Circuit versus CNF Reasoning for Equivalence Checking

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Example of Tseitin Transformation: Circuit to CNF

\[ o \land (x \leftrightarrow a) \land (x \rightarrow a) \land (x \rightarrow c) \land (x \leftarrow a \land c) \land \ldots \]

\[ o \land (\overline{x} \lor a) \land (\overline{x} \lor c) \land (x \lor \overline{a} \lor \overline{c}) \land \ldots \]
Preprocessing SAT

- general idea:
  - simplify CNF before applying complete DPLL algorithm
  - heuristic: simpler CNF is easier to solve
  - metric for simpler: smaller, e.g. less clauses or literals

- for instance failed literal rule
  - assume one literal \( l \) by setting it to \( true \)
  - perform boolean constraint propagation (BCP)
  - if BCP generates conflict (empty clause), then permanently add \( l \)
  - continue until no more literals are added \( \Rightarrow \) saturate
Resolution
[Robinson]

- method to derive logically implied clauses
  - applicable to resolvents \((l_1 \lor \ldots \lor l_n \lor \neg v)\) and \((\neg v \lor k_1 \lor \ldots \lor k_m)\)
  - with matching variable \(v\)
  - implied resolvent \((l_1 \lor \ldots \lor l_n \lor k_1 \lor \ldots \lor k_m)\) can be added

- special cases
  - trivial resolvent: \((a \lor b \lor v) \otimes (\neg v \lor \neg a) \equiv (a \lor b \lor \neg a) \equiv 1\)
  - unit resolution: \((l_1 \lor \ldots \lor l_n \lor v) \otimes (\neg v) \equiv (l_1 \lor \ldots \lor l_n)\)
  - resolution of empty clause: \((v) \otimes (\neg v)\)
Elimination of Variables by Resolution (Clause Distribution)

[DavisPutnam’60]

original clauses in which $v$ or $\overline{v}$ occurs:

\[
\begin{align*}
\neg r & \lor v \\
  s & \lor v \\
x \lor y \lor v
\end{align*}
\]

add non-trivial resolvents:

\[
\begin{align*}
(s \lor r), & \quad (x \lor y \lor r), & \quad \text{and} & \quad (s \lor \neg x \lor \neg y \lor r)
\end{align*}
\]

remove original clauses
Issues with Clause Distribution

- number of added clauses \textbf{quadratic} in worst case
  - solvers using only clause distribution explode in space

- still useful for preprocessing
  - resolution may generate trivial clause \((a \lor b \lor \overline{a})\)
  - or even better units \((a \lor v) \otimes (\overline{v} \lor a) \equiv a\)
  - empirically generates many subsumed clauses
    \( (a \lor b) \text{ subsumes } (a \lor b \lor c) \)
  - trivial and subsumed clauses do not have to be added
Subsumption

- **backward subsumption**
  - new clause being added to CNF **subsumes** clause already in CNF
  - old subsumed clause can be removed after adding new clause
  - search clause in CNF containing **all** literals of new clause

- **forward subsumption**
  - new clause **is subsumed** by clause already in CNF
  - new clause does not have to be added
  - search clause in CNF made of a **subset** of literals of new clause
Signature based Subsumption Techniques for Propositional CNF
[Biere’04, ÉenBiere’05]

- signature bit for each literal \( h(l) \in \{0, \ldots, 31\} \)
  - signature of literal is a 32-bit word: \( \sigma(l) = 2^{h(l)} \) (1 << \( h(l) \) in \( C \))
  - signature of clause is a 32-bit word: \( \sigma(C) = \bigcup_{l \in C} \sigma(l) \)
  - necessary condition: \( C \) subsumes \( D \) \( \Rightarrow \) \( \sigma(C) \subseteq \sigma(D) \)

- backward subsumption
  - traverse clauses of a single literal of new clause
  - signature subset check avoids full literal subset check in many cases
Signature based Subsumption Techniques for Propositional CNF cont.

- originally implemented in QBF solver Quantor [Biere’04]
  - fast subsumption essential for resolution based variable elimination

- SAT Preprocessor Satelite [ÉenBiere’05]
  - fast subsumption has similar impact as in QBF
  - SateliteGTI = Satelite + Minisat
    (new version of Minisat by Sörensson + Éen)
  - SateliteGTI winner of all industrial categories in SAT’05 competition

- forward subsumption: add clauses in reverse order, backward subsume
  (faster way: 1-watched literal scheme [Zhang’05])
More Features in Satelite

- **self subsuming resolution:**
  - allows to remove *single* literals from clauses
  - beside clause distribution and fw/bw subsumption most effective
  - resolvent subsumes one resolvee: \((a \lor \neg v \lor c) \otimes (a \lor v) \equiv (a \lor c)\)

- efficient scheduler for clause distribution and self subsumption

- functional substitution of gates (cheaper than clause distribution)

- **hyper unary resolution:** \((a \lor b \lor c) \otimes (\overline{a} \lor b) \otimes (\overline{c} \lor b) \equiv (b)\)
Second Level Signature based Subsumption Techniques
[Biere’04]

- avoids traversing occurrence list in many cases

- signature sum of a literal: \( \Sigma(l) = \bigcup \{ \sigma(D) \mid D \in CNF \text{ and } l \in D \} \)

- necessary condition for new clause \( D \) to subsume an old clause:
  \[ \sigma(C) \subseteq \Sigma(l) \quad \text{for all} \quad l \in C \]

- removing clauses
  - it is sound to keep old signature
  - recalculate accurate signature sums after many removals

- technique can be extended to extract gates and hyper unary resolution
## Experiments for Second Level Signatures

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</table>
Why Hyper Unary Resolution?

original CNF including clauses modelling an AND gate \( a = b \land c \land d \)

\[
(\overline{a} \lor b) \land (\overline{a} \lor c) \land (\overline{a} \lor d) \land \left( a \lor \overline{b} \lor \overline{c} \lor \overline{d} \right)
\]

new clause \( (b \lor \overline{c} \lor \overline{d}) \)

backward subsumes base clause of AND gate and prevents gate extraction

however hyper resolution with the binary side clauses of the AND gate

\[
(\overline{a} \lor b) \otimes (\overline{a} \lor c) \otimes (\overline{a} \lor d) \otimes (\overline{b} \lor \overline{c} \lor \overline{d}) \equiv a
\]

results in unit clause

similar techniques for other subsumptions of base or side clauses
Automatic Test Pattern Generation (ATPG)

- need to test chips **after** manufacturing
  - manufacturing process introduces faults (< 100% yield)
  - faulty chips cannot be sold (should not)
  - generate all test patterns from functional logic description

- simplified failure model
  - at most one wire has a fault
  - fault results in fixing wire to a logic constant:
    - “stuck at zero fault” (s-a-0)
    - “stuck at one fault” (s-a-1)
ATPG with D-Algorithm

[Roth’66]

- adding logic constants $D$ and $\overline{D}$ allows to work with only one circuit

  0 represents 0 in fault free and 0 in faulty circuit

  1 represents 1 in fault free and 1 in faulty circuit

  $D$ represents 1 in fault free and 0 in faulty circuit

  $\overline{D}$ represents 0 in fault free and 1 in faulty circuit

- otherwise obvious algebraic rules (propagation rules)

  $1 \land D \equiv D$  \quad $0 \land D \equiv 0$  \quad $\overline{D} \land D \equiv 0$  \quad etc.

- new conflicts: e.g. variable/wire can not be 0 and $D$ at the same time
Fault Injection for S-A-0 Fault

assume opposite value 1 before fault
(both for fault free and faulty circuit)

assume difference value $D$ after fault
D-Algorithm Example: Fault Injection
D-Algorithm Example: Path Sensitation
D-Algorithm Example: Propagation

test vector \((c, t, e) = (1, 1, 0)\)
generate partial input vector to justify 1

only backward propagation, remaining unassigned inputs can be arbitrary
Observation

extend partial input vector to propagate $D$ or $\overline{D}$ to output

forward propagation of $D$ and $\overline{D}$, backward propagation of 0 and 1
Dominator and Path Sensitation

- idea: use circuit topology for additional necessary conditions
  - assign and propagate these conditions after fault injection

- gate dominates fault iff every path from fault to output goes through it
  - more exactly we determine wires (input to gates) that dominate a fault

- if input dominates a fault assign other inputs to non-controlling value

\[ s\text{–}a\text{–}0 \text{ dominator} \]
\[ \begin{array}{c}
1 \\
\times
\end{array} \]
\[ D \]
\[ \text{dominator} \]
\[ \text{implied} \]
\[ D \]
\[ \text{only path to output} \]

implied non-controlling value
Redundancy Removal with D-Algorithm: Fault Injection
Redundancy Removal with D-Algorithm: Path Sensitation
Redundancy Removal with D-Algorithm: 1st Propagation
Redundancy Removal with D-Algorithm: 2nd Propagation
Redundancy Removal with D-Algorithm: Untestable
Redundancy Removal with D-Algorithm: Assume Fault

\[ c \rightarrow t \rightarrow e \rightarrow 0 \rightarrow o \]
Redundancy Removal with D-Algorithm: Simplified Circuit
Redundancy Removal for SAT

- assume CNF is generated via Tseitin transformation from formula/circuit
  - formula = model constraints + negation of property
  - CNF consists of gate input/output consistency constraints
  - plus additional unit forcing output \( o \) of whole formula to be 1

- remove redundancy in formula under assumption \( o = 1 \)

- propagation of \( D \) or \( \overline{D} \) to \( o \) does not make much sense
  - not interested in \( o = 0 \)
  - check simply for unsatisfiability \( \Rightarrow \) no need for \( D, \overline{D} \) (!?)
Variable Instantiation
"AnderssonBjesseCookHanna DAC’02" and Oepir SAT solver

- satisfiability preserving transformation

- motivated by original pure literal rule:
  - if a literal $l$ does not occur negatively in CNF $f$
  - then replace $l$ by 1 in $f$ (continue with $f[l \rightarrow 1]$)

- generalization to variable instantiation:
  - if $f[l \rightarrow 0] \rightarrow f[l \rightarrow 1]$ is valid
  - then replace $l$ by 1 in $f$ (continue with $f[l \rightarrow 1]$)
Why is Variable Instantiation a Generalization of the Pure Literal Rule?

Let \( f \equiv f' \land f_0 \land f_1 \) with

\( f' \) \( l \) does not occur

\( f_0 \) \( l \) occurs negatively

\( f_1 \) \( l \) occurs positively

further assume \( \) (assumption of pure literal rule)

\( f_0 \equiv 1 \)

then

\[ f[l \mapsto 0] \iff f' \land f_1[l \mapsto 0] \quad \implies \quad f' \iff f[l \mapsto 1] \]
Variable Instantiation Implementation

We have

\[ f[l \mapsto 1] \iff f' \land f_1[l \mapsto 0] \land f_0[l \mapsto 1] \iff f' \land f_0[l \mapsto 1] \iff f' \land \bigwedge_{i=1}^n C_i \]

and since \( f[l \mapsto 0] \implies f' \) we only need show the validity of

\[ f[l \mapsto 0] \rightarrow \bigwedge_{i=1}^n C_i \]

which is equivalent to the unsatisfiability of

\[ f[l \mapsto 0] \land \overline{C_i} \quad \text{for } i = 1 \ldots n \]

which again is equivalent to the unsatisfiability of

\[ f \land \overline{l} \land \overline{C_i} \quad \text{for } i = 1 \ldots n \]

This can be done directly on the CNF and needs \( n \) unsatisfiability checks.
Variable Instantiation for Tseitin Encodings

\[\begin{align*}
&(\overline{a} \lor c) & (c \lor \overline{e}) \\
&(\overline{b} \lor c) & (d \lor \overline{e}) \\
&(a \lor b \lor \overline{c}) & (\overline{c} \lor d \lor e)
\end{align*}\]

\[
\begin{aligned}
\neg f & \land \overline{c} \land \overline{(a \lor b)} \\
\neg f & \land \overline{c} \land \overline{(d \lor e)}
\end{aligned}
\implies \text{add } c \text{ as unit}
\]

requires two satisfiability checks while ATPG for \(c\) s-a-1 needs just one run
Stålmarck’s Method and Recursive Learning

- originally Stålmarck’s Method works on “sea of triplets” [Stålmarck’89]
  
  \[ x = x_1 \& \ldots \& x_n \] with \& boolean operator

  - equivalence reasoning + structural hashing + test rule

  - test rule translated to CNF \( f: \quad f \Rightarrow (BCP(f \& x) \cap BCP(f \& \overline{x})) \)

  add to \( f \) units that are implied by both cases \( x \) and \( \overline{x} \)

- Recursive Learning [KunzPradhan 90ties]

  - originally works on circuit structure

  - idea is to analyze all ways to justify a value, intersection is implied

  - translated to CNF \( f \) which contains clause \( (l_1 \lor \ldots \lor l_n) \)

  BCP on all \( l_i \) separately and add intersection of derived units
Further CNF Simplification Techniques

- failed literals, various forms of equivalence reasoning

- hyper binary resolution  \([\text{BacchusWinter'03,GershmanStrichman'05}]\)
  - add binary clauses obtained through hyper resolution
  - avoid adding full transitive closure of implication chains
  - equivalence reasoning through SCC detection in binary clause graph
  - as Stålmarck’s procedure subsumes structural hashing

- variable and clause elimination
  - autarkies and blocked clauses  \([\text{Kullman}]\)
Circuit based Simplification vs. CNF simplification

- Circuit reasoning/simplification can use structure of circuit
  - graph structure (dominators)
  - notion of direction (forward and backward propagation)
  - partial models (some inputs do not need to be assigned)

- CNF simplification does not rely on circuit structure
  - orthogonal: can for instance remove individual clauses

- Adapt ideas from circuit reasoning to SAT
  (e.g. avoid multiple SAT checks for redundancy removal in CNF)