Local Minimization of And-Inverter Graphs

Armin Biere

Institute for Formal Models and Verification
Johannes Kepler University Linz, Austria

Guangzhou Symposium on Satisfiability and
Logic-Based Modeling 2006

Sun Yat-sen University
Guangzhou, China
September 2006

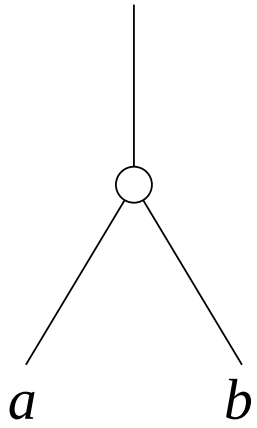
Joint Work with Robert Brummayer

And-Inverter Graphs (AIGs)

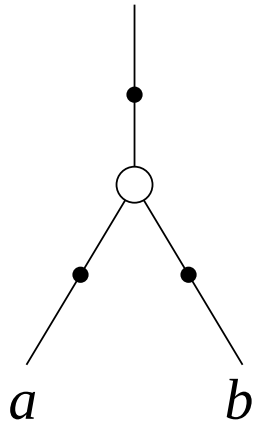
A. Biere, FMV, JKU Linz

[KühlmannParuthiKrohmGanai-TCAD02]

Name	Function	Representation by two-input AND and inversion
Inversion	$\neg a$	$\neg a$
Conjunction	$a \wedge b$	$a \wedge b$
Disjunction	$a \vee b$	$\neg(\neg a \wedge \neg b)$
Implication	$a \rightarrow b$	$\neg(a \wedge \neg b)$
Equivalence	$a \leftrightarrow b$	$\neg(a \wedge \neg b) \wedge \neg(\neg a \wedge b)$
Xor	$a \oplus b$	$\neg(\neg(a \wedge \neg b) \wedge \neg(\neg a \wedge b))$



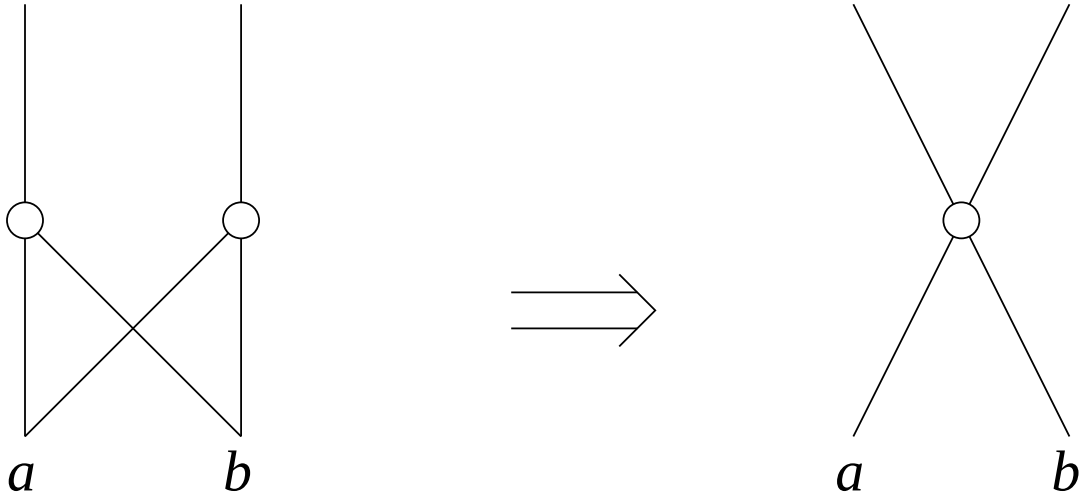
$a \wedge b$



$a \vee b$

Structural Hashing – Algebraic Reduction

A. Biere, FMV, JKU Linz



Motivation

- intermediate representation for synthesis and verification
  - compact, fast algorithms (compared to BDDs)
  - SAT: last step before CNF generation
  - combinational or sequential
- related data structures:
  - Boolean Expression Diagrams (BED) [AndersenHulgard-LICS97]
  - Reduced Boolean Circuits (RBC) [AbdullaBjesseEén-TACAS00]
  - XOR operator in addition and (originally) explicit NOT operator

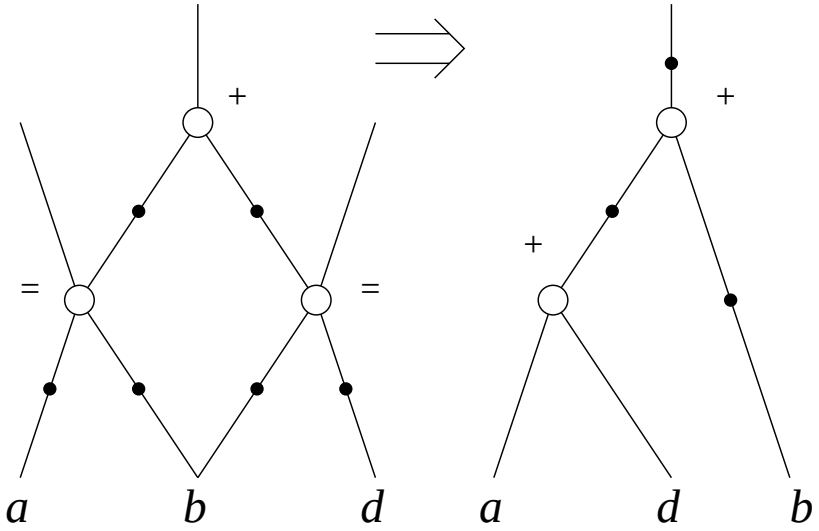
Local Minimization Through Local Normalization

A. Biere, FMV, JKU Linz

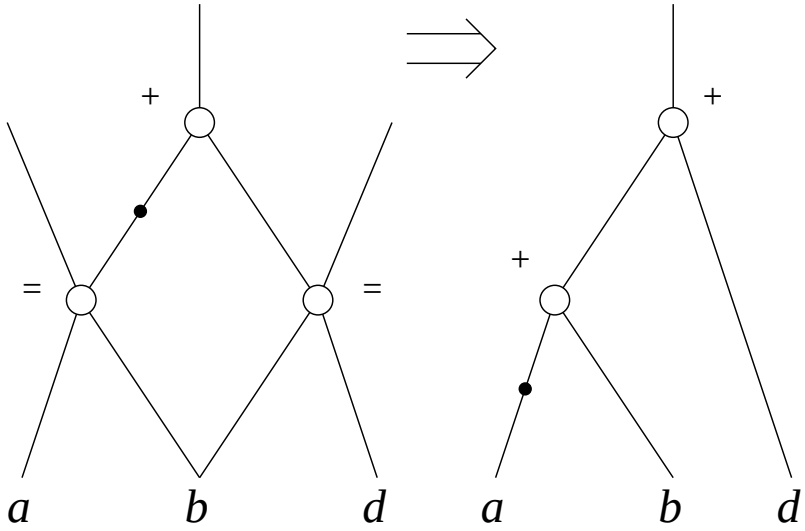
- original approach for BEDs, RBCs and AIGs
- generate **all** AIGs of height 2
  - by enumeration through a program
  - use of symmetry possible, but not necessary in this case
- some are equivalent as boolean functions
  - only 742 out of 2^{16} boolean functions have an AIG of height 2
  - pick one representative of minimum size
- always rewrite new nodes

Local Minimization $\neq$ Global Minimization

A. Biere, FMV, JKU Linz



distributivity law



harmful substitution variant

$$(a \vee b) \wedge (b \vee d) \equiv (a \wedge d) \vee b$$

$$(\neg a \vee \neg b) \wedge (b \wedge d) \equiv (\neg a \wedge b) \wedge d$$

Global Rewriting

- instead of one representative try all [BjesseBorälv-ICCAD04]
  - minimize global reference counts when rewriting
  - iterate and rebuild AIGs multiple times
- use arbitrary 4-cut [MishchenkoChatterjeeBrayton-DAC06]
  - impressive result for synthesis
  - simpler, faster, more efficient than SIS
- unclear whether global rewriting necessary?

Local Minimization without Blowup

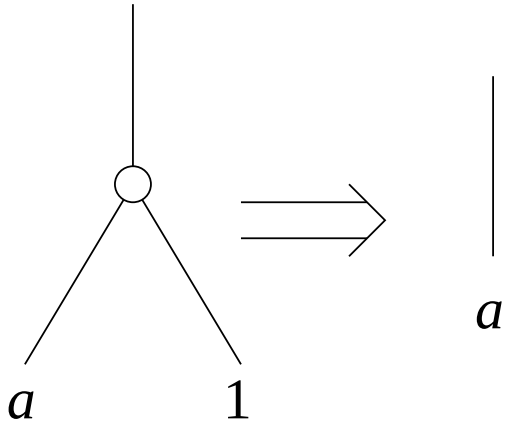
A. Biere, FMV, JKU Linz

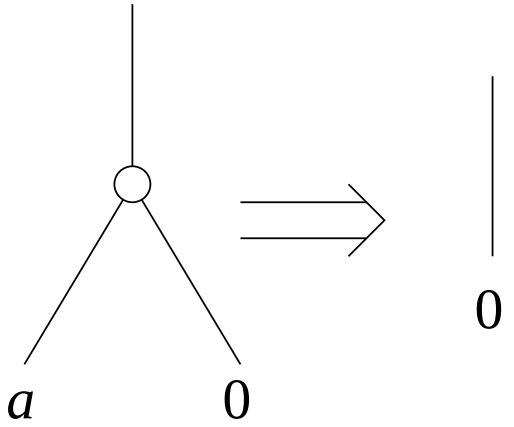
[Brummayer'06, BrummayerBiere'06]

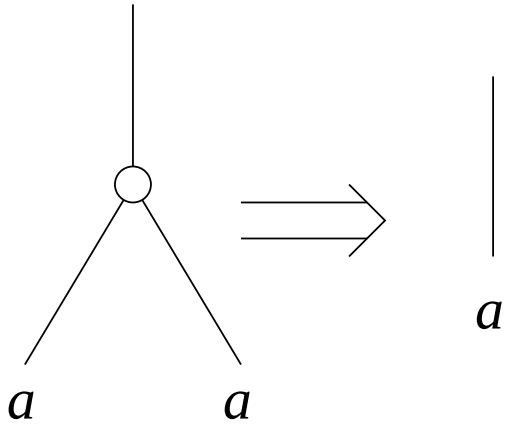
Name	LHS	RHS	O	S	Condition
Neutrality	$a \wedge 1$	a	1	S	
Boundedness	$a \wedge 0$	0	1	S	
Idempotence	$a \wedge b$	a	1	S	$a = b$
Contradiction	$a \wedge \neg b$	0	1	S	$a = b$
Contradiction	$(a \wedge b) \wedge c$	0	2	A	$(a \neq c) \vee (b \neq c)$
Contradiction	$(a \wedge b) \wedge (c \wedge d)$	0	2	S	$(a \neq c) \vee (a \neq d) \vee (b \neq c) \vee (b \neq d)$
Subsumption	$\neg(a \wedge b) \wedge c$	c	2	A	$(a \neq c) \vee (b \neq c)$
Subsumption	$\neg(a \wedge b) \wedge (c \wedge d)$	$c \wedge d$	2	S	$(a \neq c) \vee (a \neq d) \vee (b \neq c) \vee (b \neq d)$
Idempotence	$(a \wedge b) \wedge c$	$a \wedge b$	2	A	$(a = c) \vee (b = c)$
Resolution	$\neg(a \wedge b) \wedge \neg(c \wedge d)$	$\neg a$	2	S	$(a = d) \wedge (b \neq c)$
Substitution	$\neg(a \wedge b) \wedge c$	$\neg a \wedge b$	3	A	$b = c$
Substitution	$\neg(a \wedge b) \wedge (c \wedge d)$	$\neg a \wedge (c \wedge d)$	3	S	$b = c$
Idempotence	$(a \wedge b) \wedge (c \wedge d)$	$(a \wedge b) \wedge d$	4	S	$(a = c) \vee (b = c)$
Idempotence	$(a \wedge b) \wedge (c \wedge d)$	$a \wedge (c \wedge d)$	4	S	$(b = c) \vee (b = d)$
Idempotence	$(a \wedge b) \wedge (c \wedge d)$	$(a \wedge b) \wedge c$	4	S	$(a = d) \vee (b = d)$
Idempotence	$(a \wedge b) \wedge (c \wedge d)$	$b \wedge (c \wedge d)$	4	S	$(a = c) \vee (a = d)$

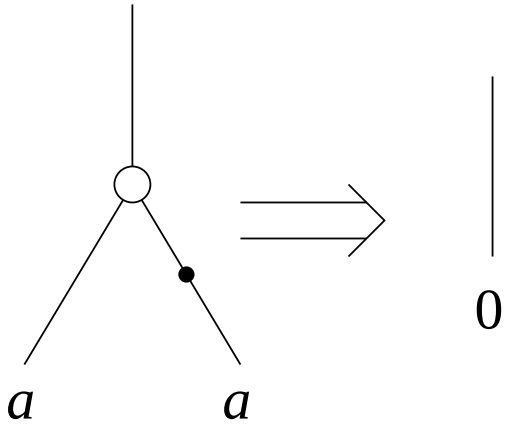
One-Level AIG Optimization (O1)

A. Biere, FMV, JKU Linz









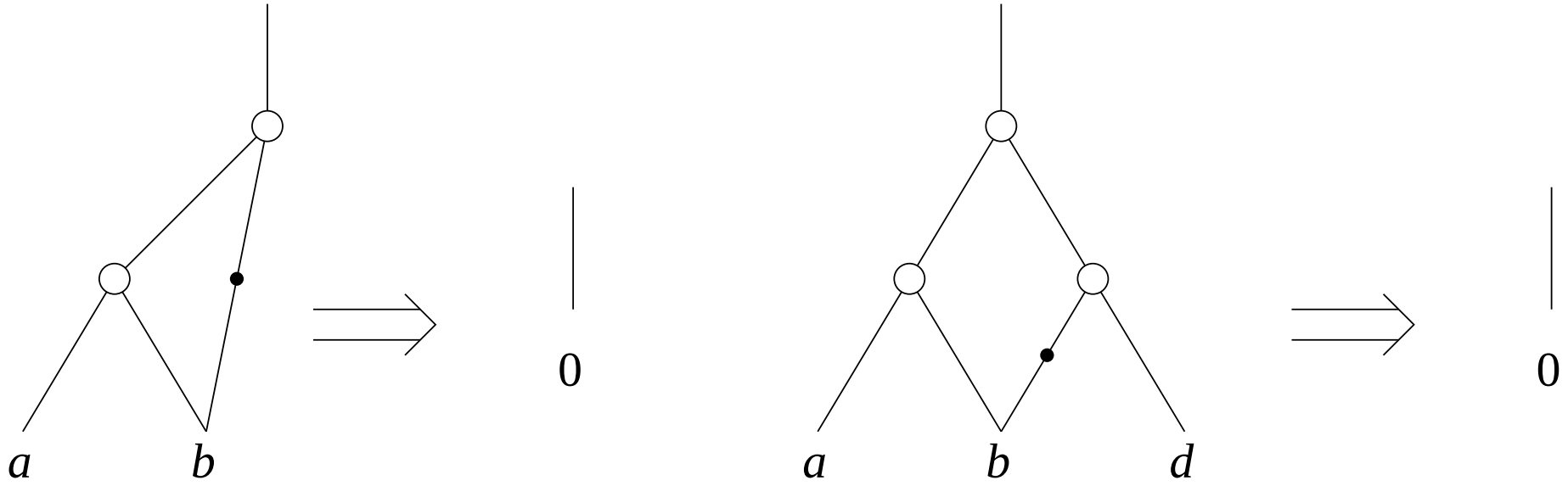
Two-Level AIG Optimizations

A. Biere, FMV, JKU Linz

- two-level AIG as left-hand side (LHS) of rewrite rule
- right-hand side (RHS) has less nodes than LHS locally smaller
- RHS has at most one new node globally not size increasing
- asymmetric (2 ANDs) or symmetric shape (3 ANDs)
- O1 optimization already done (to children and LHS top level node)
- at least one grand-children shared

Contradiction (O2)

A. Biere, FMV, JKU Linz



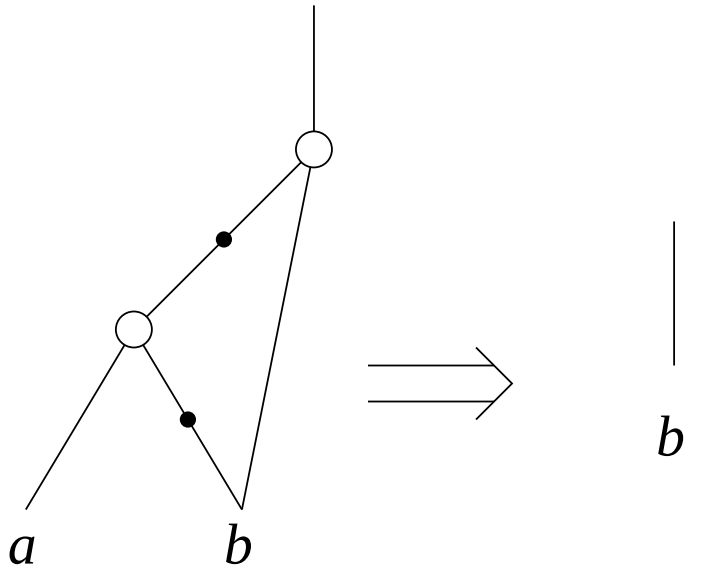
asymmetric

symmetric

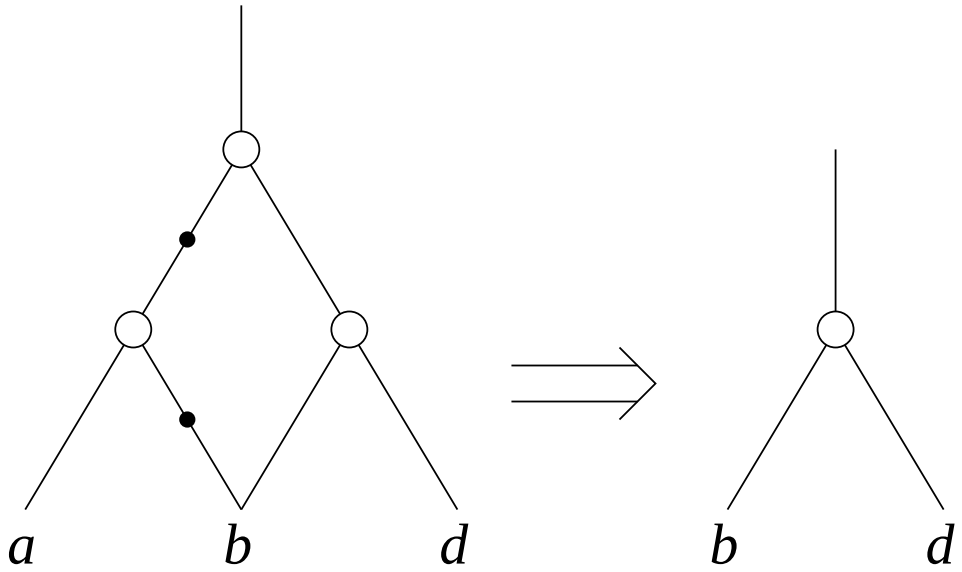
Subsumption (O2)

A. Biere, FMV, JKU Linz

actually unit subsumption



asymmetric



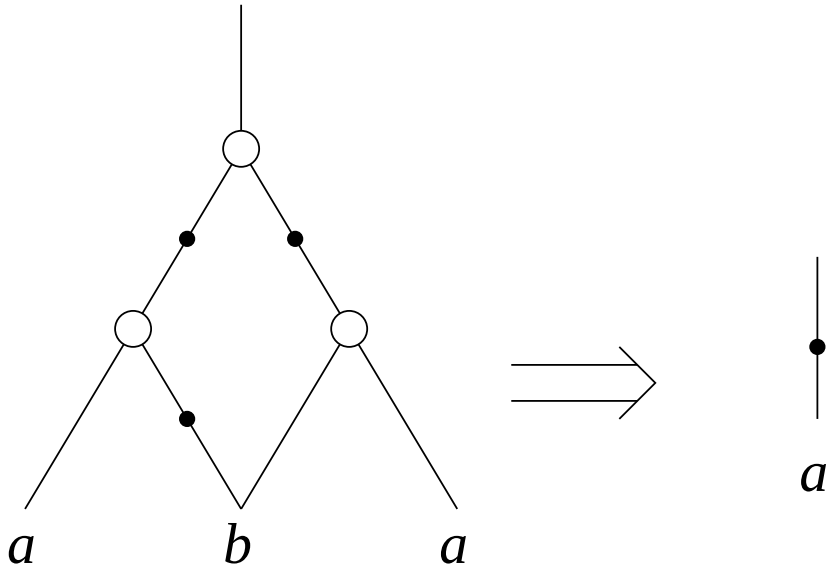
symmetric

$$(\neg a \vee b) \wedge b \equiv b$$

Resolution (O2)

A. Biere, FMV, JKU Linz

actually instance of hyper unary resolution

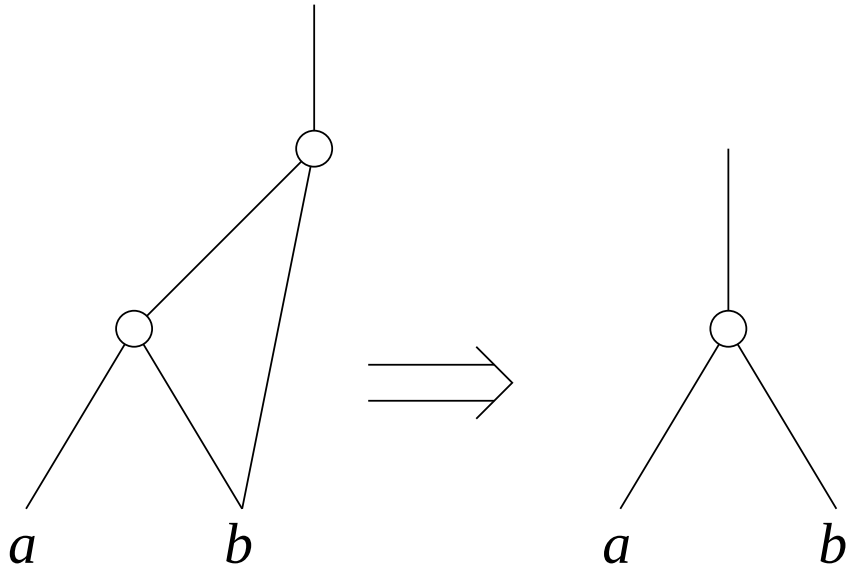


symmetric

$$(\neg a \vee b) \wedge (\neg b \vee \neg a) \equiv \neg a$$

Asymmetric Idempotence (O2)

A. Biere, FMV, JKU Linz

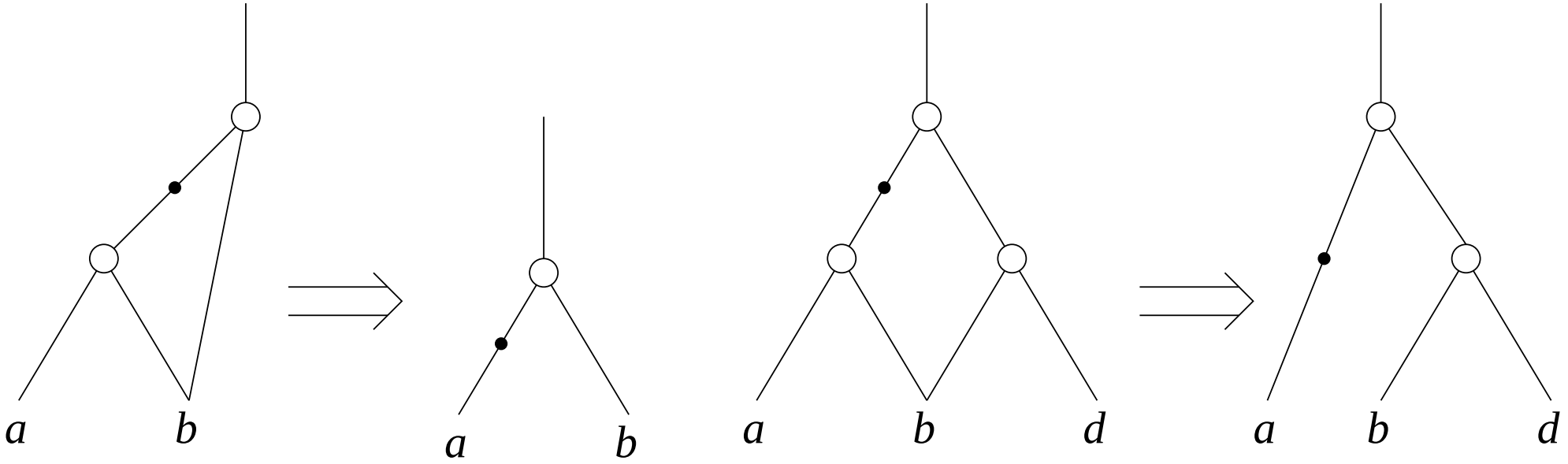


asymmetric

Substitution (O3)

A. Biere, FMV, JKU Linz

one new node



asymmetric

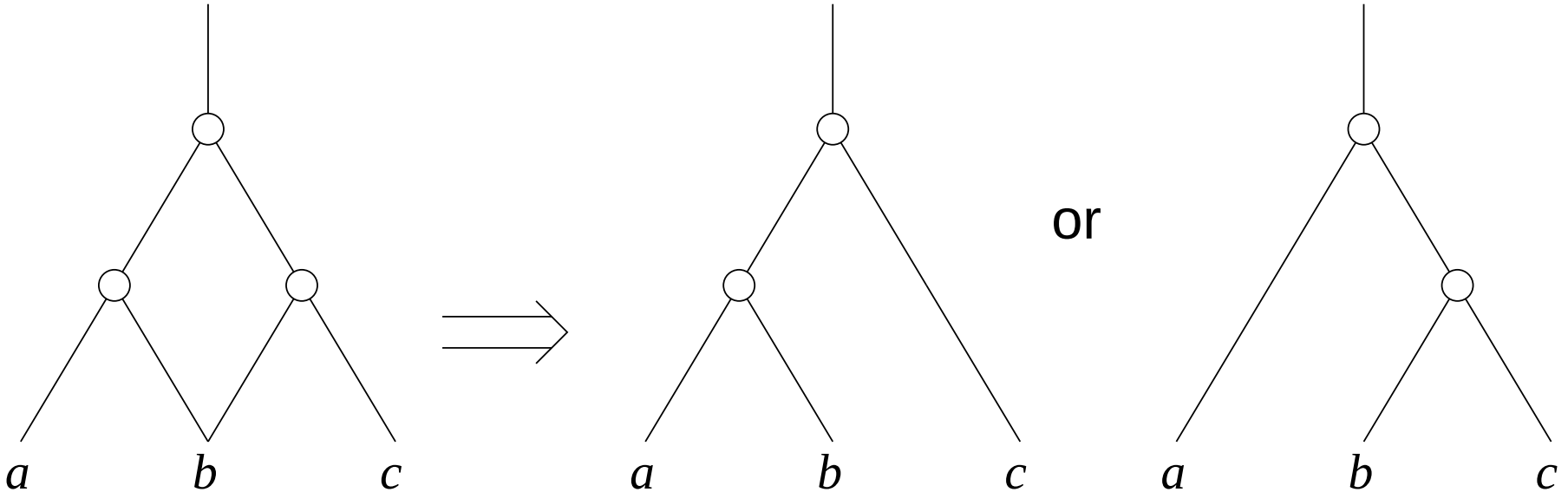
symmetric

$$(\neg a \vee \neg b) \wedge b \equiv \neg a \wedge b$$

Symmetric Idempotence (O4)

A. Biere, FMV, JKU Linz

non confluent



symmetric

Optimization Levels

A. Biere, FMV, JKU Linz

O1 greedy one-level AIG simplification/normalization

O2 local reduction without new node generation

O3 local reduction with node generation (confluent?)

O4 local reduction with node generation (non confluent!)

Og greedy Normalization [KühlmannParuthiKrohmGanai-TCAD02]

		number AIG nodes					reduction in addition to O1			
	k	O1	O2	O3	O4	Og	O2	O3	O4	Og
eijk.S298.S	58	257579	**257351**	**257351**	**257351**	260744	**0.1%**	**0.1%**	**0.1%**	-1.2%
eijk.S953.S	7	10311	**10236**	**10236**	**10236**	11791	**0.7%**	**0.7%**	**0.7%**	-14.4%
eijk.S820.S	11	18266	18111	**18071**	18091	19843	0.8%	**1.1%**	1.0%	-8.6%
eijk.S510.S	10	14519	14375	14375	**14365**	16010	1.0%	1.0%	**1.1%**	-10.3%
eijk.S832.S	11	19434	**19194**	**19194**	19215	21040	**1.2%**	**1.2%**	1.1%	-8.3%
cmu.periodic.N	96	733095	733095	**719724**	**719724**	721452	0.0%	**1.8%**	**1.8%**	1.6%
nusmv.guid7.C	27	155260	154203	**152069**	**152069**	167223	0.7%	**2.1%**	**2.1%**	-7.7%
ken.oop1.C	29	65855	64605	**63674**	**63674**	65444	1.9%	**3.3%**	**3.3%**	0.6%
nusmv.guid1.C	10	28889	28520	**27721**	**27721**	32313	1.3%	**4.0%**	**4.0%**	-11.9%
nusmv.tcas2.B	6	25999	24657	24225	**24198**	26254	5.2%	6.8%	**6.9%**	-1.0%
nusmv.tcas3.B	5	20178	19023	18644	**18618**	20409	5.7%	7.6%	**7.7%**	-1.1%
vis.prodc24.E	37	297791	289394	**257570**	**257570**	270317	2.8%	**13.5%**	**13.5%**	9.2%
vis.prodc12.E	29	204807	198235	**173352**	**173352**	183219	3.2%	**15.4%**	**15.4%**	10.5%
vis.prodc17.E	27	183883	177779	**154670**	**154670**	163807	3.3%	**15.9%**	**15.9%**	10.9%
vis.prodc15.E	23	144774	139602	**120066**	**120066**	127763	3.6%	**17.1%**	**17.1%**	11.8%
vis.prodc19.E	22	135975	131023	**112273**	**112273**	119722	3.6%	**17.4%**	**17.4%**	12.0%
texas.par2.E	2	1009	992	**813**	**813**	872	1.7%	**19.4%**	**19.4%**	13.6%
vis.prodc14.E	16	86137	82589	**69211**	**69211**	74441	4.1%	**19.7%**	**19.7%**	13.6%
vis.prodc18.E	13	64185	61385	**50755**	**50755**	54901	4.4%	**20.9%**	**20.9%**	14.5%
vis.prodc13.E	8	33849	32161	**25788**	**25788**	28195	5.0%	**23.8%**	**23.8%**	16.7%
vis.prodc16.E	5	18217	17229	**13510**	**13510**	14819	5.4%	**25.8%**	**25.8%**	18.7%

		generation time					solving time				
	k	O1	O2	O3	O4	Og	O1	O2	O3	O4	Og
eijk.S298.S	58	**1.63**	1.68	1.66	1.65	1.92	20.58	15.64	19.54	**15.02**	22.44
eijk.S953.S	7	**0.05**	**0.05**	**0.05**	**0.05**	0.06	0.41	0.40	0.38	**0.34**	0.50
eijk.S820.S	11	**0.08**	**0.08**	**0.08**	**0.08**	0.11	**0.75**	0.82	0.85	0.83	1.54
eijk.S510.S	10	**0.06**	**0.06**	**0.06**	**0.06**	0.08	0.74	0.70	**0.69**	0.78	0.97
eijk.S832.S	11	**0.09**	**0.09**	**0.09**	**0.09**	0.11	0.95	0.93	**0.84**	0.88	1.21
cmu.periodic.N	96	**4.90**	4.92	4.91	4.91	5.12	23.80	23.83	**21.83**	**21.83**	22.25
nusmv.guid7.C	27	0.90	0.90	**0.83**	**0.83**	1.18	20.57	18.43	**13.51**	16.14	17.00
ken.oop1.C	29	0.29	0.28	**0.27**	**0.27**	0.30	**9.08**	15.40	11.88	11.90	9.34
nusmv.guid1.C	10	**0.17**	**0.17**	**0.17**	**0.17**	0.22	0.66	0.63	**0.61**	0.62	0.80
nusmv.tcas2.B	6	**0.14**	**0.14**	**0.14**	**0.14**	0.17	0.57	0.53	0.51	0.52	**0.43**
nusmv.tcas3.B	5	0.11	0.11	**0.10**	**0.10**	0.12	0.52	0.48	0.46	0.46	**0.39**
vis.prodc24.E	37	3.16	3.11	**2.75**	2.76	3.15	**19.06**	23.60	19.65	20.80	23.09
vis.prodc12.E	29	2.11	2.07	**1.82**	1.84	2.07	11.34	11.20	11.44	**10.17**	11.64
vis.prodc17.E	27	1.82	1.82	1.60	**1.58**	1.83	9.69	**9.12**	10.84	11.53	13.98
vis.prodc15.E	23	1.52	1.52	**1.29**	1.30	1.47	5.61	5.79	5.59	**5.20**	7.11
vis.prodc19.E	22	1.45	1.44	**1.23**	1.26	1.41	5.33	5.47	5.64	**4.99**	6.09
texas.par2.E	2	0.48	0.48	**0.44**	**0.44**	0.49	0.03	0.03	**0.02**	**0.02**	0.03
vis.prodc14.E	16	0.88	0.87	**0.76**	**0.76**	0.87	2.39	2.43	2.42	**2.30**	2.65
vis.prodc18.E	13	0.71	0.71	**0.61**	**0.61**	0.71	1.73	1.60	1.64	**1.56**	1.86
vis.prodc13.E	8	0.38	0.38	**0.33**	**0.33**	0.38	0.85	0.80	**0.69**	**0.69**	0.83
vis.prodc16.E	5	0.24	0.24	**0.20**	0.21	0.23	0.45	0.45	**0.36**	**0.36**	0.42

Conclusion

- set of locally minimizing AIG rewrite rules
  - guaranteed to be globally non-size increasing
  - complete (no more such rules for two-level AIGs exist)
  - efficient in reducing size and speeding up SAT solvers
- future work
  - experimental comparison to global rewriting
  - more levels?

AIGER Commercial

A. Biere, FMV, JKU Linz

[http://fmv.jku.at/aiger]

- file format and utilities for AIGs
  - can be generated by several tools:

    C32SAT, SMV2AIG, SMV2QBF, CNF2AIG
  - generic reader and writer implementation (in C)
  - ASCII and binary format (with conversion utilities)
- used in Model Checking Competition 2006
- structural SAT and QBF?