity and effectively apply QBF to real problems

• and do not forget **testing, debugging, simulation**