Consistency Checking of All Different Constraints over Bit-Vectors within a SAT Solver

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Simple Path Constraints

- bounded model checking:  \[\text{[BiereCimattiClarkeZhu'99]}\]

\[
I(s_1) \land T(s_1, s_2) \land \ldots \land T(s_{k-1}, s_k) \land \bigvee_{0 \leq i \leq k} B(s_i) \text{ satisfiable?}
\]

- reoccurrence diameter checking:  \[\text{[BiereCimattiClarkeZhu'99]}\]

\[
T(s_1, s_2) \land \ldots \land T(s_{k-1}, s_k) \land \bigwedge_{1 \leq i < j \leq k} s_i \neq s_j \text{ unsatisfiable?}
\]

- \(k\)-induction base case:  \[\text{[SheeranSinghStålmarch'00]}\]

\[
I(s_1) \land T(s_1, s_2) \land \ldots \land T(s_{k-1}, s_k) \land B(s_k) \land \bigwedge_{0 \leq i < k} \neg B(s_i) \text{ satisfiable?}
\]

- \(k\)-induction induction step:  \[\text{[SheeranSinghStålmarch'00]}\]

\[
T(s_1, s_2) \land \ldots \land T(s_{k-1}, s_k) \land B(s_k) \land \bigwedge_{0 \leq i < k} \neg B(s_i) \land \bigwedge_{1 \leq i < j \leq k} s_i \neq s_j \text{ unsatisfiable?}
\]
All Different Constraints (ADC)

• classical concept in constraint programming:
  – $k$ variables over a domain of size $m$ supposed to have different values
  – for instance $k$-queen problem

• propagation algorithms to establish arc-consistency
  – explicit propagators: [Régin’94]
    * $O(k \cdot m)$ space
    * $O(k^2 \cdot m^2)$ time
  – symbolic propagators: [GentNightingale’04] also [MarquesSilvaLynce’07]
    * one-hot CNF encoding with $\Omega(k \cdot m)$ boolean variables

• in model checking $k \ll m$ typically $k < 1000 \quad m = 2^n > 2^{100}$ $n$ latches
Symbolic ADCs for Large Domains

- encoding bit-vector inequalities directly:
  - let $u, v$ be two $n$-bit vectors, $d_0, \ldots, d_{n-1}$ fresh boolean variables
    
    $u \neq v$ is equisatisfiable to $\left( \bigvee_{i=0}^{n-1} d_i \right) \land \bigwedge_{j=0}^{n-1} \left( u_j \lor v_j \lor \overline{d_j} \right) \land \left( \overline{u_j} \lor \overline{v_j} \lor \overline{d_j} \right)$

  - can be extended to encode Ackermann Constraints + McCarthy Axioms

  - either eagerly encode all $s_i \neq s_j$ quadratic in $k$

  - or refine adding bit-vector inequalities on demand [EénSörensson’03]

- natively handle ADCs within SAT solver: our main contribution

  - similar to theory consistency checking in lazy SMT vs. “lemmas on demand”

  - can be extended to also perform theory propagation

- sorting networks ineffective in our experience [KröningStrichman’03, JussilaBiere’06]
All Different Objects (ADOs)

watch

ADO for $u$

$u_2 \quad u_1 \quad u_0$

ADO for $v$

$v_2 \quad v_1 \quad v_0$

ADO for $w$

$w_2 \quad w_1 \quad w_0$

hash
All Different Objects (ADOs)

\[ v_2 \quad v_1 \quad v_0 \quad \text{ADO for } v \]

assign

\[ w_2 \quad w_1 \quad w_0 \quad \text{ADO for } w \]

hash

\[ u_0 \quad u_1 \quad u_0 \quad \text{ADO for } u \]
All Different Objects (ADOs)

ADO for $u$

$\begin{array}{c}
0 \\
u_1 \\
1
\end{array}$

ADO for $v$

$\begin{array}{c}
v_2 \\
v_1 \\
v_0
\end{array}$

ADO for $w$

$\begin{array}{c}
w_2 \\
w_1 \\
w_0
\end{array}$

hash
All Different Objects (ADOs)

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All Different Objects (ADOs)

ADO for $u$

0 | $u_1$ | 1

ADO for $v$

0 | 1 | $v_0$

ADO for $w$

$w_2$ | $w_1$ | $w_0$

hash
All Different Objects (ADOs)

<table>
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ADO for \( u \):

\[
\begin{array}{c|c|c}
0 & u_1 & 1 \\
\end{array}
\]

ADO for \( v \):

\[
\begin{array}{c|c|c}
0 & 1 & v_0 \\
\end{array}
\]

ADO for \( w \):

\[
\begin{array}{c|c|c}
w_2 & 1 & 1 \\
\end{array}
\]
All Different Objects (ADOs)

hash

complete

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ADO for \( u \)

<table>
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<th>( v_0 )</th>
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ADO for \( v \)

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ADO for \( w \)
All Different Objects (ADOs)

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All Different Objects (ADOs)

ADO for $u$

ADO for $v$

ADO for $w$

hash

lookup
All Different Objects (ADOs)

ADO for $u$

conflict  $u = v$

ADO for $v$

ADO for $w$

hash

$u = v$ conflict
• ADO key is calculated from concrete bit-vector
  – by for instance XOR’ing bits word by word

• ADOs watched by variables (not literals)
  – during backtracking all inserted ADOs need to be removed from hash table
  – save whether variable assignment forced ADO to be inserted
  – stack like insert/remove operations on hash table allow open addressing

• conflict analysis
  – all bits of the bit-vectors in conflict are followed
  – can be implemented by temporarily generating a pseudo clause

\[(u_2 \lor \overline{u}_1 \lor \overline{u}_0 \lor v_2 \lor \overline{v}_1 \lor \overline{v}_0)\]
Overall Results on all 344 HWMCC’07 Benchmarks

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<th>space GB</th>
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only checked up to $k = 100$ (at most 100 steps per instance)

three possible outcomes: inconclusive, satisfiable, or unsatisfiable
Symbolic ADCs versus Refine

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Mixed Approach versus Refine Only

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Conclusion

• symbolic consistency checker for ADCs over bit-vectors
  – successfully applied to simple path constraints in model checking
  – similar to theory consistency checking in lazy SMT solvers
  – combination with eager refinement approach lemmas on demand

• future work: ADC based BCP for bit-vectors
  – aka theory propagation in lazy SMT solvers
  – extensions to handle Ackermann constraints or even McCarthy axioms
  – one-way to get away from pure bit-blasting in BV