Evaluating CDCL Restart Schemes

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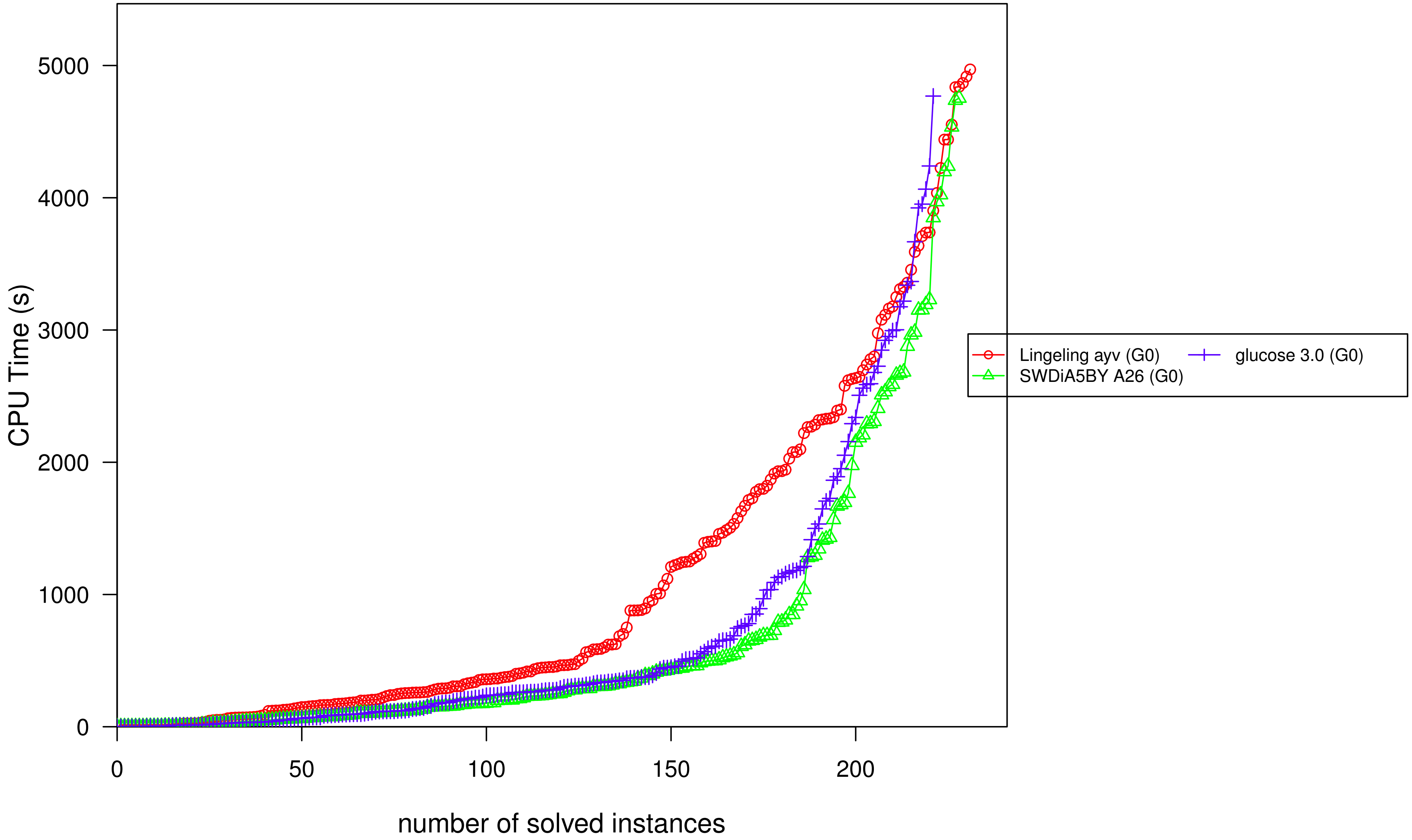
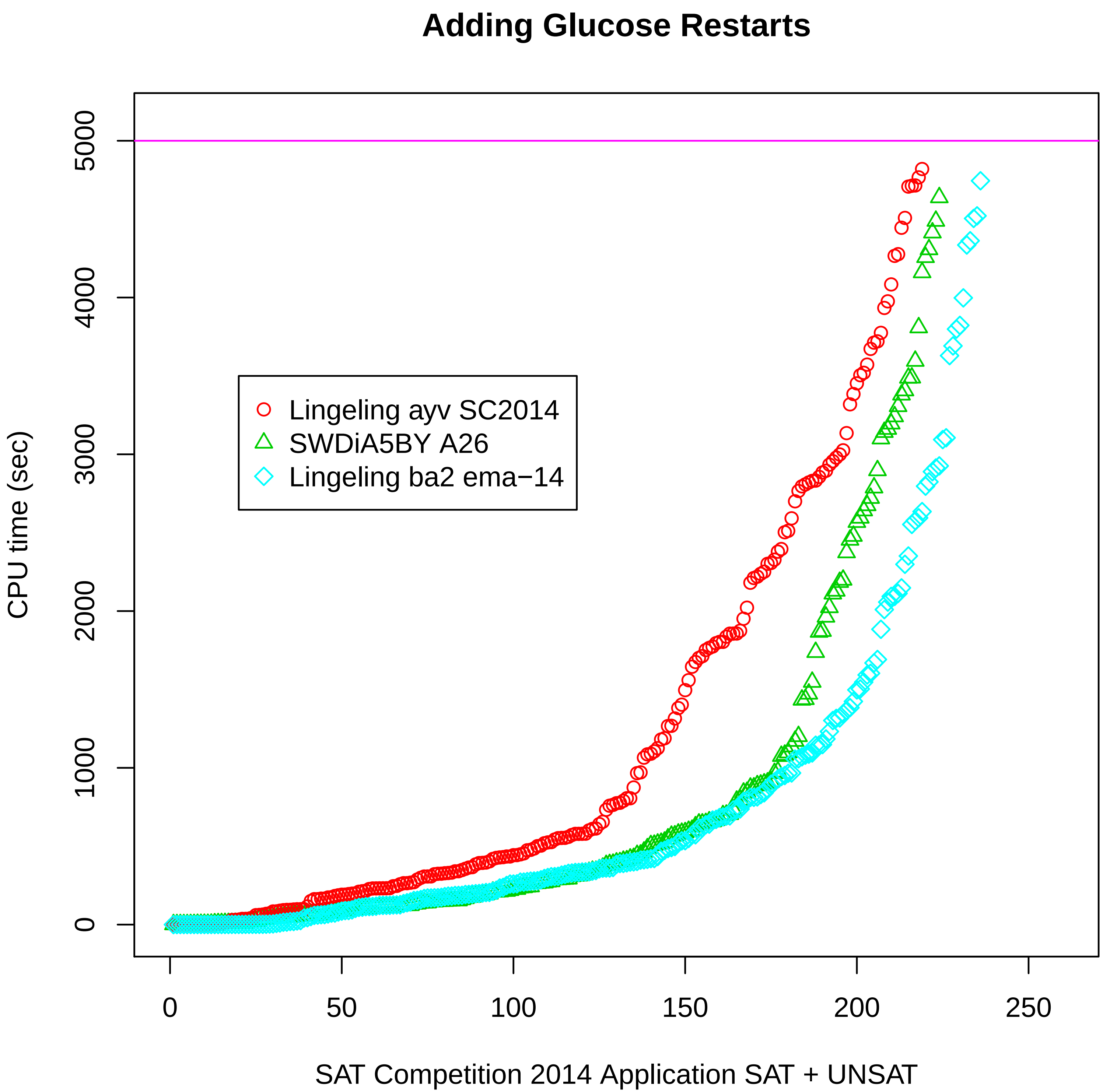
Original SAT Competition 2014 Application SAT+UNSAT Cactus

Number of solved instances within a given amount of CPU time



http://satcompetition.org/edacc/sc14



Adding Glucose Restarts
Lingeling ayv SC2014
SWDiA5BY A26
Lingeling ba2 ema−14
CPU time (sec)
0
1000
2000
3000
4000
5000
0
50
100
150
200
250
SAT Competition 2014 Application SAT + UNSAT

- Lingeling actually barely won
  - only for long time limit of 5000 seconds
  - for 900 seconds: no chance
- two main reasons
  - selected benchmark biased towards decendants of Glucose / MiniSAT
  - but Glucose restarts are important for many (selected) benchmarks
- this paper is about lessons learned while
  - porting the Glucose restart scheme to Lingeling
  - and **simplifying** by

    using *exponential moving averages* (*EMA*) application track instances clustered in buckets (by the organizers):

2d-strip-packing (4), argumentation (20), bio (11),

crypto-aes (8), crypto-des (7), crypto-gos (9),
crypto-md5 (21), crypto-sha (29), crypto-vpmc (4),

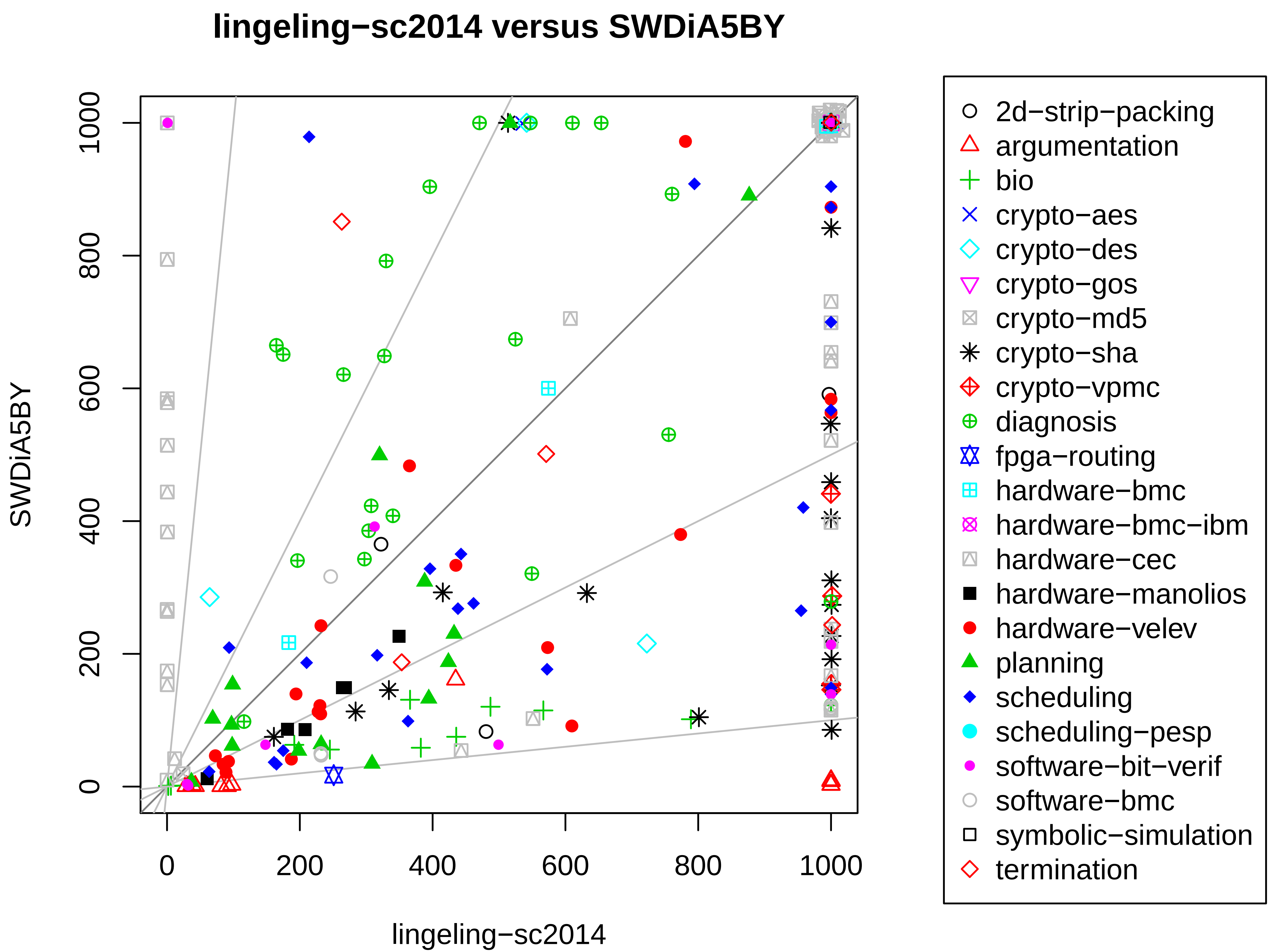
diagnosis (28), fpga-routing (1),

hardware-bmc (4), hardware-bmc-ibm (18), hardware-cec (30),
hardware-manolios (6), hardware-velev (27),

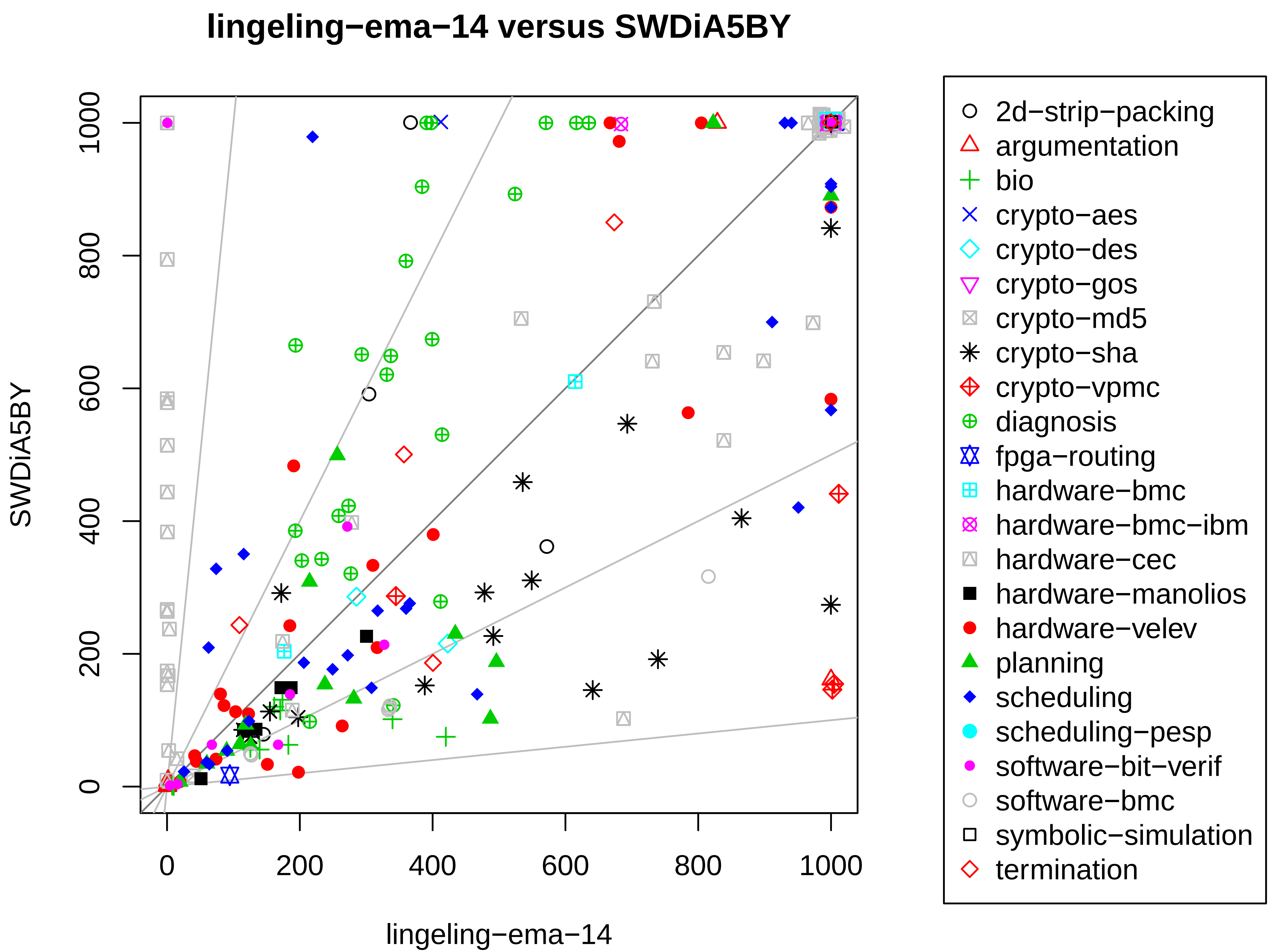
planning (19), scheduling (30), scheduling-pesp (3),

software-bit-verif (9), software-bmc (6), symbolic-simulation (1), termination (5)

in total **300** instances clustered in **23** buckets



lingeling−sc2014 versus SWDiA5BY
SWDiA5BY
lingeling−sc2014
0
200
400
600
800
1000
2d−strip−packing
argumentation
bio
crypto−aes
crypto−des
crypto−gos
crypto−md5
crypto−sha
crypto−vpmc
diagnosis
fpga−routing
hardware−bmc
hardware−bmc−ibm
hardware−cec
hardware−manolios
hardware−velev
planning
scheduling
scheduling−pesp
software−bit−verif
software−bmc
symbolic−simulation
termination



lingeling−ema−14 versus SWDiA5BY
SWDiA5BY
lingeling−ema−14
0
200
400
600
800
1000
2d−strip−packing
argumentation
bio
crypto−aes
crypto−des
crypto−gos
crypto−md5
crypto−sha
crypto−vpmc
diagnosis
fpga−routing
hardware−bmc
hardware−bmc−ibm
hardware−cec
hardware−manolios
hardware−velev
planning
scheduling
scheduling−pesp
software−bit−verif
software−bmc
symbolic−simulation
termination

Restarts in CDCL


```
Status run_CDCL_loop_with_restarts () {
  for (;;) {
    if (bcp ()) {
          if (restarting ()) restart ();
      else if (!decide ()) return SATISFIABLE;
    } else {
      conflicts++;
      if (!analyze ()) return UNSATISFIABLE;
    }
  }
}
```

- run BCP and conflict analysis (including learning) until completion
- restart if restart policy implemented in `restarting` says so
- usually based on a global `conflicts` counter
- otherwise pick next decision (unless all are assigned)

Restart Scheme Examples


```
bool restarting () {
  return conflicts >= limit;
}

void static_uniform_restart () {
  restarts++;
  limit = conflicts + interval;
  backtrack (0);
}

void static_geometric_restart () {
  limit = conflicts + interval * pow (1.5, ++restarts);
  backtrack (0);
}

void luby_restart () {
  limit = conflicts + interval * luby (++restarts);
  backtrack (0);
}
```

Restart Scheme Classification

- **static schemes**
  - fixed schedule of restarts only based on conflicts counter
    - **uniform intervals**: wait a fixed number of conflicts after each restart
    - **non-uniform restart intervals**
      - number of performed restarts determines next interval (in terms of conflicts)
      - arithmetically or geometrically increasing actual interval
      - Luby scheme (also known as reluctant doubling)
      - inner-outer scheme
- **dynamic schemes**
  - *agility* based restart blocking
  - *local restarts* (not discussed in the paper nor the talk)
  - *reusing the trail* implicitly also blocks restarts (even partially)
  - *Glucose restart* scheme (focus here)

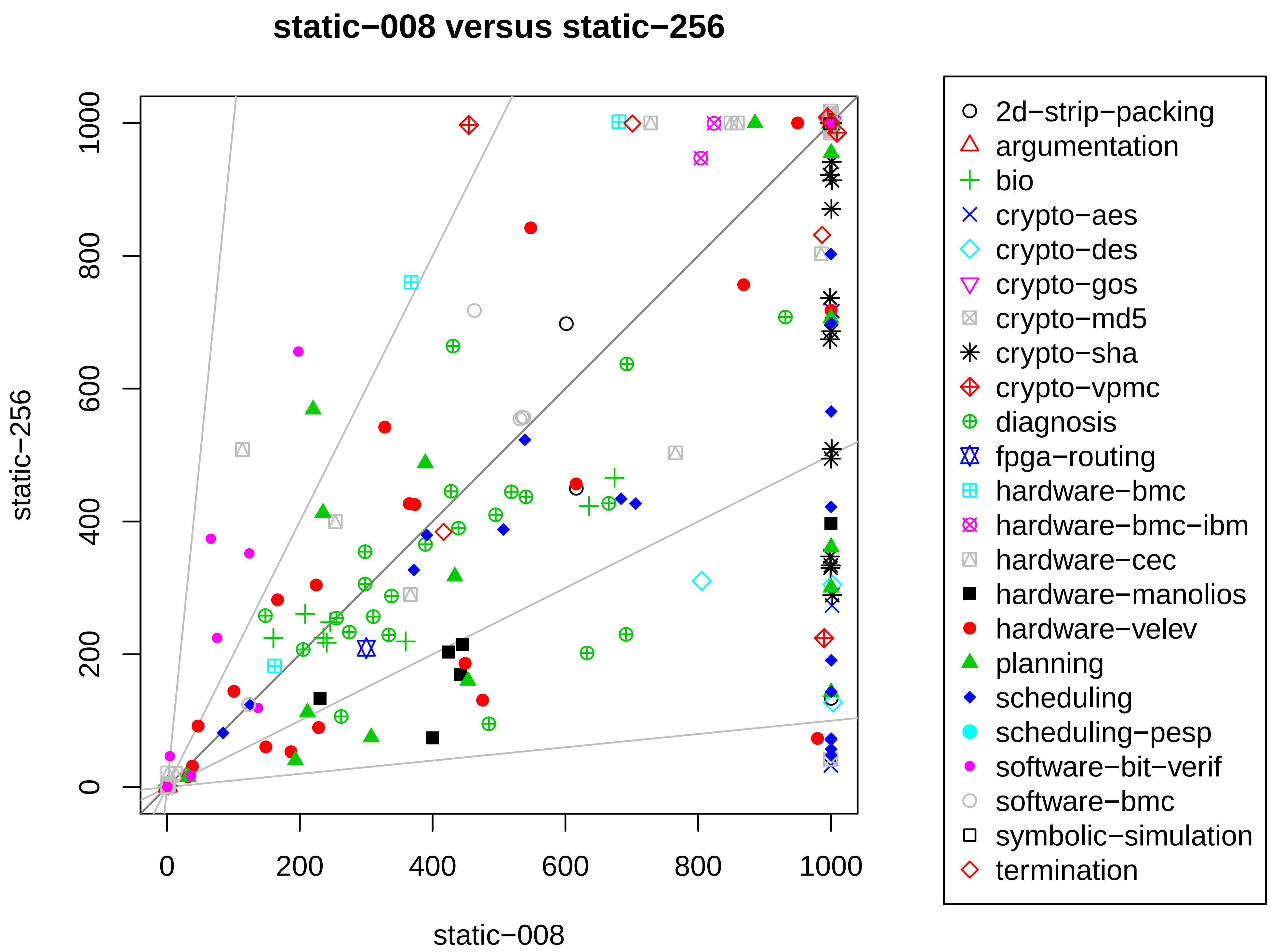
Comparing Static Uniform Restart Schemes

r	002	004	008	016	032	064	128	256	512
tot	122	127	139	144	144	161	163	**168**	158
sat	40	41	49	56	56	73	76	**83**	79
uns	82	86	**90**	88	88	88	87	85	79

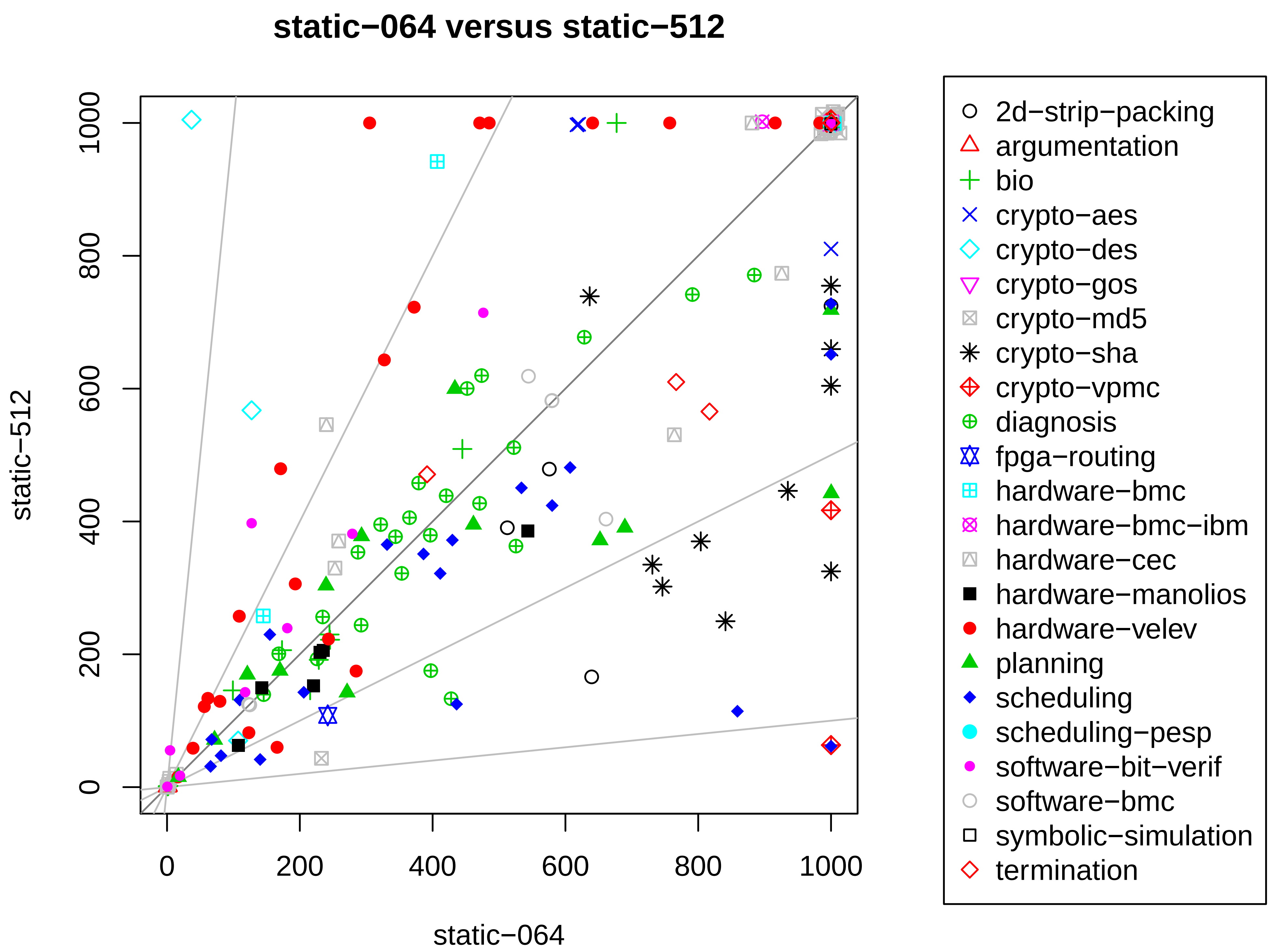
r	002	004	008	016	032	064	128	256	512
2d-strip-packing	0/2	0/2	0/2	0/2	0/2	1/2	1/2	1/2	**2/2**
crypto-sha	0/0	0/0	0/0	0/0	1/0	7/0	11/0	**13/0**	10/0
hardware-cec	0/22	0/23	**0/24**	0/22	0/22	0/23	0/22	0/21	0/21
hardware-manolios	0/4	0/5	0/5	0/5	**0/6**	**0/6**	**0/6**	**0/6**	**0/6**
hardware-velev	5/9	6/10	8/11	8/12	8/12	**8/13**	8/12	8/11	8/6
planning	6/3	6/5	7/4	7/4	8/4	9/3	9/4	**11/4**	10/4
scheduling	1/7	0/7	1/7	4/7	6/7	9/7	9/7	11/7	**12/7**

SAT / UNSAT

underlined best



static−008 versus static−256
static−256
static−008
0
200
400
600
800
1000
0
200
400
600
800
1000
2d−strip−packing
argumentation
bio
crypto−aes
crypto−des
crypto−gos
crypto−md5
crypto−sha
crypto−vpmc
diagnosis
fpga−routing
hardware−bmc
hardware−bmc−ibm
hardware−cec
hardware−manolios
hardware−velev
planning
scheduling
scheduling−pesp
software−bit−verif
software−bmc
symbolic−simulation
termination

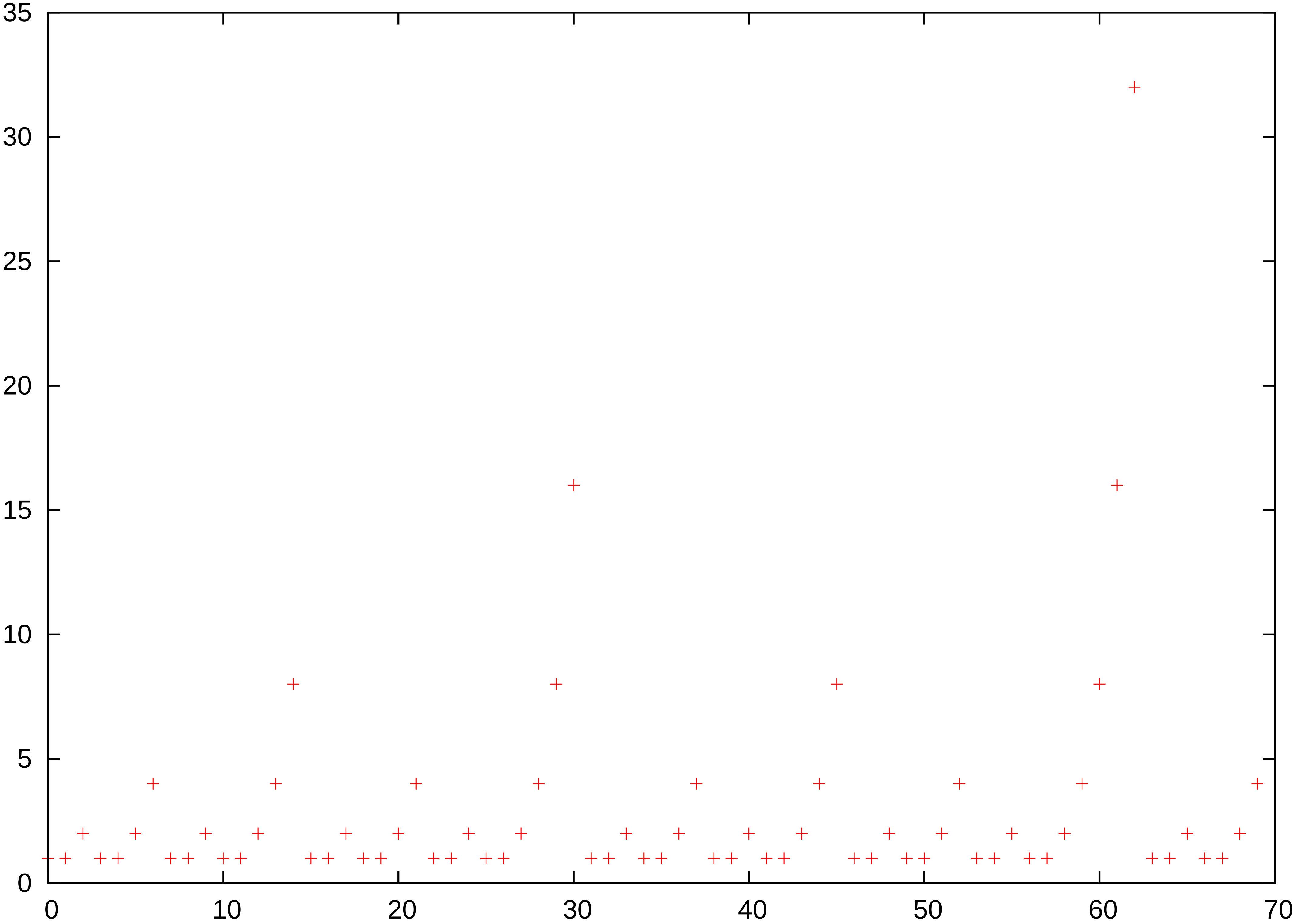


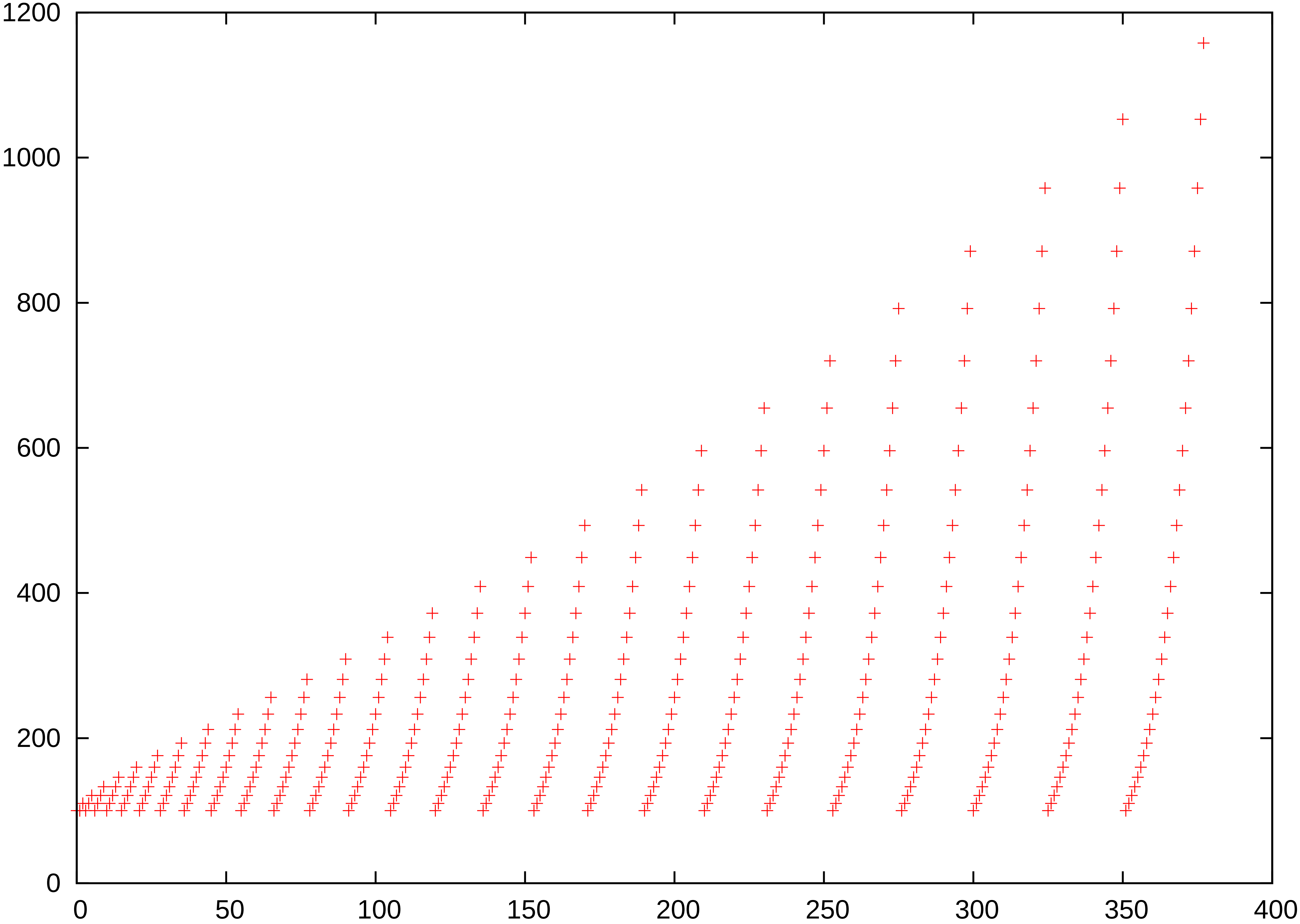
static−064 versus static−512
static−512
static−064
0
200
400
600
800
1000
0
200
400
600
800
1000
2d−strip−packing
argumentation
bio
crypto−aes
crypto−des
crypto−gos
crypto−md5
crypto−sha
crypto−vpmc
diagnosis
fpga−routing
hardware−bmc
hardware−bmc−ibm
hardware−cec
hardware−manolios
hardware−velev
planning
scheduling
scheduling−pesp
software−bit−verif
software−bmc
symbolic−simulation
termination

Comparing Static but Non-Uniform Restart Schemes

	luby-b						inner-outer					
	01	02	04	08	16	32	001	002	004	008	032	128
tot	156	**168**	161	163	160	159	**161**	**161**	158	153	154	150
sat	72	**80**	74	77	74	74	80	**81**	77	74	76	76
uns	84	**88**	87	86	86	85	**81**	80	81	79	78	74
avgc	9	17	31	58	108	203	443	509	601	732	1084	1740

avgc = average restart interval (over all instances) in conflicts


35
30
25
20
15
10
5
0
0
10
20
30
40
50
60
70



1200
1000
800
600
400
200
0
0
50
100
150
200
250
300
350
400

Glucose Restarts in a Nutshell

```
bool restarting () {
  return conflicts >= limit &&
     average_RECENT_lbds () > 1.25 * average_ALL_lbds ();
}

void glucose_restart () {  // same as static_uniform_restart
  restarts++;
  limit = conflicts + 50;
  backtrack (0);
}
```

- glucose level (LBD) of learned clause:
  - number of different decision levels in a learned clauses
  - calculated at the point the clause is learned during conflict analysis
- last 50 LBDs are stored and considered recent (explicit LBD queue)
- total average of all LBDs is simply `sum_lbd / conflicts`
- for discussion of *blocking restarts* since Glucose 2.1 see the paper

Glucose uses *simple moving average* (*SMA*) for the average of recent LBDs and *cumulative moving average* (*CMA*) for the the average of all LBDs and

simple $\quad SMA(n,w) = \frac{1}{w} \cdot (t_n + t_{n-1} + \ldots + t_{n-w+1})$ with $n \geq w \geq 1$

cumulative $\quad CMA(n) = SMA(n,n)$

$$CMA(n) = CMA(n-1) + \frac{t_n - CMA(n-1)}{n}$$

$$SMA(n,w) = SMA(n-1,w) + \frac{t_n}{w} - \frac{t_{n-w}}{w}$$

requires $\quad SMA(n,50) > 1.25 \cdot CMA(n)$ $\quad$ to restart

and 50 conflicts have passed

we suggest to use *EMA*s instead of the "fast" *SMA* and/or "slow" *CMA*

exponential $\mathit{EMA}(n,\alpha) = \alpha \cdot t_n + (1-\alpha) \cdot \mathit{EMA}(n-1,\alpha)$ with $0 < \alpha < 1$ $a \approx \frac{2}{1+w}$

current estimate

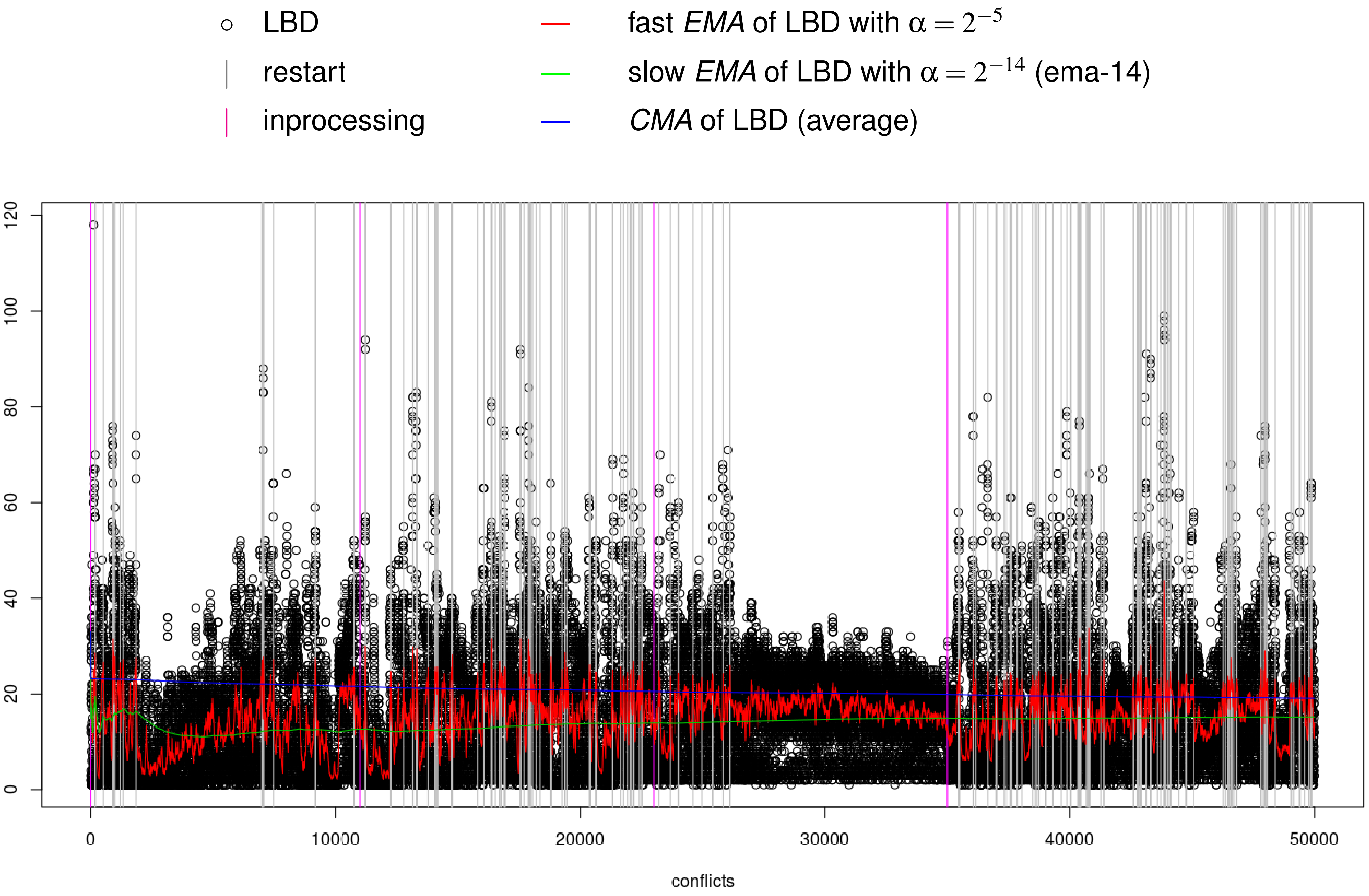
alternative $\mathit{EMA}(n,\alpha) = \mathit{EMA}(n-1,\alpha) + \alpha \cdot (t_n - \mathit{EMA}(n-1,\alpha))$

next estimate difference/error

to restart version *average* requires $\mathit{EMA}(n,2^{-5}) > 1.25 \cdot \mathit{CMA}(n)$

to restart version *ema-14* requires $\mathit{EMA}(n,2^{-5}) > 1.25 \cdot \mathit{EMA}(n,2^{-14})$

and again in both cases that a certain number of conflicts say 50 have passed



LBD
restart
inprocessing
fast EMA of LBD with $\alpha = 2^{-5}$
slow EMA of LBD with $\alpha = 2^{-14}$ (ema-14)
CMA of LBD (average)
0
20
40
60
80
100
120
0
10000
20000
30000
40000
50000
conflicts

Comparing EMAs with SMA and CMA

solver	Glucose 4.0			Lingeling ba2							
restarts	*ss*	*es*	*ee*	*avg*	*e8*	*e10*	*e12*	*e14*	*e16*	*e18*	*e20*
tot	163	163	165	178	167	170	180	**181**	180	177	171
sat	72	73	76	83	80	78	**86**	**86**	**86**	82	77
uns	91	90	89	**95**	87	92	94	**95**	94	**95**	94
avgc	192	166	167	145	230	204	195	186	172	147	108

Glucose 4.0 column *ss* correspond to the original Glucose version

column *es* to adding EMAs for only forcing restarts

column *ee* includes using EMA for blocking restarts too

column *avg* is Lingeling version *average* of Glucose version *ee*

columns *eX* correspond to Lingeling versions *ema*-X
using a slow EMA with $\alpha = 2^{-X}$ instead of CMA

```
double fast, slow;
...

bool analyze () {
  int lbd;
  ...
  slow += (lbd - slow)/(double)(1<<14);
  fast += (lbd - fast)/(double)(1<<5);
  ...
}

bool restarting () {
  return conflicts > limit && fast > 1.25 * slow;
}
```

fast 64-bit fixed point implementation avoiding floating point

inspired by Donald Knuth's implementation of our agility metric

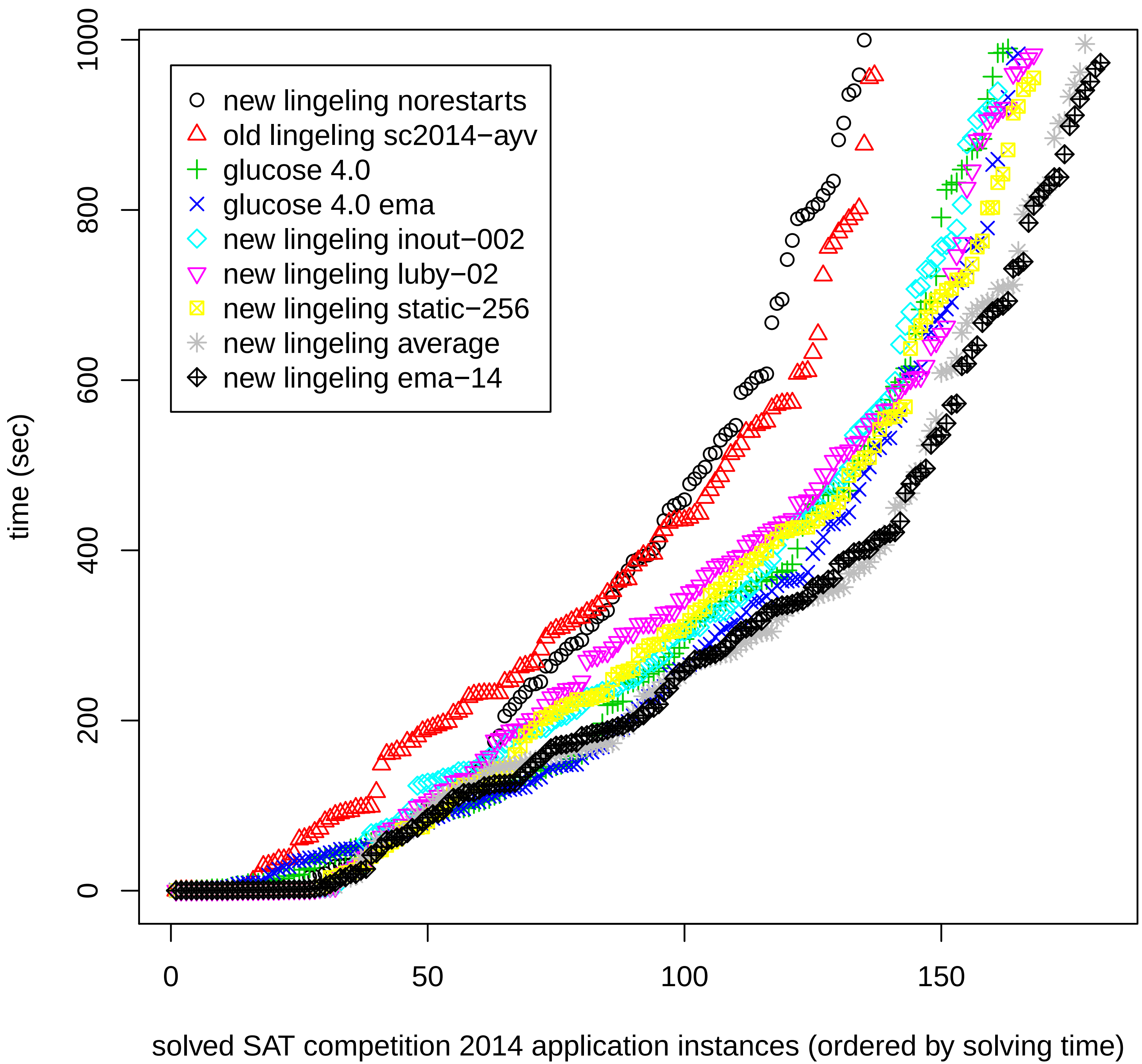
```
long fast, slow;                    // assume (sizeof (long) == 8);
...                                 // initialization code skipped ...

bool analyze () {
  int lbd;                          // assume (sizeof (int) == 4);
  ...
  fast -= fast >> 5;
  fast += lbd << (32 - 5);
  slow -= slow >> 14;
  slow += lbd << (32 - 14);
  ...
}

bool restarting () {
  return conflicts > limit && fast / 125 > slow / 100;
}
```

Conclusion

- data and source: http://fmv.jku.at/evalrestart/evalrestart.7z
- optimal restart interval varies with benchmark bucket
  - for miters fast restarts essential
  - for crypto benchmarks longer intervals necessary
  - disabling restarts completely is bad
  - Glucose restarts superior to Luby style
- presented an EMA variant of the Glucose restart scheme
  - simpler model, simpler to implement
  - similar performance (slightly faster)
- future work
  - how to improve blocking of restarts
  - restart intervals still not optimal: really need machine learning?
  - finally cross-fertilize ideas from *SAT and Stock Market Analysis*
    originally proposed title for this paper



new lingeling norestarts
old lingeling sc2014−ayv
glucose 4.0
glucose 4.0 ema
new lingeling inout−002
new lingeling luby−02
new lingeling static−256
new lingeling average
new lingeling ema−14
time (sec)
0
200
400
600
800
1000
0
50
100
150
solved SAT competition 2014 application instances (ordered by solving time)