Lazy Hyper Binary Resolution

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Background on Hyper Binary Resolution (HBR)

- one Hyper Binary Resolution step

\[
\frac{(l \lor l_1 \lor \cdots \lor l_n) \ (\overline{l_1} \lor l') \ \cdots \ (\overline{l_n} \lor l')}{(l' \lor l')}
\]

- combines multiple resolution steps into one
- special case “hyper unary resolution” where \( l = l' \)
- HBR is stronger than unit propagation if it is repeated until (confluent) closure
- equality reduction: if \( (a \lor \overline{b}), (\overline{a} \lor b) \in f \) then replace \( a \) by \( b \) in \( f \)

- can be simulated by unit propagation

\[
\text{if } \ (l \lor l') \in \text{HypBinRes}(f) \ \text{then } \ l' \in \text{UnitProp}(f \land \overline{l}) \text{ or vice versa}
\]

- implemented by repeated probing, c.f. HypBinResFast
Previous Optimizations

[BacchusWinter-SAT03][GershmanStrichman-SAT05]

• maintain acyclic and transitively-reduced binary implication graph
  
  – acyclic: (incremental) decomposition in strongly connected components (SCCs)

  \[(\bar{a} \lor b)(\bar{b} \lor c)(\bar{c} \lor a) \land R \]

  equisatisfiable to

  \[R[a/b, a/c]\]

  – transitively-reduced: remove resp. do not add transitive edges

• not all literals have to be probed

  – if \(l \in \text{UnitProp}(r)\) and \(\text{UnitProp}(r)\) does not produce anything \(\Rightarrow\) no need to probe \(l\)

  – at least with respect to units it is possible to focus on roots

• still as with failed literal probing, too expensive to run until completion
Observations

- time complexity: seems to be at least quadratic, unfortunately also in practice

- space complexity: unclear, at most quadratic, linear?

  - Are there CNFs where one transitively reduced hyper binary resolution closure is quadratic in size with respect to the size of the original CNF?

    - where \( \text{size} = \#\text{clauses} \) or \( \text{size} = \#\text{literals} \)

- hyper binary resolution simulates structural hashing for AND gates \( a \) and \( b \)

\[
F \equiv (\overline{a} \lor x)(\overline{a} \lor y)(a \lor \overline{x} \lor \overline{y}) \quad (\overline{b} \lor x)(\overline{b} \lor y)(b \lor \overline{x} \lor \overline{y}) \quad \cdots
\]

\[
\frac{(\overline{a} \lor x)(\overline{a} \lor y)(b \lor \overline{x} \lor \overline{y})}{(\overline{a} \lor b)} \quad \frac{(\overline{b} \lor x)(\overline{b} \lor y)(a \lor \overline{x} \lor \overline{y})}{(\overline{b} \lor a)}
\]

- can also be seen by \( b \in \text{UnitProp}(F \land a) \) and \( a \in \text{UnitProp}(F \land b) \)

- can not simulate structural hashing of XOR or ITE gates
Lazy Hyper Binary Resolution (LHBR)

- learn binary clauses lazily or on-the-fly
  - in BCP
  - during preprocessing with failed literal probing
  - or during search

- whenever a large clause \((a_1 \lor \cdots \lor a_m \lor c)\) with \(m \geq 2\) becomes a reason for \(c\)
  - for the partial assignment \(\sigma\) we have \(\sigma(a_i) = 0\) and \(\sigma(c) = 1\)
  - check whether there is a literal \(d\) which dominates all \(\overline{a_i}\)
  - in the implication graph restricted to binary clauses

- learn \((\overline{d} \lor c)\) if such a dominator exists
1. trail contains assigned literals

2. set $n_2$ and $n_3$ to the trail level of those literals that still need to be propagated

3. while $0 \leq n_3 \leq n_2 < |\text{trail}|$ and there is no conflict
   
   (a) if $n_2 < |\text{trail}|$
      
      i. pick literal $l$ at position $n_2$, increment $n_2$ and visit binary clauses with $\bar{l}$
      
      ii. assign literals forced through these binary clauses first

   (b) otherwise (necessarily $n_3 < |\text{trail}|$)
      
      i. pick literal $l$ at position $n_3$, increment $n_3$ and visit large clauses with $\bar{l}$
      
      ii. assign literals forced through these large clauses
How-To Check Existence of Dominators

• for each assigned literal \( l \) calculate **one** dominator \( \text{bindom}(l) \)

• in the implication graph restricted to binary clauses

• for decisions \( l \) set \( \text{bindom}(l) = l \)

• for binary implications \( (a_1 \lor c) \) with \( \sigma(a_1) = 0, \sigma(c) = 1 \) set \( \text{bindom}(c) = \text{bindom}(\overline{a_1}) \)

• necessary / sufficient for the \( \overline{a}_i \) in large \( (m \geq 2) \) reasons to have a common dominator:
  \[
  (a_1 \lor \cdots \lor a_m \lor c) \quad \text{bindom}(\overline{a_1}) = \cdots = \text{bindom}(\overline{a_m})
  \]

• if this condition triggers, actually use least common ancestor (closest dominator)

• use \( (d \lor c) \) as **new reason** instead of \( (a_1 \lor \cdots \lor a_m \lor c) \)
PrecoSAT Integration

- Interleave search and preprocessing
  - Bound time spent in search to roughly 80%
  - Measured in number of propagations / resolutions

- BCP during search learns binary clauses through LHBR (search LHBR)

- During preprocessing / simplification on the top level
  - Unit propagation on the top-level does LHBR (top-level LHBR)
  - Failed literal probing learns most binary clauses through LHBR (probing LHBR)

- Effectiveness of LHBR reduced due to “lifting” in failed literal probing (also in PicoSAT)
Experiments

- rerunning SAT’09 competition with competition version 236 of PrecoSAT
  - 900 seconds time out
  - roughly twice as fast machines

- PrecoSAT without LHBR solves 6 less instances
  - 171 instead of 177 out of 292

- statistics
  - LHBR learned 48 million binary clause
  - on 292 instances that is 181 thousand learned binary clauses on average
  - additionally 202 million learned clauses through conflict analysis
  - 19% learned (binary) clauses due to LHBR
Summary

- no measurable overhead doing LHBR during BCP
  - so at least not harmful (in contrast to many other “optimizations”)
  - except for rare cases where it produces too many clauses
  - most recent version limits number of learned binary clauses to 5 million

- unpublished but implemented in PrecoSAT
  - source code of PrecoSAT available under MIT license

- it actually performs a limited version of on-the-fly strengthening / subsumption

[HanSomenzi-SAT09] [HamadiJabbourSais’09]
Future Work

- how to run LHBR and / or failed literal probing until completion
- transitive reduction (initially and after equality reduction)
- incremental SCC decomposition and equality reduction
- determine time / space complexity for the problem
- how to simulate structural hashing of XOR / ITE gates
- get more experimental data on
  - how often this actually happens and for which benchmarks
  - the empirical relation to on-the-fly subsumption