Circuit versus CNF Reasoning for Equivalence Checking

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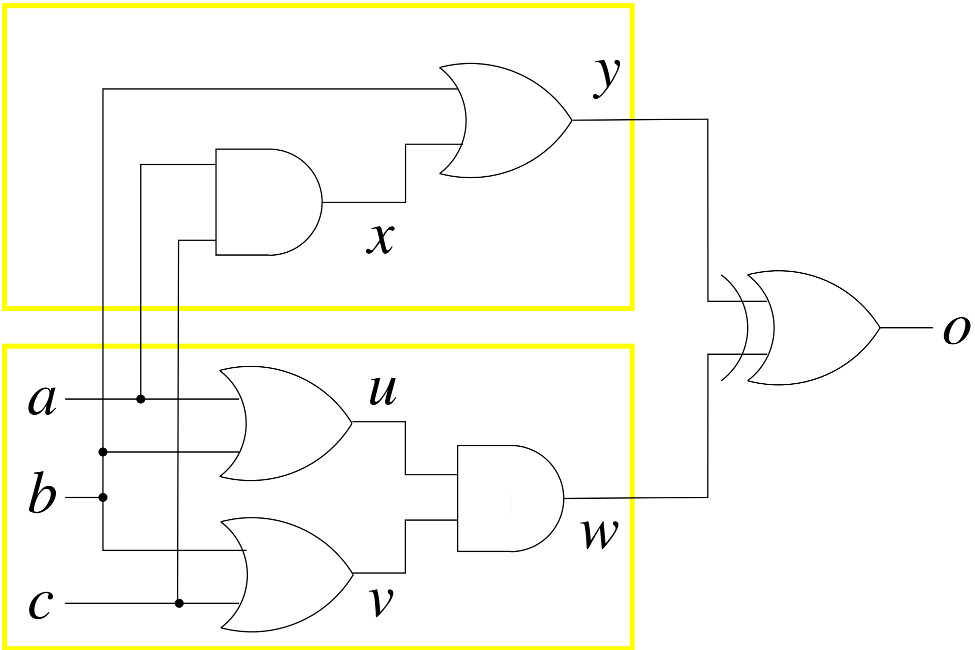
Example of Tseitin Transformation: Circuit to CNF

[Tseitin'68]



$$
\begin{array}{r}
o \; \wedge \\
(x \leftrightarrow a \wedge c) \; \wedge \\
(y \leftrightarrow b \vee x) \; \wedge \\
(u \leftrightarrow a \vee b) \; \wedge \\
(v \leftrightarrow b \vee c) \; \wedge \\
(w \leftrightarrow u \wedge v) \; \wedge \\
(o \leftrightarrow y \oplus w)
\end{array}
$$

$$o \wedge (x \rightarrow a) \wedge (x \rightarrow c) \wedge (x \leftarrow a \wedge c) \wedge \ \ldots$$

$$o \wedge (\overline{x} \vee a) \wedge (\overline{x} \vee c) \wedge (x \vee \overline{a} \vee \overline{c}) \wedge \ \ldots$$

Preprocessing SAT

- general idea:
  - simplify CNF before applying complete DPLL algorithm
  - heuristic: simpler CNF is easier to solve
  - metric for simpler: smaller, e.g. less clauses or literals
- for instance failed literal rule
  - *assume* one literal l by setting it to *true*
  - perform boolean constraint propagation (BCP)
  - if BCP generates conflict (empty clause), then permanently add l
  - continue until no more literals are added $\Rightarrow$ saturate

Resolution

[Robinson]

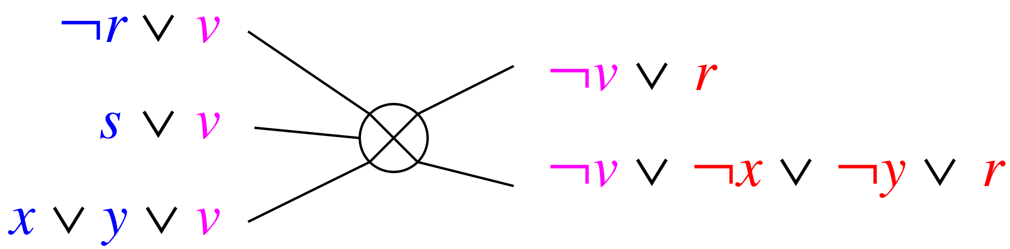
- method to derive logically implied clauses
  - applicable to **resolvees** $(l_1 \vee \ldots \vee l_n \vee v)$ and $(\neg v \vee k_1 \vee \ldots \vee k_m)$
  - with matching variable v
  - implied **resolvent** $(l_1 \vee \ldots \vee l_n \vee k_1 \vee \ldots \vee k_m)$ can be added

- special cases
  - trivial resolvent: $(a \vee b \vee v) \otimes (\bar{v} \vee \bar{a}) \equiv (a \vee b \vee \bar{a}) \equiv 1$
  - unit resolution: $(l_1 \vee \ldots \vee l_n \vee v) \otimes (\bar{v}) \equiv (l_1 \vee \ldots \vee l_n)$
  - resolution of empty clause: $(v) \otimes (\bar{v})$

Elimination of Variables by Resolution (Clause Distribution)

[DavisPutnam'60]

original clauses in which v or $\overline{v}$ occurs:



add non-trivial resolvents:

$(s \vee r)$, $(x \vee y \vee r)$, and $(s \vee \neg x \vee \neg y \vee r)$

remove original clauses

Issues with Clause Distribution

- number of added clauses quadratic in worst case
  - solvers using only clause distribution explode in space
- still useful for preprocessing
  - resolution may generate trivial clause $(a \vee b \vee \overline{a})$
  - or even better units $(a \vee v) \otimes (\neg v \vee a) \equiv a$
  - empirically generates many subsumed clauses

    $(a \vee b)$ subsumes $(a \vee b \vee c)$
  - trivial and subsumed clauses do not have to be added

Subsumption

- backward subsumption
  - new clause being added to CNF **subsumes** clause already in CNF
  - old subsumed clause can be removed after adding new clause
  - search clause in CNF containing **all** literals of new clause
- forward subsumption
  - new clause **is subsumed** by clause already in CNF
  - new clause does not have to be added
  - search clause in CNF made of a **subset** of literals of new clause

Signature based Subsumption Techniques for Propositional CNF

[Biere'04,ÉenBiere'05]

- signature bit for each literal $h(l) \in \{0, \ldots, 31\}$
  - signature of literal is a 32-bit word: $\sigma(l) = 2^{h(l)}$ ($1 << h(l)$ in C)
  - signature of clause is a 32-bit word: $\sigma(C) = \bigcup_{l \in C} \sigma(l)$
  - necessary condition: C subsumes D $\Rightarrow$ $\sigma(C) \subseteq \sigma(D)$
- backward subsumption
  - traverse clauses of a single literal of new clause
  - signature subset check avoids full literal subset check in many cases

Signature based Subsumption Techniques for Propositional CNF cont.

- originally implemented in QBF solver Quantor [Biere'04]
  - fast subsumption essential for resolution based variable elimination
- SAT Preprocessor Satelite [ÉenBiere'05]
  - fast subsumption has similar impact as in QBF
  - SateliteGTI = Satelite + Minisat

    (new version of Minisat by Sörenssen + Éen)
  - SateliteGTI winner of all industrial categories in SAT'05 competition
- forward subsumption: add clauses in reverse order, backward subsume

  (faster way: 1-watched literal scheme [Zhang'05])

More Features in Satelite

- self subsuming resolution:
  - allows to remove *single* literals from clauses
  - beside clause distribution and fw/bw subsumption most effective
  - resolvent subsumes one resolvee: $(a \vee \overline{v} \vee c) \otimes (a \vee v) \equiv (a \vee c)$
- efficient scheduler for clause distribution and self subsumption
- functional substitution of gates (cheaper than clause distribution)
- hyper unary resolution: $(a \vee b \vee c) \otimes (\overline{a} \vee b) \otimes (\overline{c} \vee b) \equiv (b)$

Second Level Signature based Subsumption Techniques
[Biere'04]

- avoids traversing occurrence list in many cases

- signature sum of a literal: $\Sigma(l) = \bigcup\{\sigma(D) \mid D \in CNF \text{ and } l \in D\}$

- necessary condition for new clause D to subsume an old clause:

$$\sigma(C) \subseteq \Sigma(l) \quad \text{for all} \quad l \in C$$

- removing clauses
  - it is sound to keep old signature
  - recalculate *accurate* signature sums after many removals

- technique can be extended to *extract gates* and *hyper unary resolution*

Experiments for Second Level Signatures

	sec	v	v'	red	c	c'	red	l	l'	red	sub	2nd hit	1st miss
1	1.05	9	2	73%	55	21	61%	149	68	54%	28	76.6%	62.0%
2	0.15	2	0	98%	11	0	97%	28	1	96%	8	70.1%	39.6%
3	1.43	14	4	73%	71	33	54%	187	107	43%	30	81.0%	55.1%
4	2.27	28	3	89%	135	25	81%	352	83	76%	95	72.9%	43.7%
5	0.69	9	0	93%	39	4	89%	100	15	84%	30	69.6%	44.1%
6	8.98	51	5	90%	356	87	76%	972	259	73%	1091	31.5%	16.8%
7	0.59	7	0	100%	40	0	100%	102	0	100%	29	68.9%	43.7%
8	6.22	58	13	76%	277	142	49%	714	453	36%	165	72.8%	69.7%
9	7.04	63	15	76%	307	162	47%	794	520	34%	180	72.7%	70.6%
10	6.40	59	9	84%	322	73	77%	850	240	72%	322	64.6%	33.0%
11	2.78	32	7	76%	149	53	64%	393	179	54%	75	80.5%	56.5%
12	3.95	39	11	70%	193	89	54%	511	302	41%	84	80.5%	55.2%
13	1.34	13	3	74%	65	29	55%	172	97	44%	25	79.8%	68.1%

Why Hyper Unary Resolution?

original CNF including clauses modelling an AND gate $a = b \wedge c \wedge d$

$$(\overline{a} \vee b) \wedge (\overline{a} \vee c) \wedge (\overline{a} \vee d) \wedge \underbrace{(a \vee \overline{b} \vee \overline{c} \vee \overline{d})}_{\text{base clause}}$$

new clause $(\overline{b} \vee \overline{c} \vee \overline{d})$

backward subsumes base clause of AND gate and prevents gate extraction

however hyper resolution with the binary side clauses of the AND gate

$$(\overline{a} \vee b) \otimes (\overline{a} \vee c) \otimes (\overline{a} \vee d) \otimes (\overline{b} \vee \overline{c} \vee \overline{d}) \equiv a$$

results in unit clause

similar techniques for other subsumptions of base or side clauses

Automatic Test Pattern Generation (ATPG)

- need to test chips **after** manufacturing
  - manufacturing process introduces faults (< 100% yield)
  - faulty chips can not be sold (should not)
  - generate all test patterns from functional logic description

- simplified failure model
  - at most one wire has a fault
  - fault results in fixing wire to a logic constant:

    “stuck at zero fault” (s-a-0) “stuck at one fault” (s-a-1)

ATPG with D-Algorithm

[Roth'66]

- adding logic constants D and $\overline{D}$ allows to work with only one circuit

0	represents	0	in fault free and	0	in faulty circuit
1	represents	1	in fault free and	1	in faulty circuit
D	represents	1	in fault free and	0	in faulty circuit
$\overline{D}$	represents	0	in fault free and	1	in faulty circuit

- otherwise obvious algebraic rules (propagation rules)

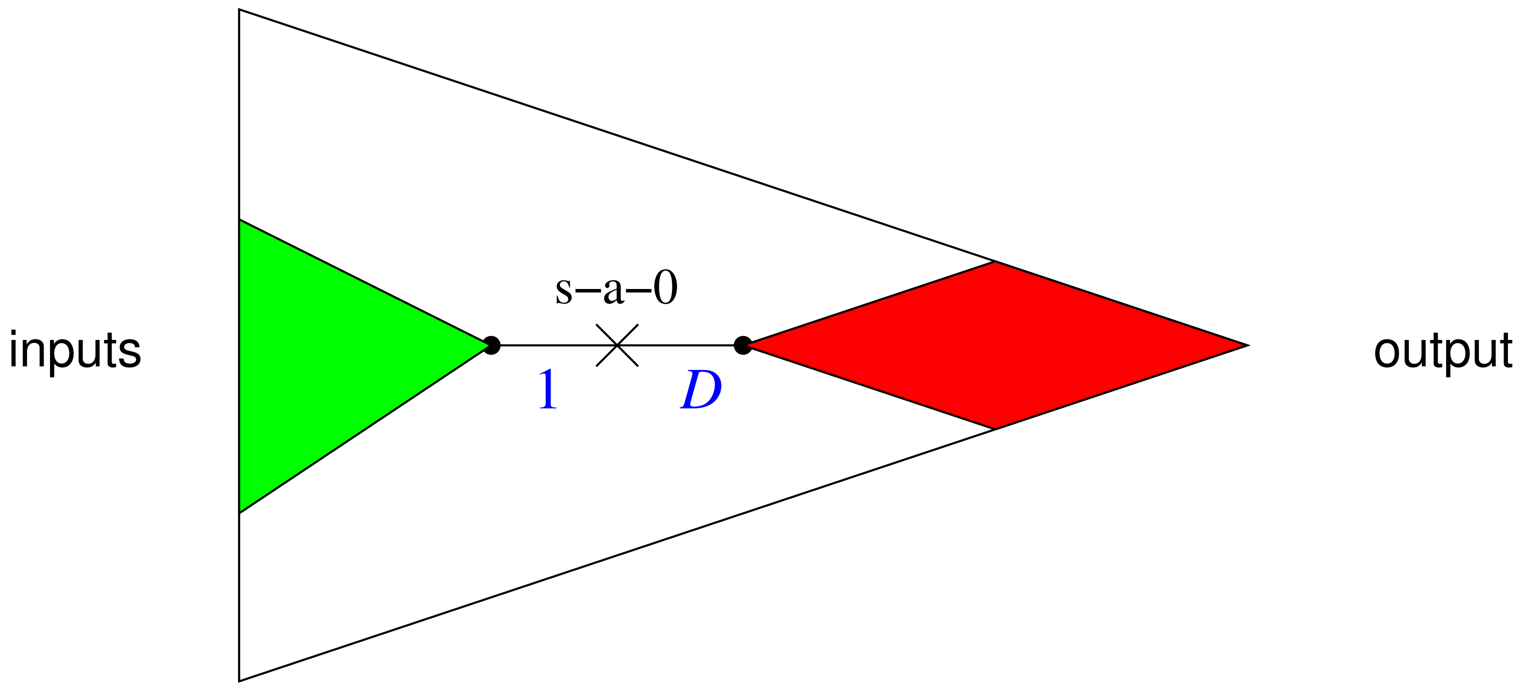
$$1 \wedge D \equiv D \qquad 0 \wedge D \equiv 0 \qquad \overline{D} \wedge D \equiv 0 \qquad \text{etc.}$$

- new conflicts: e.g. variable/wire can not be 0 and D at the same time

Fault Injection for S-A-0 Fault

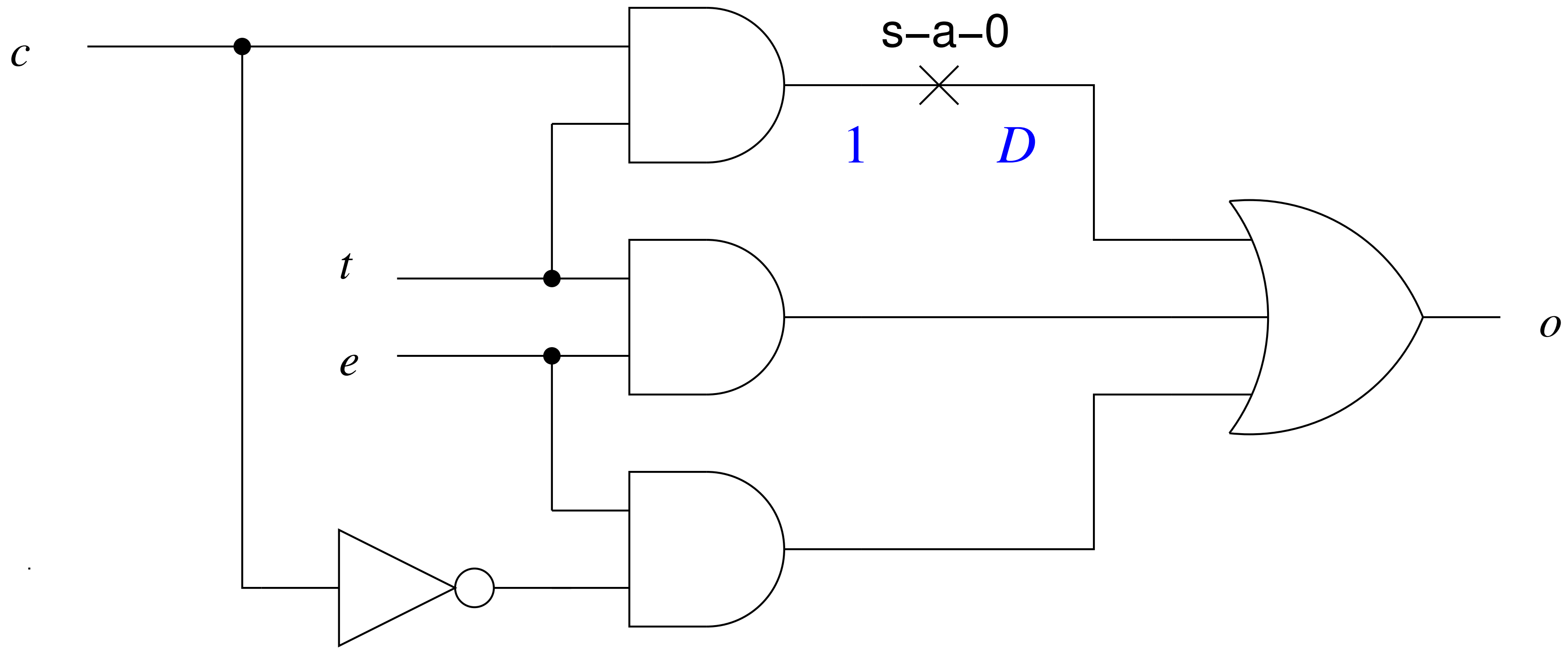
assume opposite value 1 before fault

(both for fault free and faulty circuit)

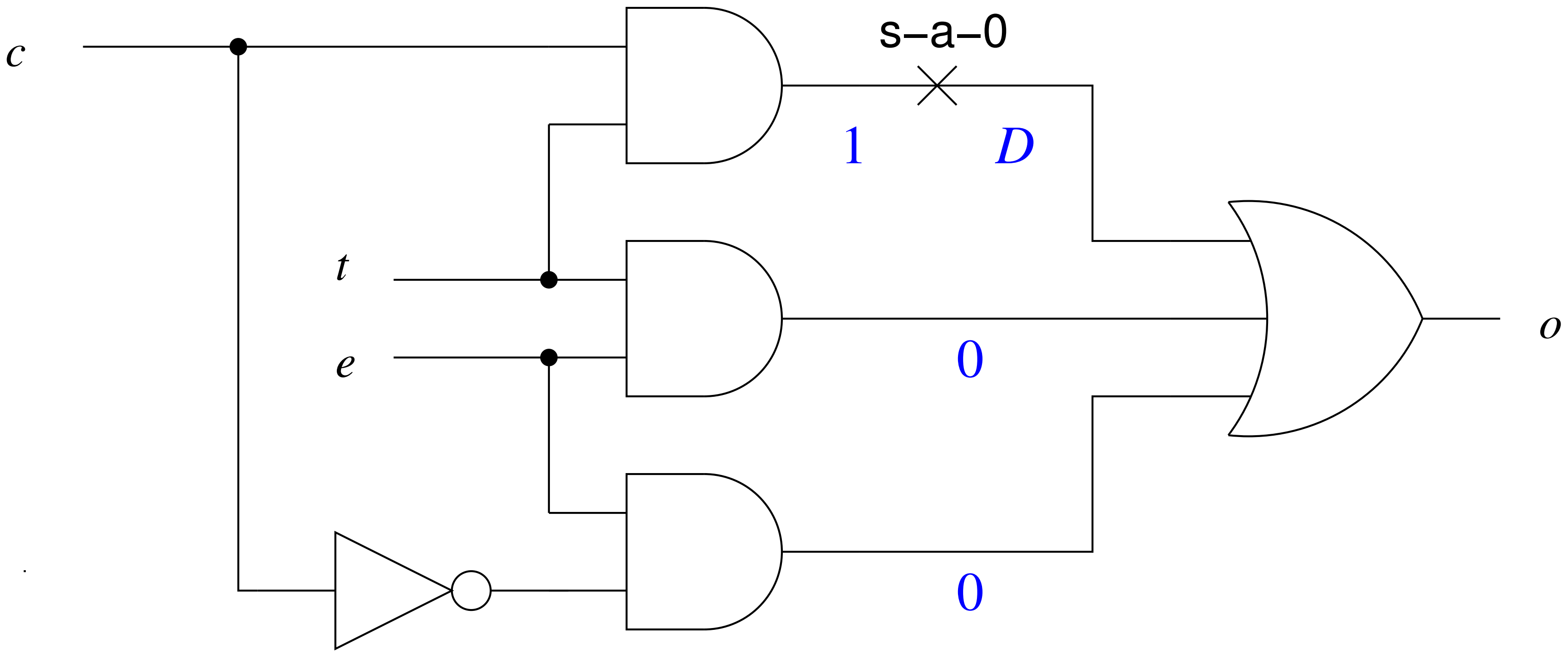


assume difference value D after fault

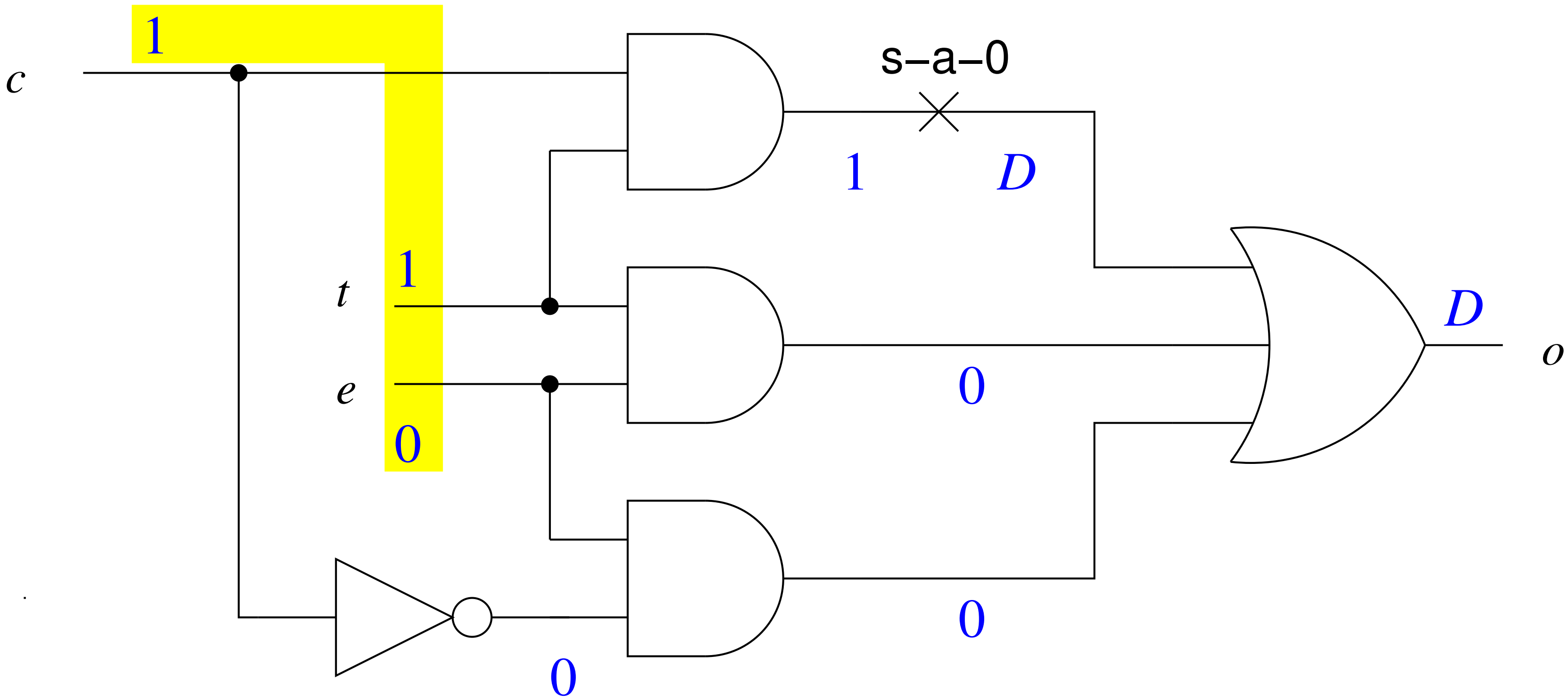
D-Algorithm Example: Fault Injection



D-Algorithm Example: Path Sensitation



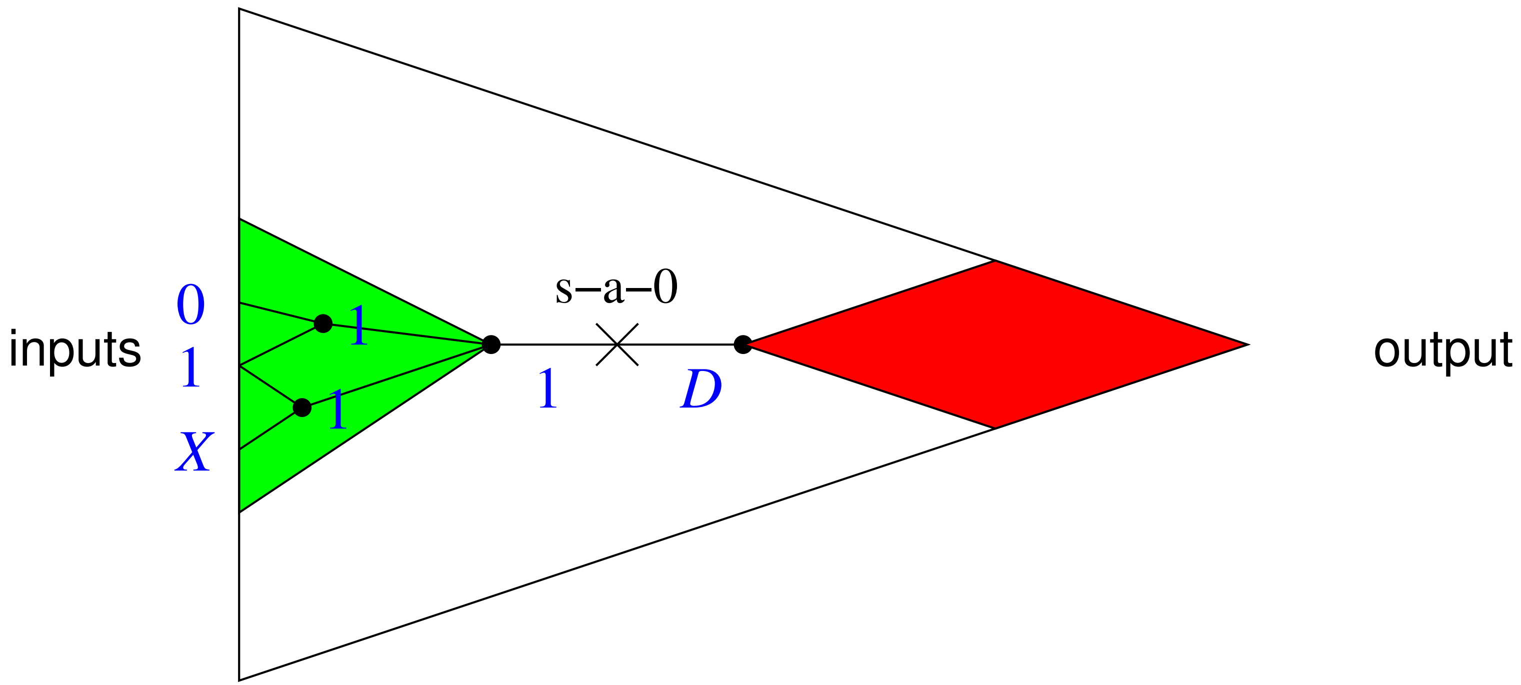
D-Algorithm Example: Propagation



test vector $(c, t, e) = (1, 1, 0)$

Justification

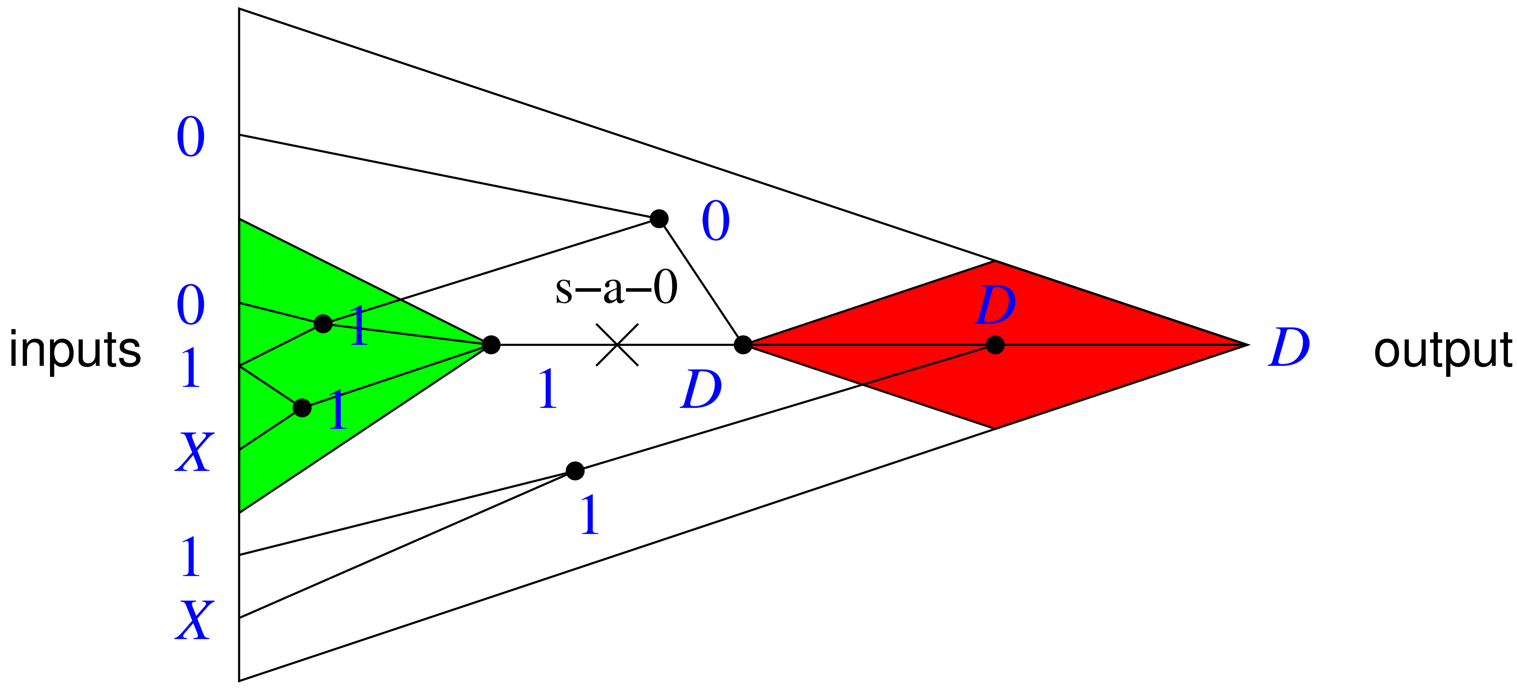
generate partial input vector to justify 1



only backward propagation, remaining unassigned inputs can be arbitrary

Observation

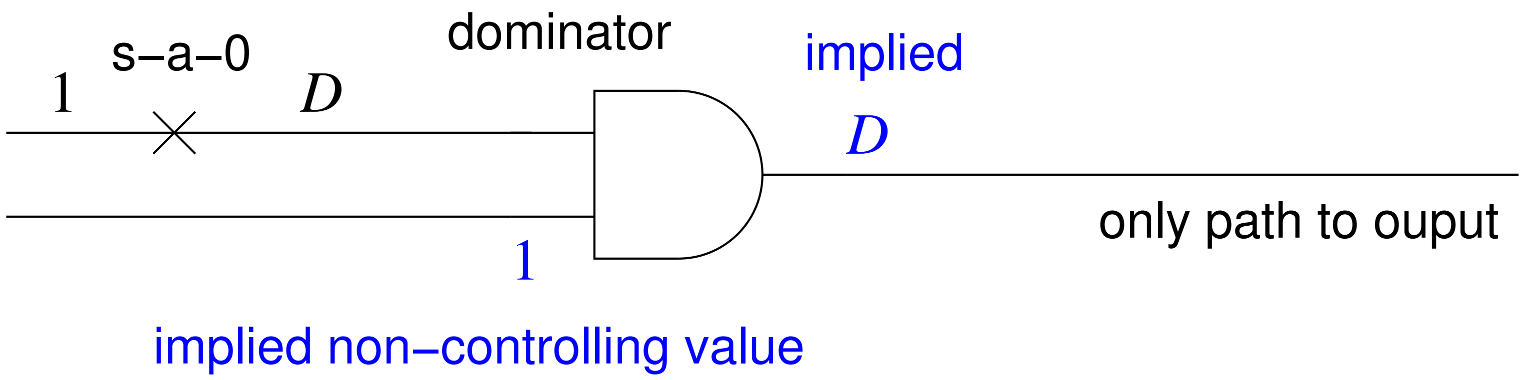
extend partial input vector to propagate D or $\overline{D}$ to output



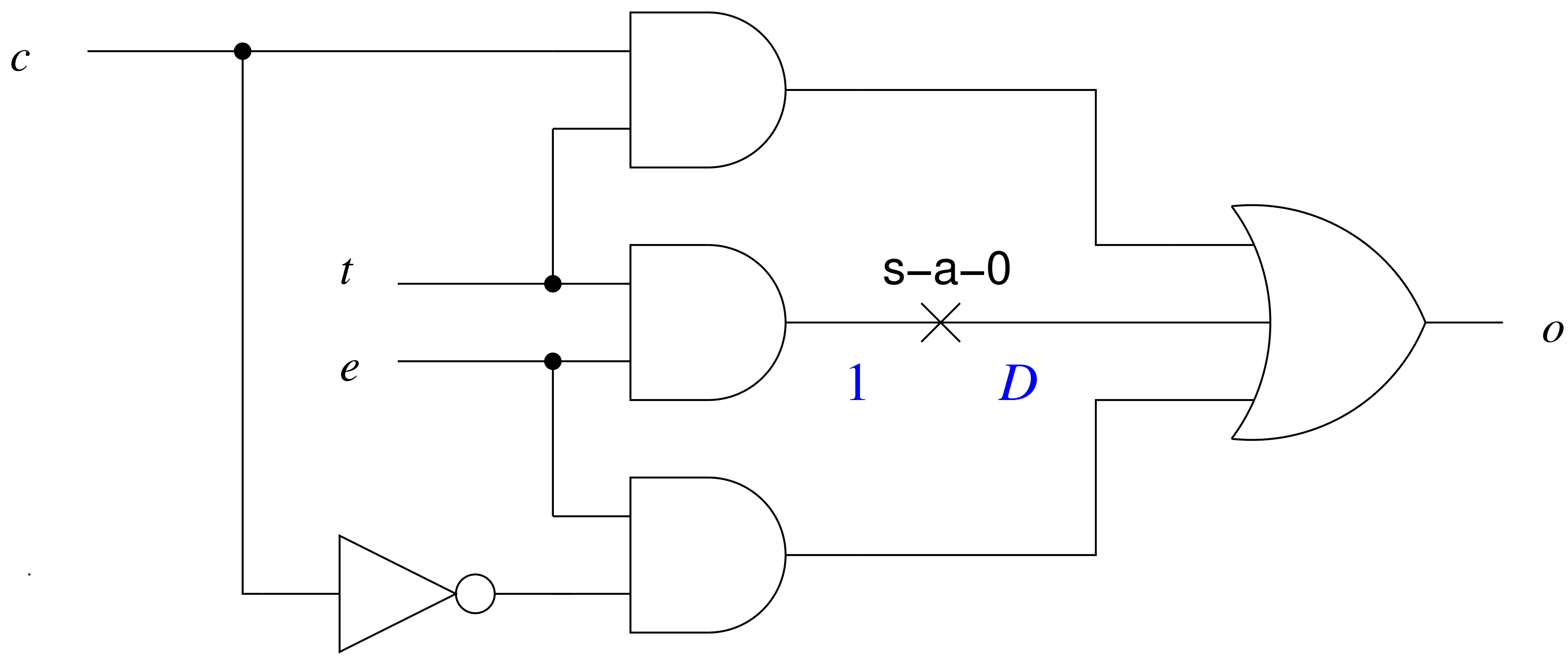
forward propagation of D and $\overline{D}$, backward propagation of 0 and 1

Dominators and Path Sensitation

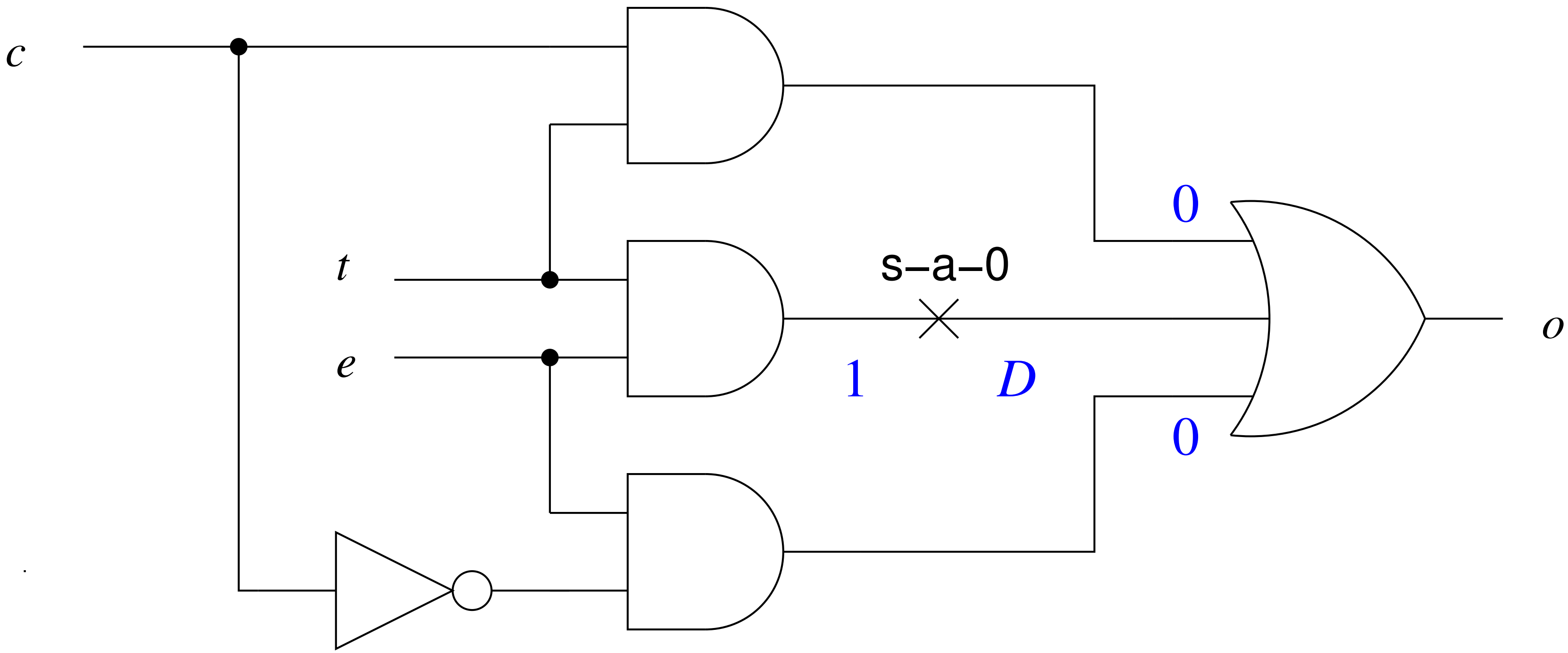
- idea: use circuit topology for additional necessary conditions
  - assign and propagate these conditions after fault injection
- gate dominates fault iff every path from fault to output goes through it
  - more exactly we determine wires (input to gates) that dominate a fault
- if input dominates a fault assign other inputs to non-controlling value



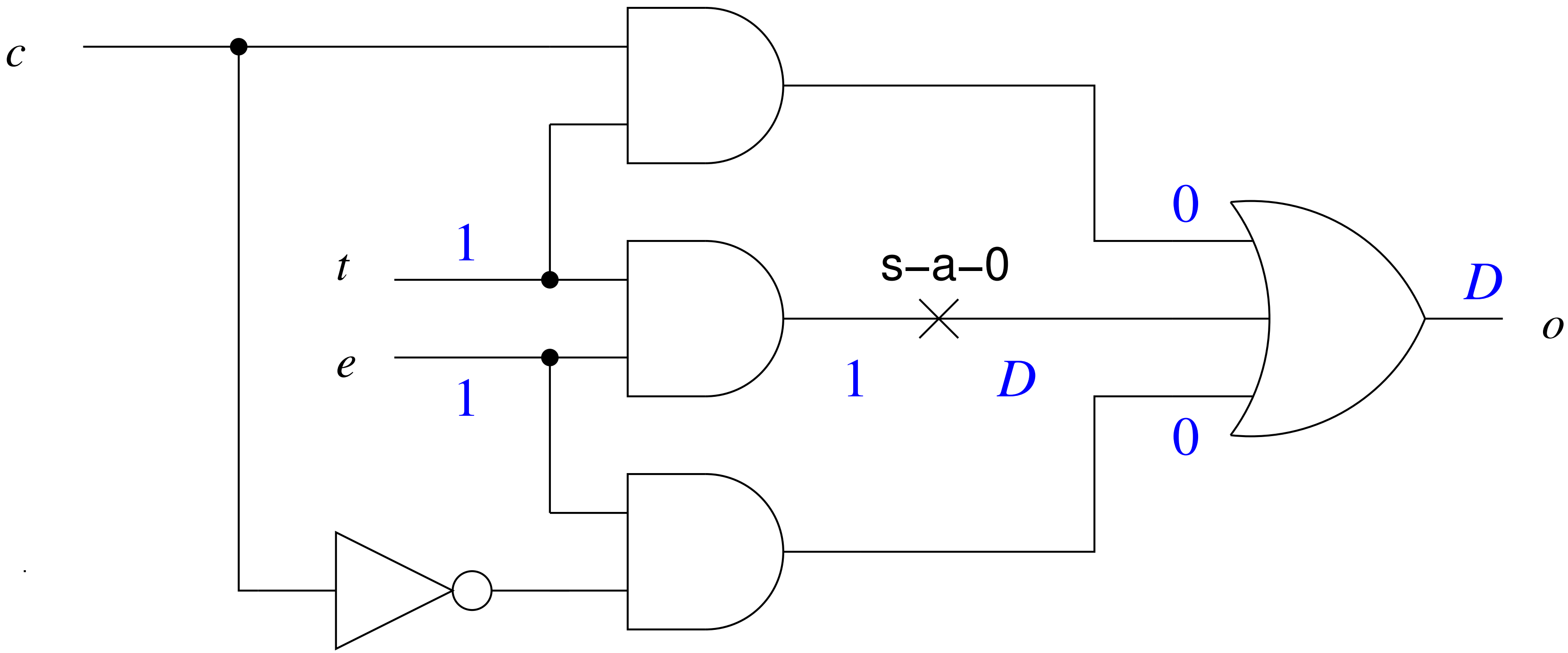
Redundancy Removal with D-Algorithm: Fault Injection



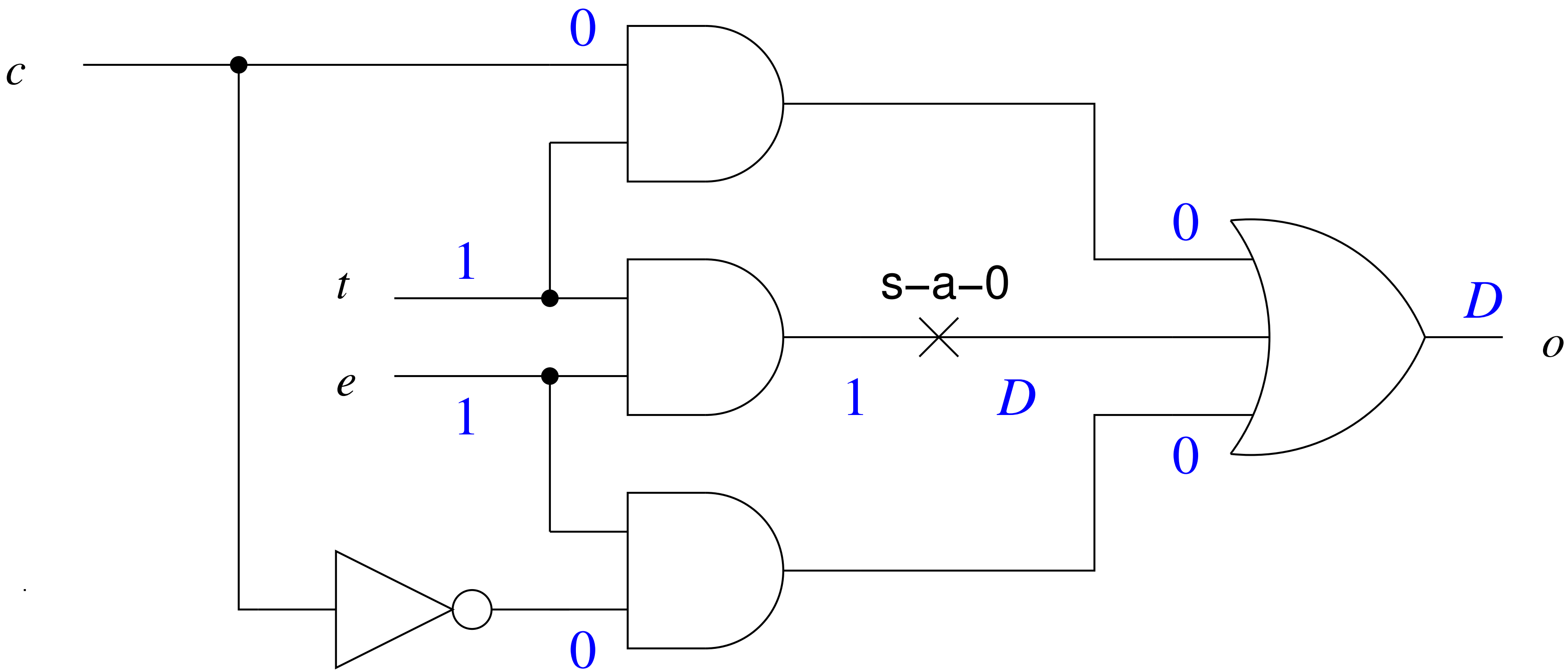
Redundancy Removal with D-Algorithm: Path Sensitation



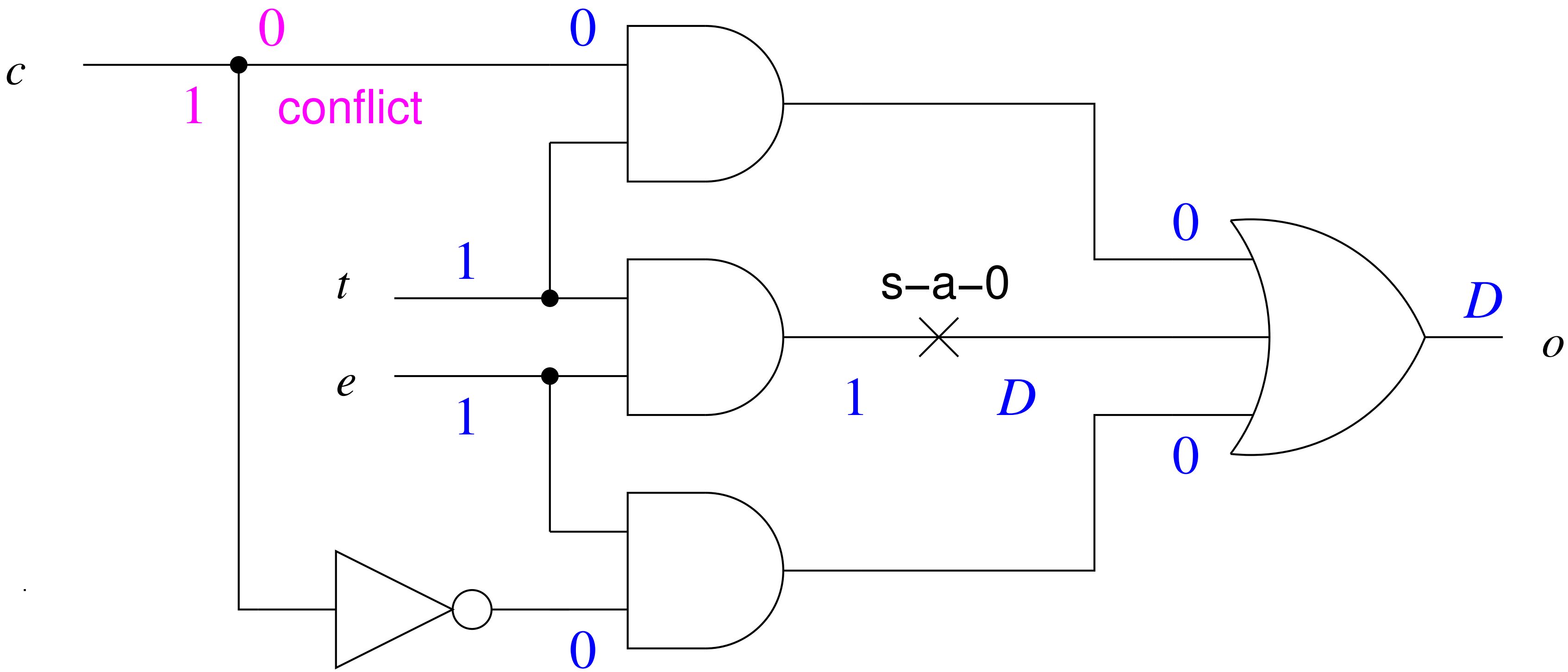
Redundancy Removal with D-Algorithm: 1st Propagation



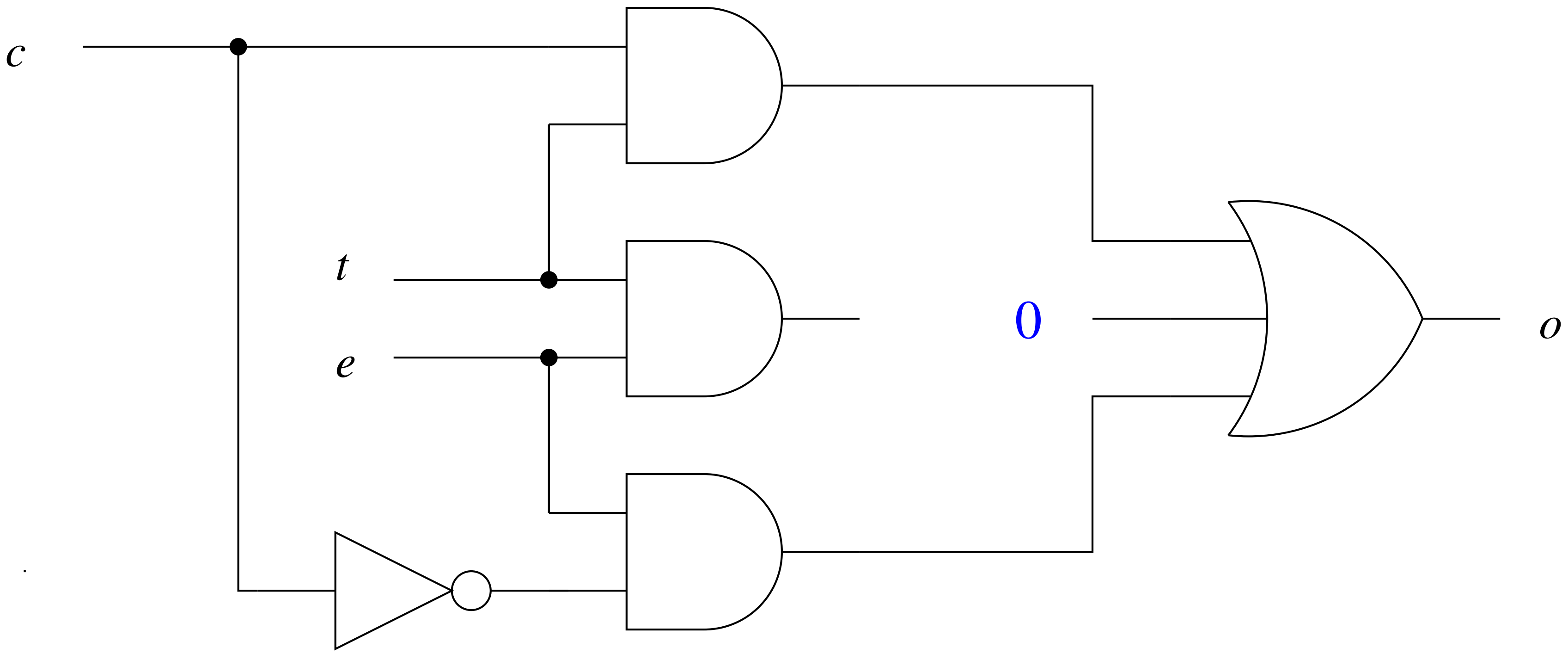
Redundancy Removal with D-Algorithm: 2nd Propagation



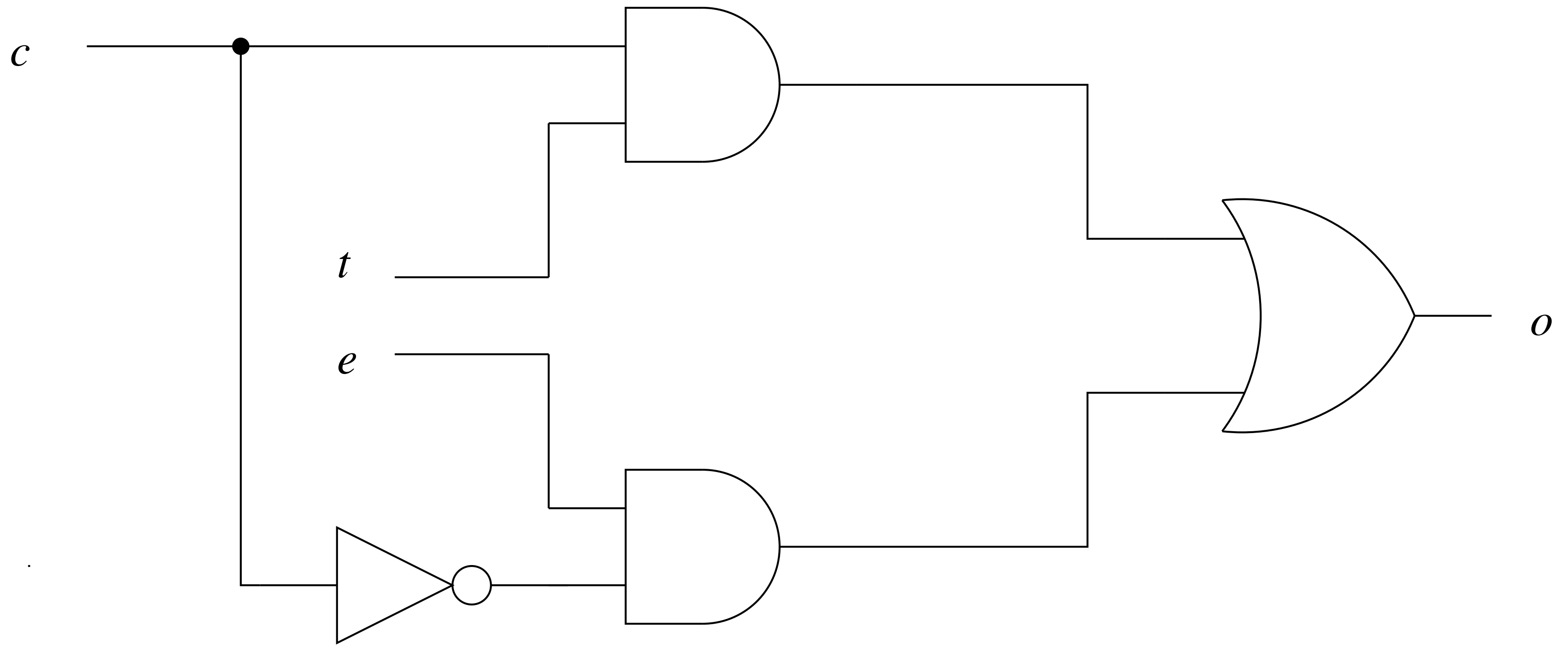
Redundancy Removal with D-Algorithm: Untestable



Redundancy Removal with D-Algorithm: Assume Fault



Redundancy Removal with D-Algorithm: Simplified Circuit



Redundancy Removal for SAT

- assume CNF is generated via Tseitin transformation from formula/circuit
  - formula = model constraints $+$ negation of property
  - CNF consists of gate input/output consistency constraints
  - plus additional unit forcing output o of whole formula to be 1
- remove redundancy in formula under assumption $o = 1$
- propagation of D or $\overline{D}$ to o does not make much sense
  - not interested in $o = 0$
  - check simply for unsatisfiability $\Rightarrow$ no need for D, $\overline{D}$ (!?)

Variable Instantiation

[AnderssonBjesseCookHanna DAC'02] and Oepir SAT solver

- satisfiability preserving transformation

- motivated by original pure literal rule:
  - if a literal l does not occur negatively in CNF f
  - then replace l by 1 in f (continue with $f[l \mapsto 1]$)

- generalization to variable instantiation:
  - if $f[l \mapsto 0] \rightarrow f[l \mapsto 1]$ is valid
  - then replace l by 1 in f (continue with $f[l \mapsto 1]$)

Why is Variable Instantiation a Generalization of the Pure Literal Rule?

Let $f \equiv f' \wedge f_0 \wedge f_1$ with

f' l does not occur

f_0 l occurs negatively

f_1 l occurs positively

further assume (assumption of pure literal rule)

$f_0 \equiv 1$

then

$$f[l \mapsto 0] \quad \Leftrightarrow \quad f' \wedge f_1[l \mapsto 0] \quad \overset{!}{\Rightarrow} \quad f' \quad \Leftrightarrow \quad f[l \mapsto 1]$$

Variable Instantiation Implementation

We have

$$f[l \mapsto 1] \quad \Leftrightarrow \quad f' \wedge \underbrace{f_1[l \mapsto 1]}_{1} \wedge f_0[l \mapsto 1] \quad \Leftrightarrow \quad f' \wedge f_0[l \mapsto 1] \quad \Leftrightarrow \quad f' \wedge \overbrace{\bigwedge_{i=1}^{n} C_i}^{f_0[l \mapsto 1]}$$

and since $f[l \mapsto 0] \Rightarrow f'$ we only need show the validity of

$$f[l \mapsto 0] \ \rightarrow \ \bigwedge_{i=1}^{n} C_i$$

which is equivalent to the unsatisfiability of

$$f[l \mapsto 0] \ \wedge \ \overline{C_i} \qquad \text{for } i = 1 \ldots n$$

which again is equivalent to the unsatisfiability of

$$f \ \wedge \ \bar{l} \ \wedge \ \overline{C_i} \qquad \text{for } i = 1 \ldots n$$

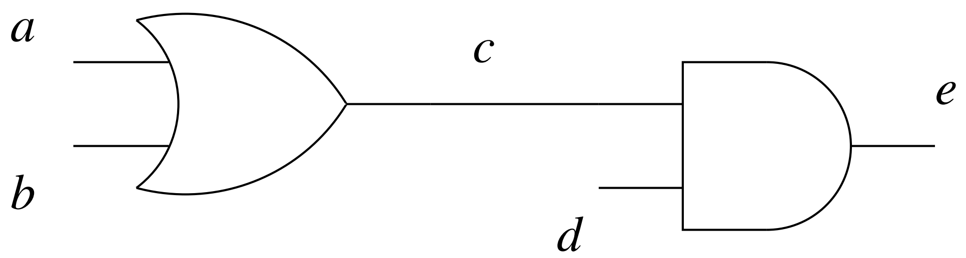
This can be done directly on the CNF and needs n unsatisfiability checks.

Variable Instantiation for Tseitin Encodings

$$(\overline{a} \vee c) \qquad (c \vee \overline{e})$$

$$(\overline{b} \vee c) \qquad (d \vee \overline{e})$$

$$(a \vee b \vee \overline{c}) \qquad (\overline{c} \vee \overline{d} \vee e)$$



$$\left.\begin{array}{l} \not\models \quad f \wedge \overline{c} \wedge \overline{(a \vee b)} \\ \not\models \quad f \wedge \overline{c} \wedge \overline{(\overline{d} \vee e)} \end{array}\right\} \Rightarrow \quad \text{add } c \text{ as unit}$$

requires two satisfiability checks while ATPG for c s-a-1 needs just one run

Stålmarck's Method and Recursive Learning

- orginally Stålmarck's Method works on "sea of triplets" [Stålmarck'89]

  $$x = x_1 @ \dots @ x_n \qquad \text{with } @ \text{ boolean operator}$$

  - equivalence reasoning + structural hashing + test rule
  - test rule translated to CNF f: $\quad f \Rightarrow (BCP(f \wedge x) \cap BCP(f \wedge \overline{x}))$

    add to f units that are implied by both cases x and $\overline{x}$

- Recursive Learning [KunzPradhan 90ties]

  - originally works on circuit structure
  - idea is to analyze all ways to justify a value, intersection is implied
  - translated to CNF f which contains clause $(l_1 \vee \dots \vee l_n)$

    BCP on all l_i separately and add intersection of derived units

Further CNF Simplification Techniques

- failed literals, various forms of equivalence reasoning

- hyper binary resolution [BacchusWinter'03,GershmanStrichman'05]
  - add binary clauses obtained through hyper resolution
  - avoid adding full transitive closure of implication chains
  - equivalence reasoning through SCC detection in binary clause graph
  - as Stålmarck's procedure subsumes structural hashing

- variable and clause elimination
  - autarkies and blocked clauses [Kullman]

Circuit based Simplification vs. CNF simplification

- circuit reasoning/simplification can use structure of circuit
  - graph structure (dominators)
  - notion of direction (forward and backward propagation)
  - partial models (some inputs do not need to be assigned)

- CNF simplification does not rely on circuit structure
  - orthogonal: can for instance remove individual clauses

- adapt ideas from circuit reasoning to SAT

  (e.g. avoid multiple SAT checks for redundancy removal in CNF)