Consistency Checking of All Different Constraints over Bit-Vectors within a SAT Solver

Armin Biere, Robert Brummayer

Institute for Formal Models and Verification
Johannes Kepler University
Linz, Austria

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- bounded model checking: [BiereCimattiClarkeZhu'99]

$$I(s_1) \wedge T(s_1, s_2) \wedge \ldots \wedge T(s_{k-1}, s_k) \wedge \bigvee_{0 \leq i \leq k} B(s_i)$$ satisfiable?

- reoccurrence diameter checking: [BiereCimattiClarkeZhu'99]

$$T(s_1, s_2) \wedge \ldots \wedge T(s_{k-1}, s_k) \;\wedge \bigwedge_{1 \leq i < j \leq k} s_i \neq s_j$$ unsatisfiable?

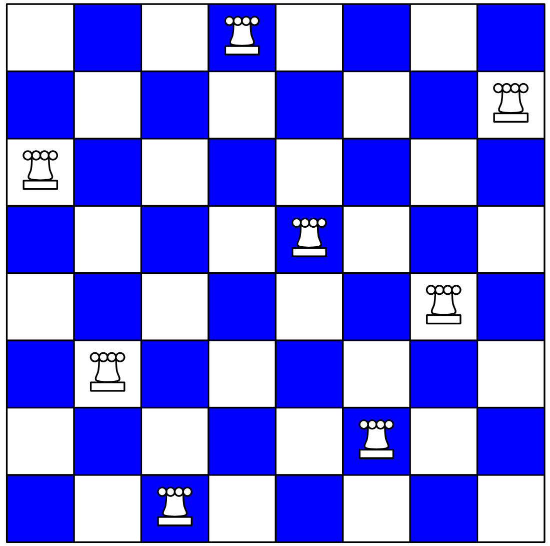
- k-induction base case: [SheeranSinghStålmarck'00]

$$I(s_1) \wedge T(s_1, s_2) \wedge \ldots \wedge T(s_{k-1}, s_k) \;\wedge B(s_k) \wedge \bigwedge_{0 \leq i < k} \neg B(s_i)$$ satisfiable?

- k-induction induction step: [SheeranSinghStålmarck'00]

$$T(s_1, s_2) \wedge \ldots \wedge T(s_{k-1}, s_k) \;\wedge B(s_k) \wedge \bigwedge_{0 \leq i < k} \neg B(s_i) \;\wedge \bigwedge_{1 \leq i < j \leq k} s_i \neq s_j$$ unsatisfiable?

- classical concept in constraint programming:
  - k variables over a domain of size m supposed to have different values
  - for instance k-queen problem

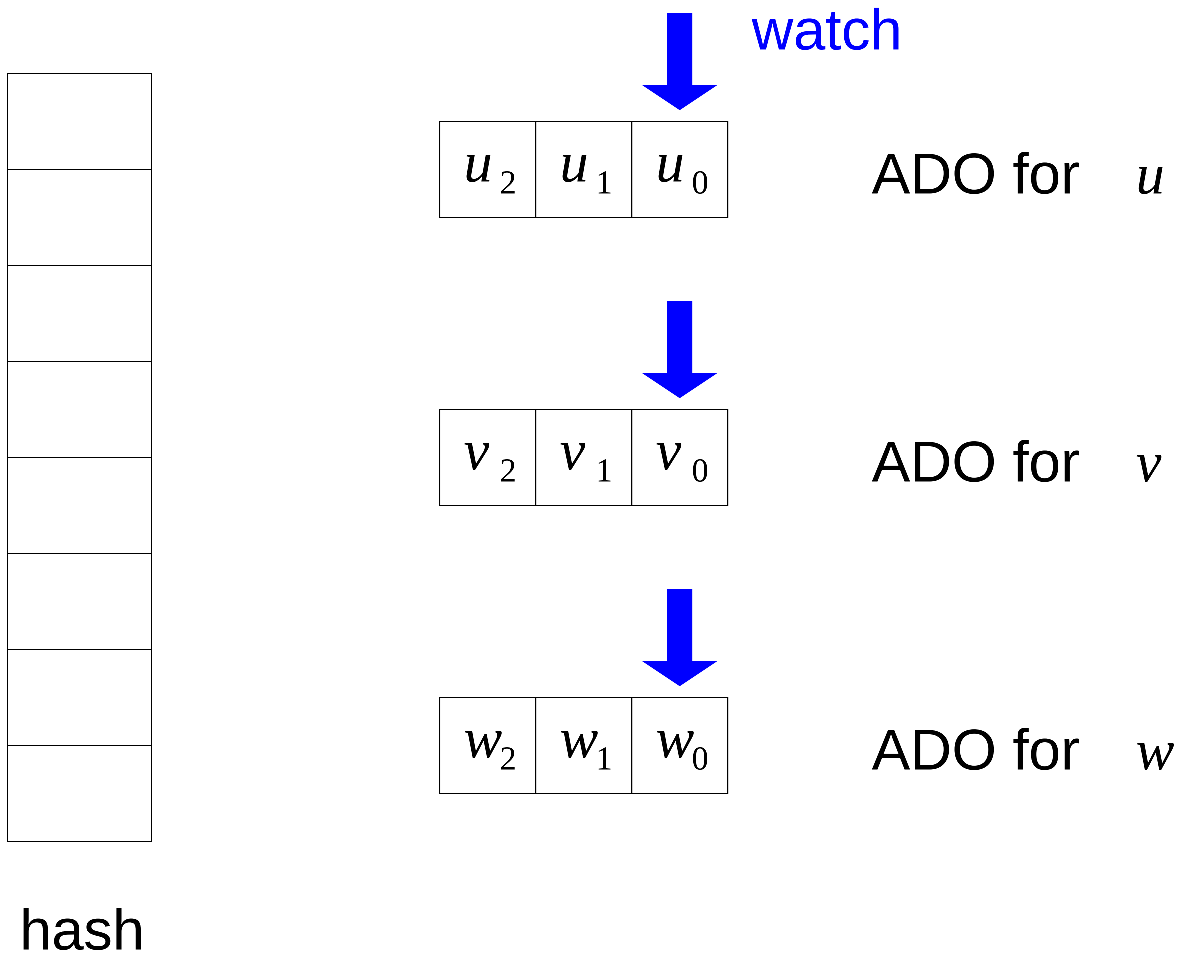


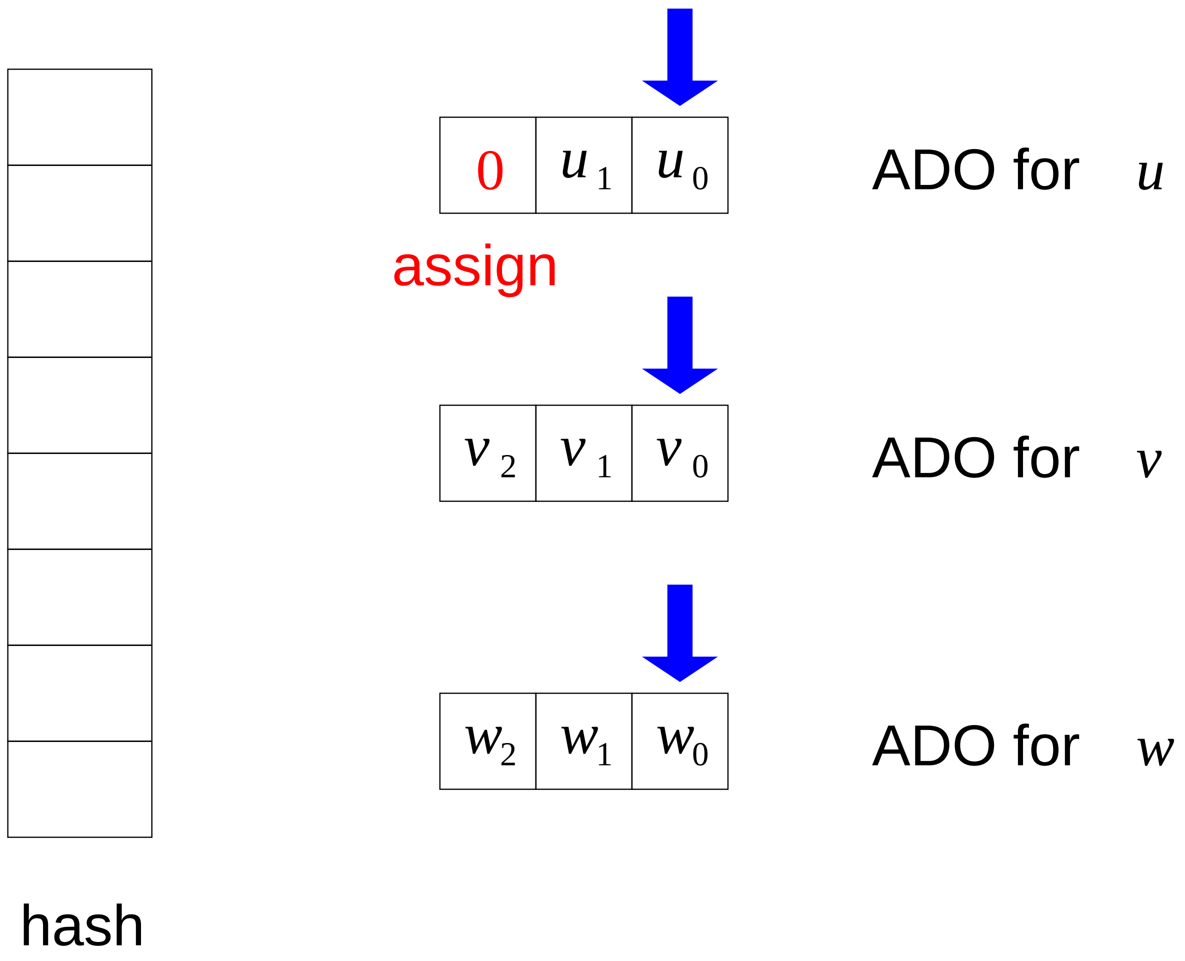
- propagation algorithms to establish arc-consistency
  - explicit propagators: [Régin'94]
    - $\mathrm{O}(k \cdot m)$ space
    - $\mathrm{O}(k^2 \cdot m^2)$ time
  - symbolic propagators: [GentNightingale'04] also [MarquesSilvaLynce'07]
    - one-hot CNF encoding with $\Omega(k \cdot m)$ boolean variables
- in model checking $k \ll m$ typically $k < 1000 \quad m = 2^n > 2^{100}$ n latches

- encoding bit-vector inequalities directly:
  - let u, v be two n-bit vectors, $d_0, \ldots, d_{n-1}$ fresh boolean variables

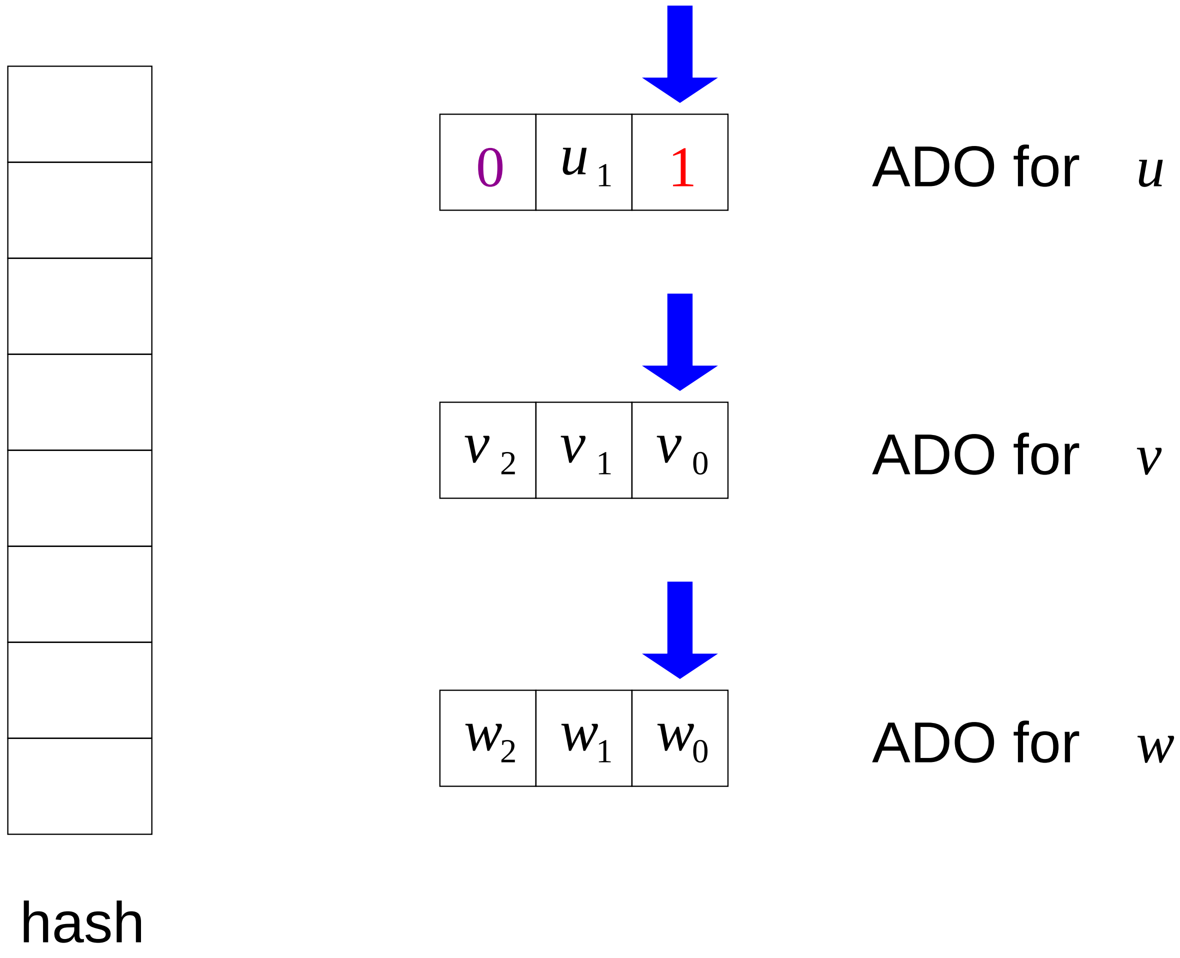
    $u \neq v$ is equisatisfiable to $(d_0 \vee \cdots \vee d_{n-1}) \wedge \bigwedge_{j=0}^{n-1} (u_j \vee v_j \vee \overline{d_j}) \wedge (\overline{u_j} \vee \overline{v_j} \vee \overline{d_j})$
  - can be extended to encode Ackermann Constraints + McCarthy Axioms
  - either **eagerly** encode all $s_i \neq s_j$ quadratic in k
  - or **refine** adding bit-vector inequalities on demand [EénSörensson'03]
- natively handle ADCs within SAT solver: our main contribution
  - similar to theory consistency checking in lazy SMT vs. “lemmas on demand”
  - can be extended to also perform theory propagation
- sorting networks ineffective in our experience [KröningStrichman'03,JussilaBiere'06]

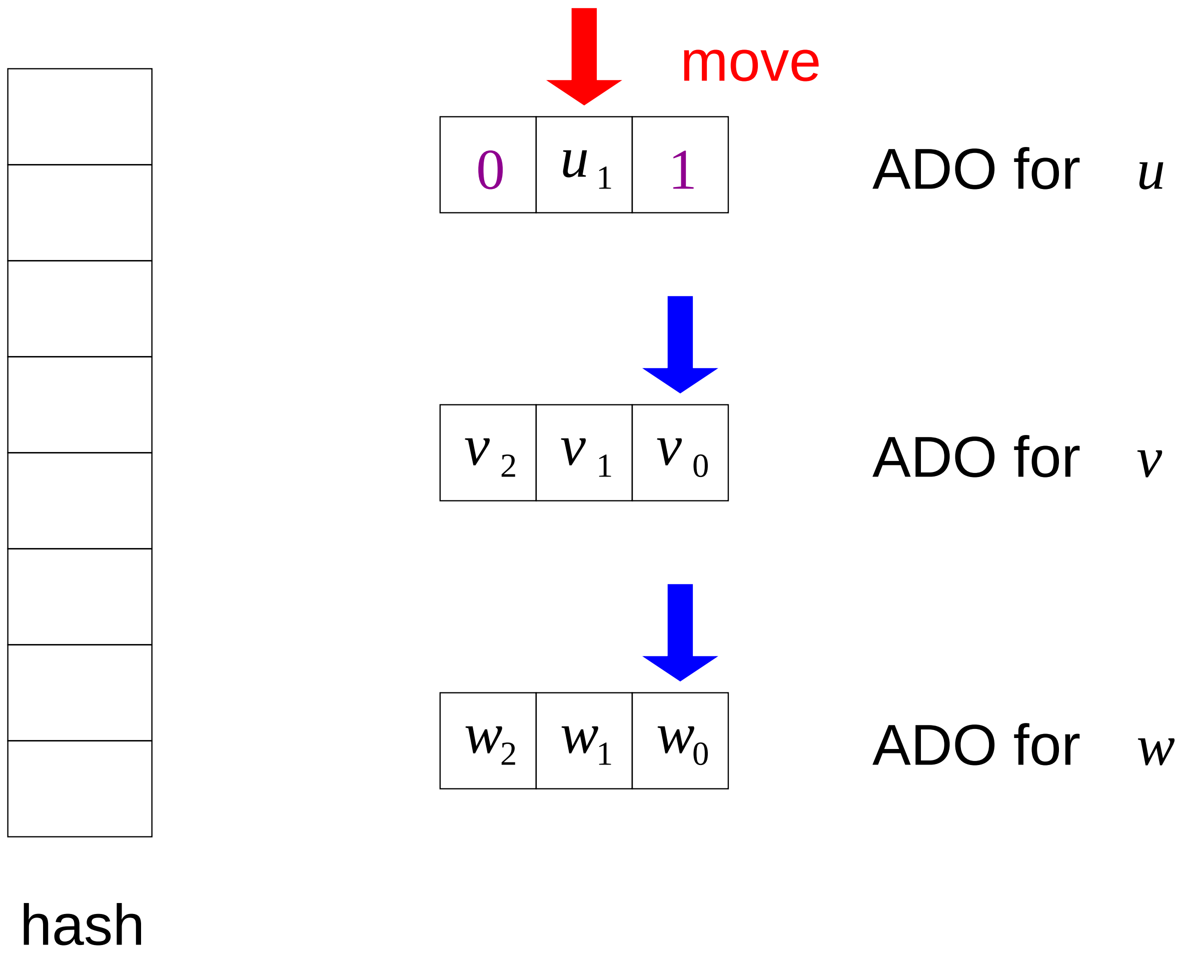
All Different Objects (ADOs)

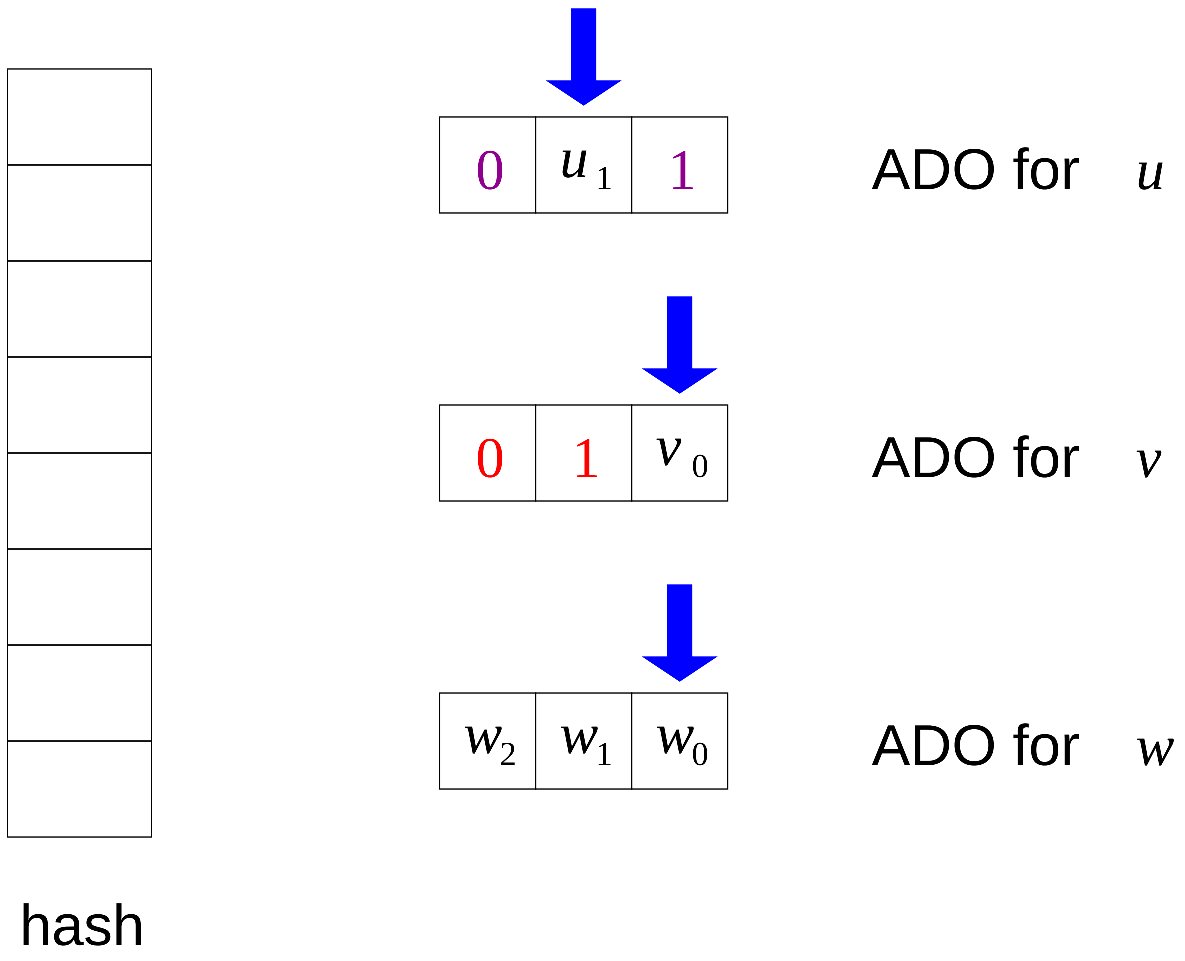


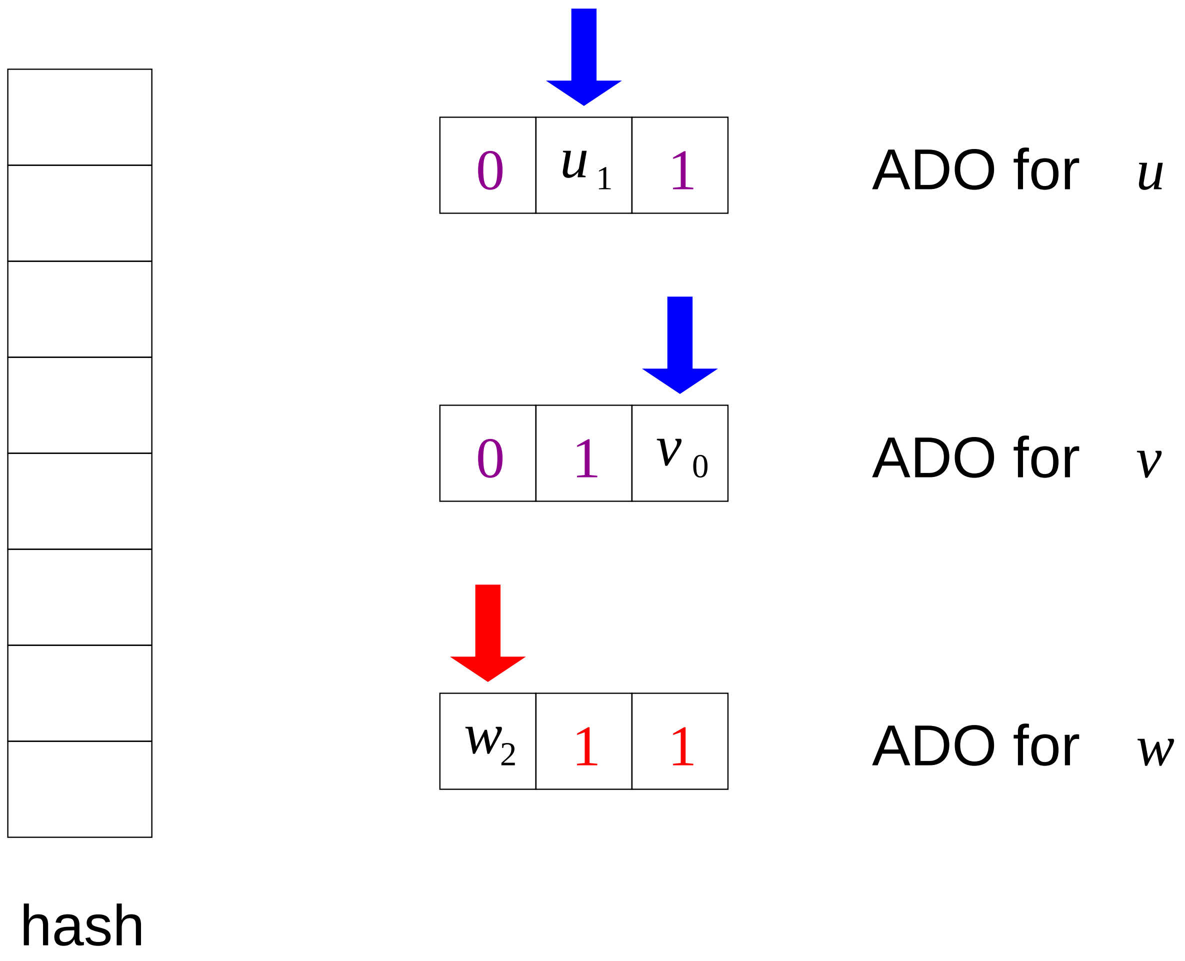

0
u_1
u_0
ADO for u
assign
v_2
v_1
v_0
ADO for v
w_2
w_1
w_0
ADO for w
hash

All Different Objects (ADOs)

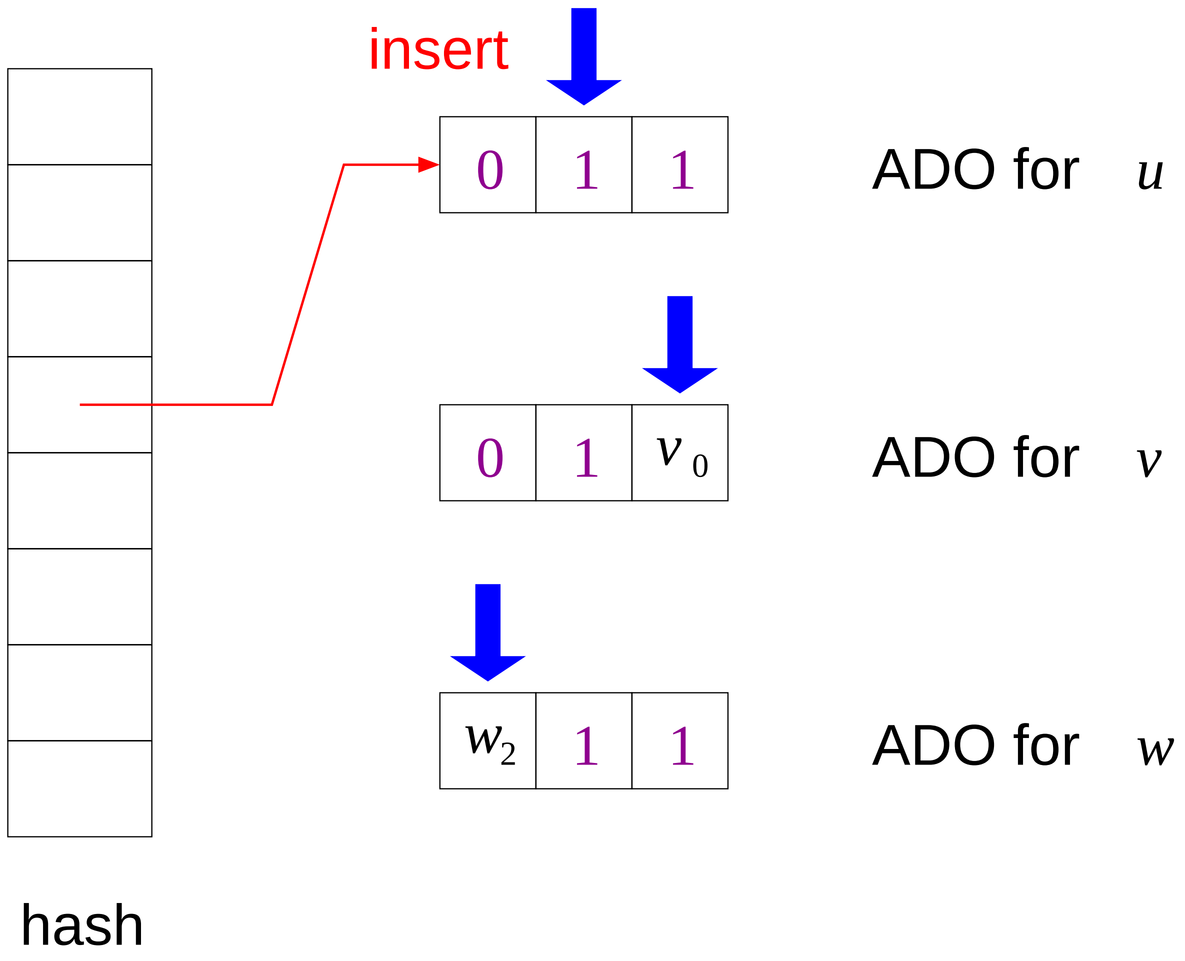



move
0
u_1
1
ADO for u
v_2
v_1
v_0
ADO for v
w_2
w_1
w_0
ADO for w
hash

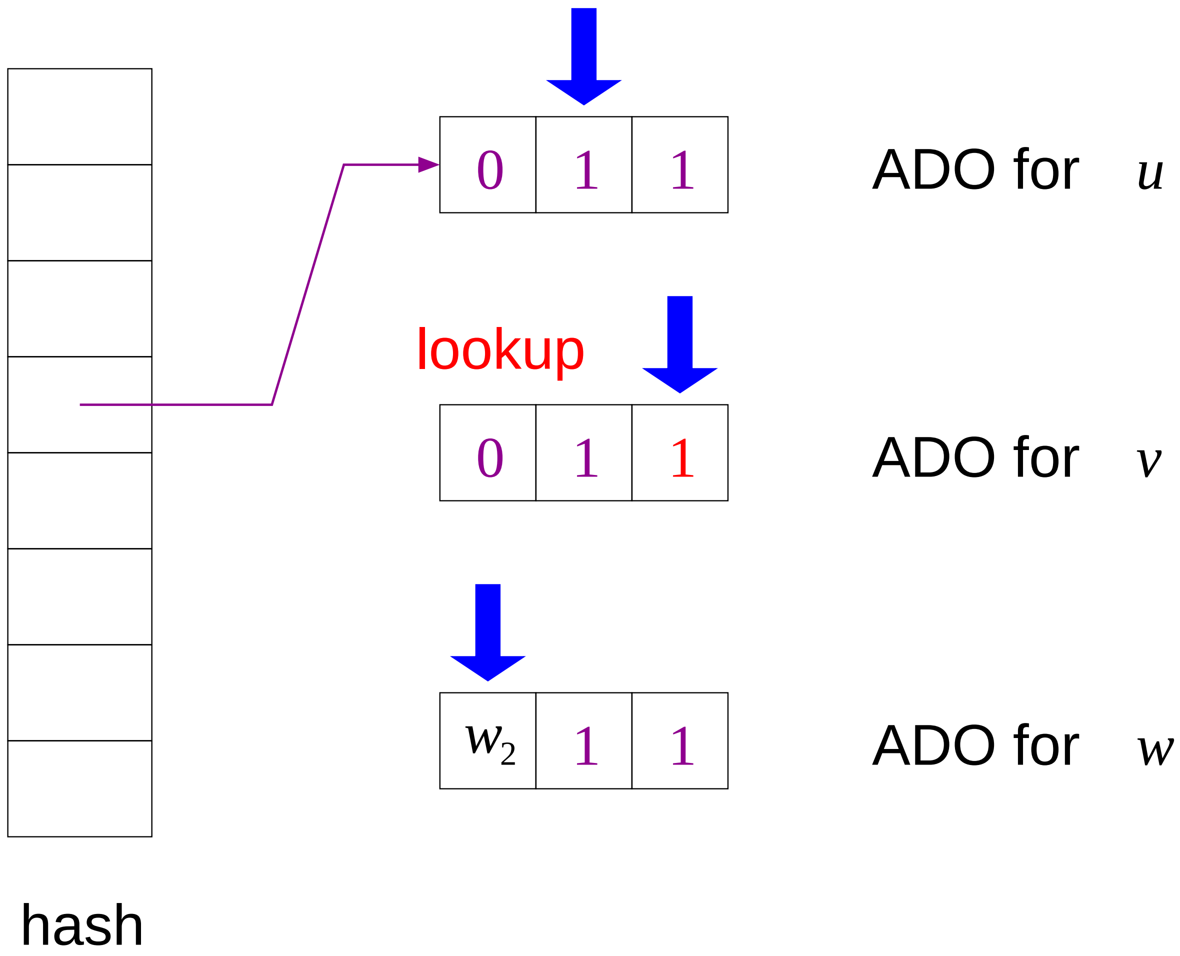

0
u_1
1
ADO for u
0
1
v_0
ADO for v
w_2
w_1
w_0
ADO for w
hash

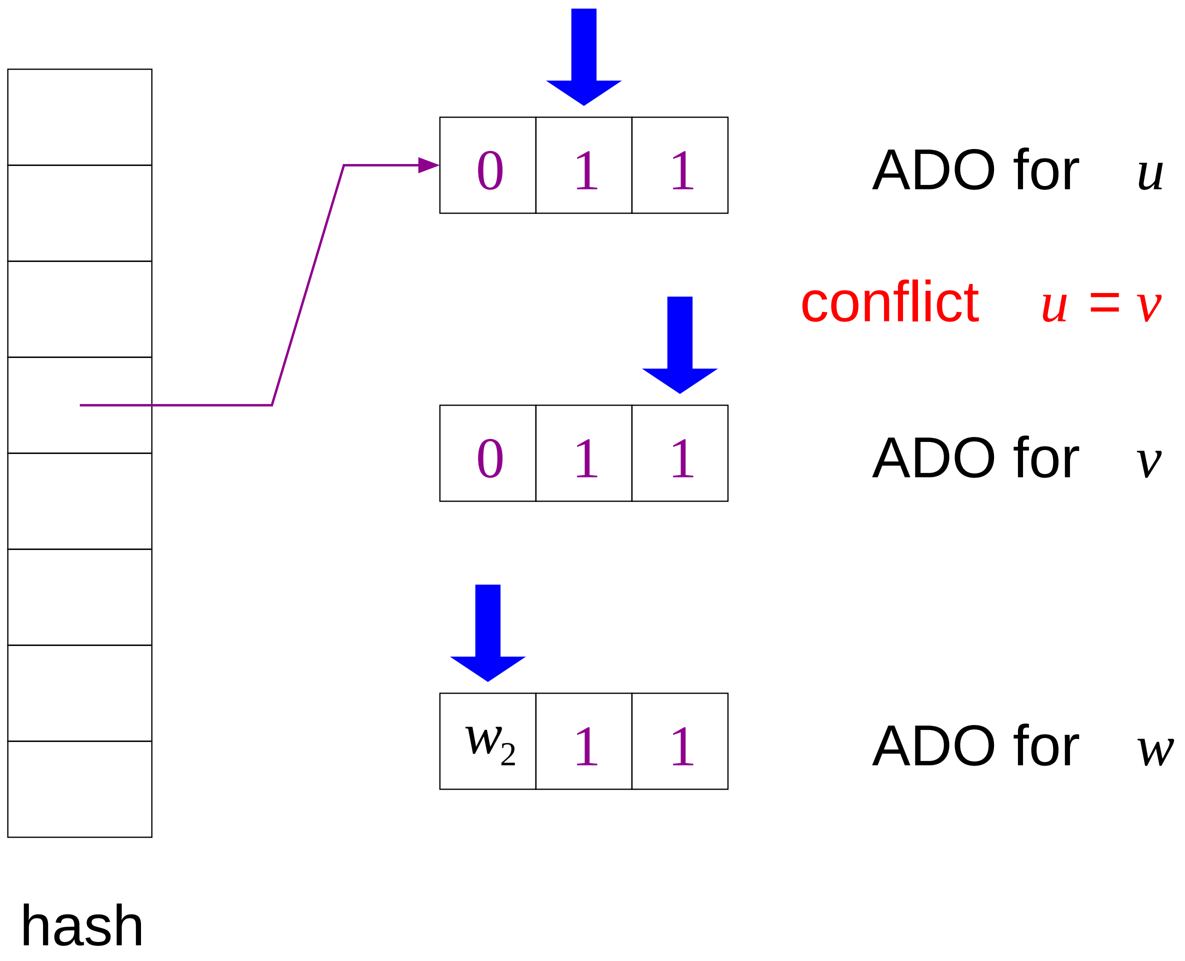

0
u_1
1
ADO for u
0
1
v_0
ADO for v
w_2
1
1
ADO for w
hash

complete
0 1 1
ADO for u
0 1 v_0
ADO for v
w_2 1 1
ADO for w
hash


insert
0
1
1
ADO for u
0
1
v_0
ADO for v
w_2
1
1
ADO for w
hash

All Different Objects (ADOs)




0 1 1
ADO for u
conflict $u = v$
0 1 1
ADO for v
w_2 1 1
ADO for w
hash

Implementation Details

- ADO key is calculated from concrete bit-vector
  - by for instance XOR'ing bits word by word

- ADOs watched by variables (not literals)
  - during backtracking all inserted ADOs need to be removed from hash table
  - save whether variable assignment forced ADO to be inserted
  - stack like insert/remove operations on hash table allow open addressing

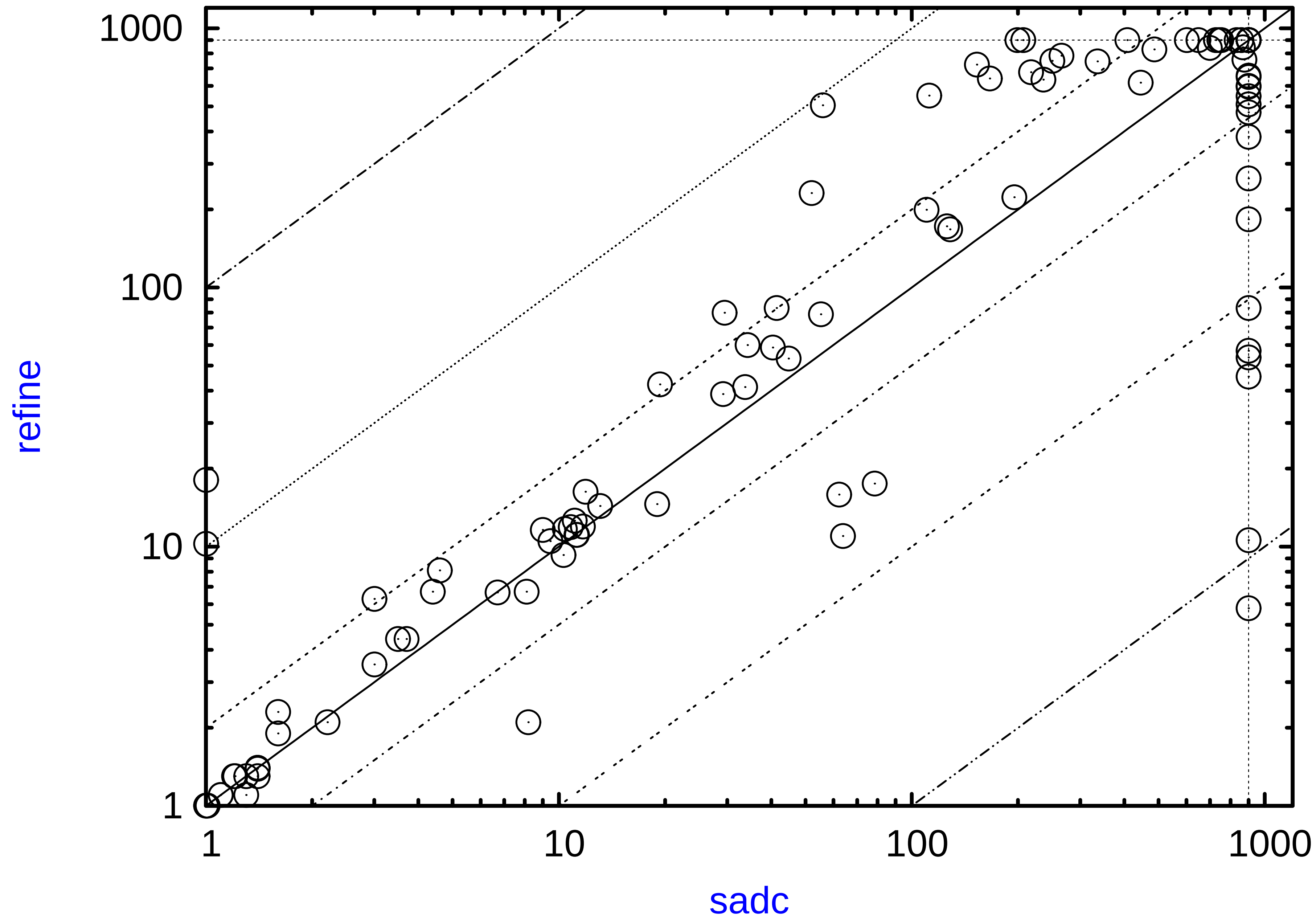
- conflict analysis
  - all bits of the bit-vectors in conflict are followed
  - can be implemented by temporarily generating a pseudo clause

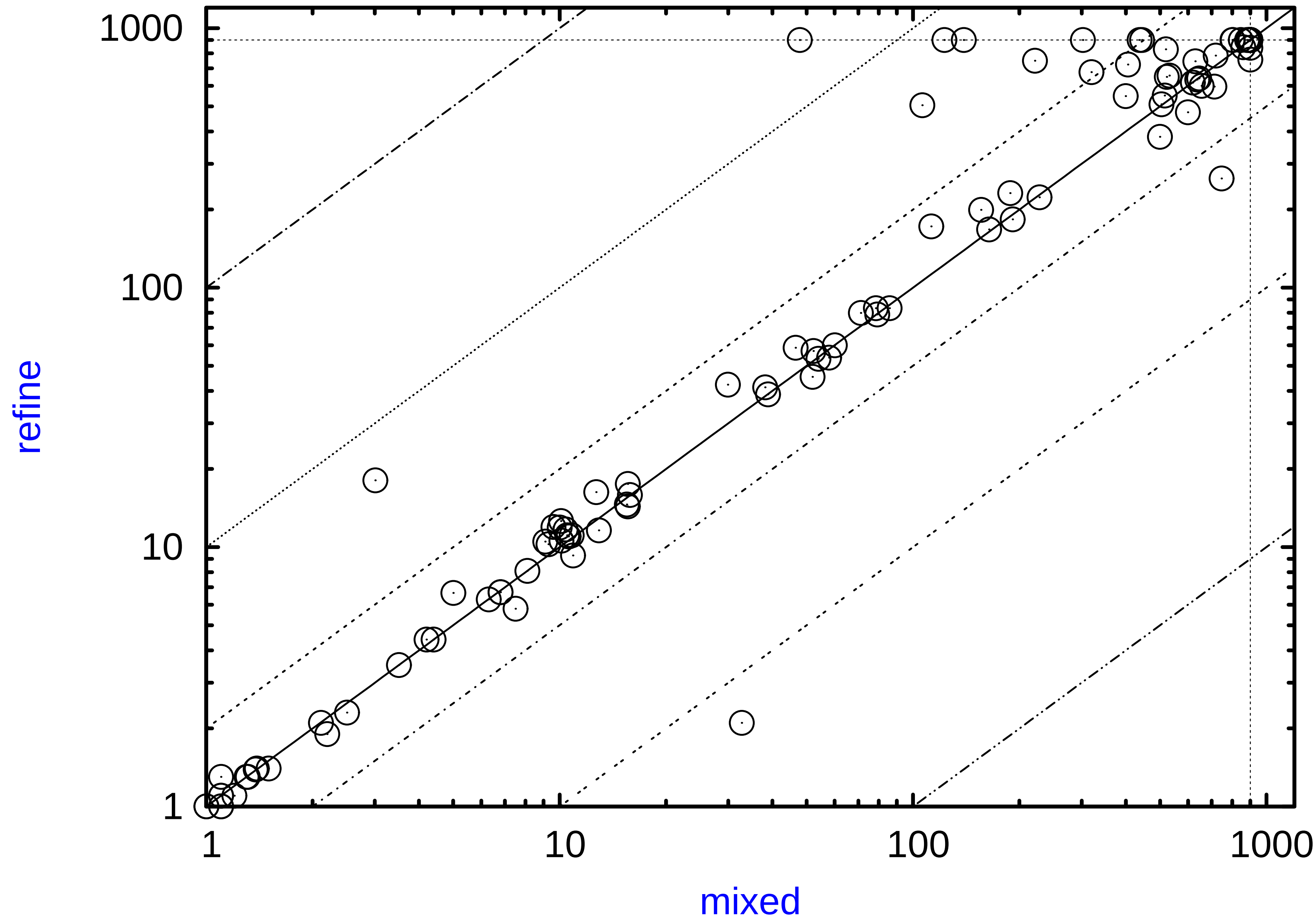
$$(u_2 \vee \overline{u}_1 \vee \overline{u}_0 \vee v_2 \vee \overline{v}_1 \vee \overline{v}_0)$$

		solved		*inconclusive*		*unsatisfiable*	*time*	*space*	*steps*
	complete		*unsolved*		*satisfiable*		10^3sec	GB	
mixed	y	259	85	38	182	39	96	23.2	9736
refine	y	250	94	32	179	39	101	23.0	9698
sadc	y	244	100	36	171	37	103	17.2	9131
eager	y	242	102	27	177	38	102	31.9	9438
none	n	267	77	56	179	32	87	16.6	10877
base	n	283	61	96	187	0	70	28.9	15187

only checked up to $k = 100$ (at most 100 steps per instance)

three possible outcomes: *inconclusive*, *satisfiable*, or *unsatisfiable*


refine
1000
100
10
1
1
10
100
1000
sadc


1000
100
10
1
1
10
100
1000
refine
mixed

- symbolic consistency checker for ADCs over bit-vectors
  - successfully applied to simple path constraints in model checking
  - similar to theory consistency checking in lazy SMT solvers
  - combination with eager refinement approach lemmas on demand
- future work: ADC based BCP for bit-vectors
  - aka theory propagation in lazy SMT solvers
  - extensions to handle Ackermann constraints or even McCarthy axioms
  - one-way to get away from pure bit-blasting in BV