About the SAT Solvers

Limmat, Compsat, Funex

and the QBF Solver

Quantor

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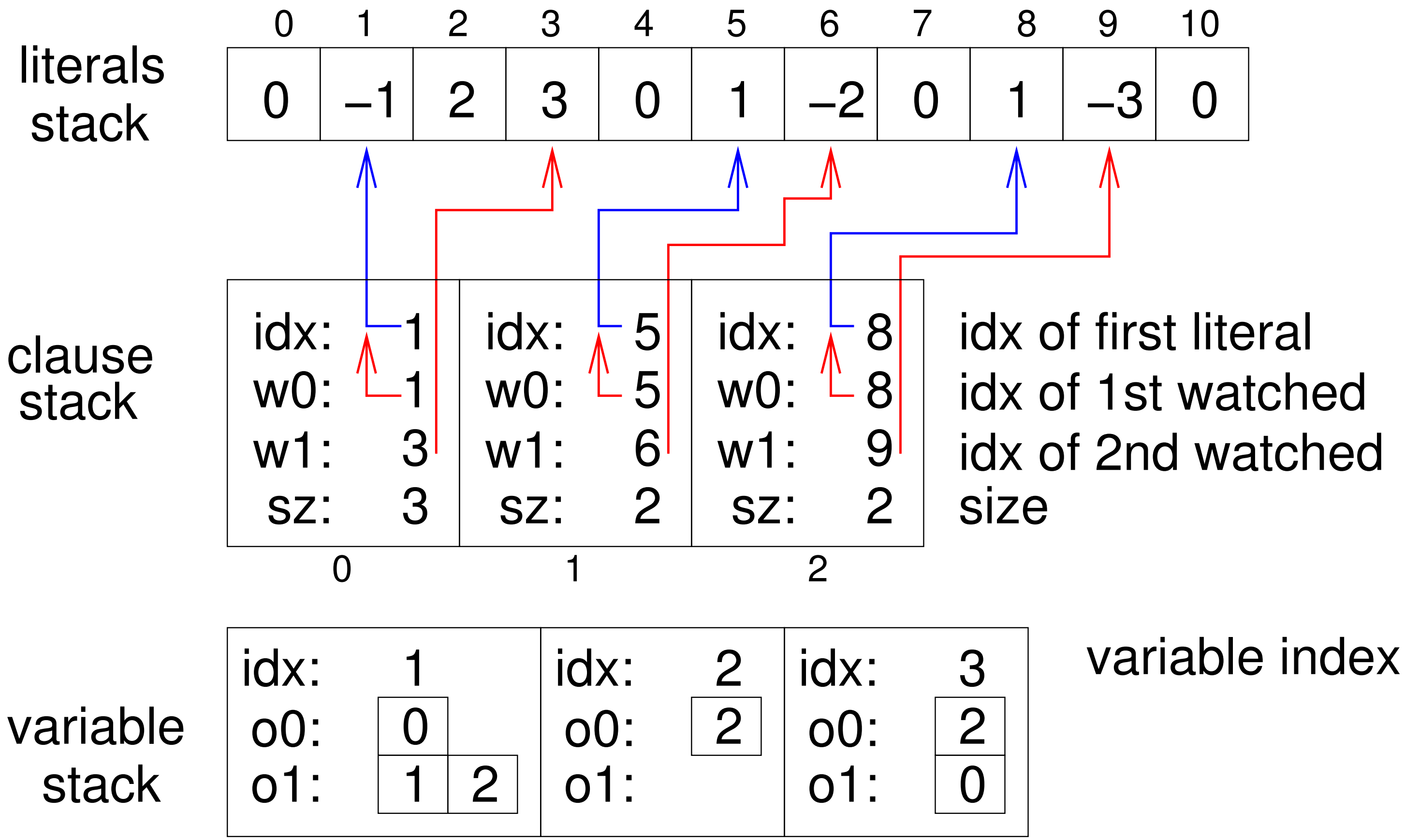
Separate Clause Structure



o0/o1 are stacks of clause indices in which resp. literal is watched

Separate Clause Structure

- implemented in both Limmat and Funex but not in Compsat

- 1st **benefit**: direct access to other watched literal

  - no traversal when assigning literals where the other watched is satisfied

- 2nd **benefit**: clause information accessible in BCP

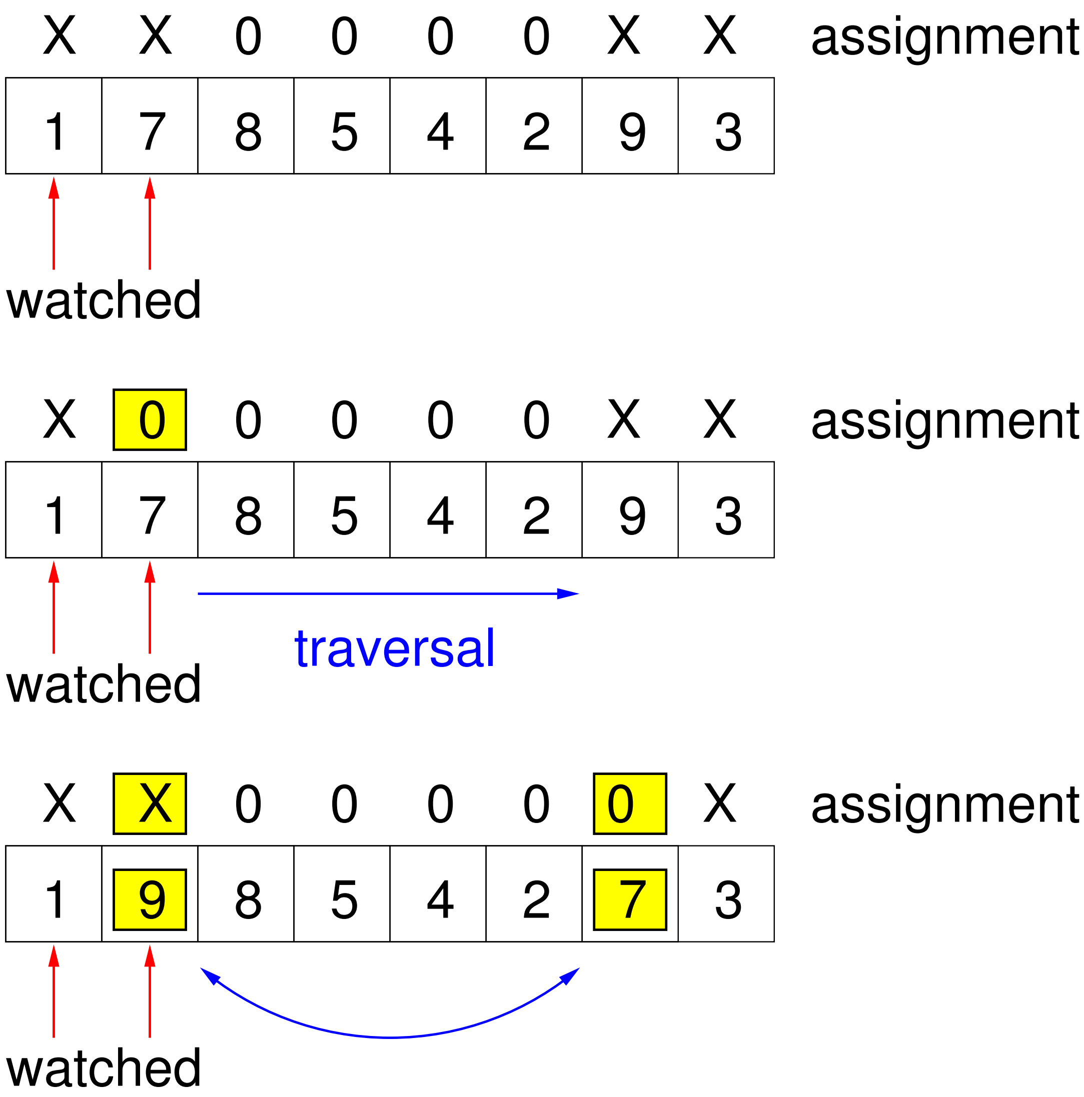
  - heuristics: shorter clauses as reasons in assignments preferred

- major **drawback**: additional indirection in occurrence lookup

  - Zchaff and Compsat: occurrence stacks store literal positions directly

Compsat: Watched-Literals Approach

eg [VanGelder]



Compsat: Additional Features

- van Gelder's approach allows very simple data structures (integers and integer stacks)

- **comp**act memory layout
  - as in BDD library ABCD: **minimize time by minimizing space**

- various space optimized integer stacks
  - compact stack with 8 Byte anchor (C++ STL requires 12 Byte anchor)
  - elements can be 16 bit or 32 bit (may change dynamically)
  - fully configurable

$\Rightarrow$ **therefore very low memory footprint**

Compsat: Flaw

- submitted version of Compsat had a serious last minute bug
- **BCP queue was not flushed after restart**
- showed up in large benchmarks only
  - at least one restart required
- bug escaped automated test suite
- one line bug fix, since flushing of BCP queue already implemented
  - added new test cases with restart intervals of length 1

Funex Additional Features

- incorporated **Berkmin** style decision function (clause linking)
  - cache of satisfied clauses reduces time spent in decision function
- selection of decision functions is specified as ω-regular expression
  - decision functions: `dlis`, `horn`, `chaff`, `berkmin`
  - default selection: `(horn.berkmin^3000)^infinity`
- dedicated BCP for *binary*, *short* and *long* clauses respectively
- fast restarts initially; restarts slow down later

$$\exists\, a,b\ [\ \boxed{\forall x\ [\ \boxed{\exists\, c,d\ [\ f(a,b,c,d,x)\ ]}\ ]}\ ]$$

$$\equiv\quad \exists\, a,b\ [\ \boxed{\boxed{\exists\, c,d\ [\ f(a,b,c,d,x)\ ]}\ \langle x/1\rangle \wedge \boxed{\exists\, c,d\ [\ f(a,b,c,d,x)\ ]}\ \langle x/0\rangle}\ ]$$

$$\equiv\quad \exists\, a,b\ [\ \exists\, c,d,x\ [\ x \wedge f(a,b,c,d,x)\ ]\ \wedge\ \exists\ \boxed{c,d}\ [\ f(a,b,\boxed{c,d},0)\ ]\ ]$$

$$\equiv\quad \exists\, a,b\ [\ \exists\, c,d,x\ [\ x \wedge f(a,b,c,d,x)\ ]\ \wedge\ \exists\ \boxed{c',d'}\ [\ f(a,b,\boxed{c',d'},0)\ ]\ ]$$

$$\equiv\quad \exists\, a,b,c,d,c',d'\ [\ x \wedge f(a,b,c,d,x) \wedge f(a,b,c',d',0)\ ]$$

Elimination of Universal Quantifiers by Copying Clauses

$$\exists\, a,b\,[\ \forall x,y\,[\ \exists\, c,d\,[\ f(a,b,c,d,x,y)\ ]\ ]\ ]$$

$$\equiv \quad \exists\, a,b\,[\,\forall y\,[\ \forall x\,[\ \exists\, c,d\,[\ f(a,b,c,d,x,y)\ ]\ ]\ ]\,]$$

$$\equiv \quad \exists\, a,b\,[\,\forall y\,[\ \exists\, c,d\,[\ f(a,b,c,d,x,y)\ ] <x/1> \wedge\ \exists\, c,d\,[\ f(a,b,c,d,x,y)\ ] <x/0>\ ]\,]$$

$$\equiv \quad \exists\, a,b\,[\,\forall y\,[\,\exists\, c,d,x\,[\,x \wedge f(a,b,c,d,x,y)\,]\ \wedge\ \exists\ c,d\ [\ f(a,b,\ c,d\ ,0,y)\,]\,]\,]$$

$$\equiv \quad \exists\, a,b\,[\,\forall y\,[\,\exists\, c,d,x\,[\,x \wedge f(a,b,c,d,x,y)\,]\ \wedge\ \exists\ c',d'\ [\ f(a,b,\ c',d'\ ,0,y)\,]\,]\,]$$

$$\equiv \quad \exists\, a,b\,[\,\forall y\,[\,\exists\, c,d,c',d'\,[\,x \wedge f(a,b,c,d,x,y) \wedge f(a,b,c',d',0,y)\,]\,]\,]$$

Features

- elimination of innermost universal variable with most occurrences (DLIS)
  - heuristically maximizes the number of removed clauses after expansion
- simplification of CNF matrix and quantifier prefix by
  - unit resolution
  - pruning of unates, zombies, and empty scopes
  - elimination of satisfied clauses
  - garbage collection of indices
- for existential problems use builtin DPLL style SAT solver

Future Work

- quantify out innermost existential variables by resolution (DP)
  - investigate when to eliminate universal or existential variables
- simplify CNF by subsumption tests
- apply look-forward strategies like learning
  - facts involving universal variables may lead to early conflicts
  - existential implications or equivalences may lead to elimination
- compare with other quantifier elimination algorithms
  - eg [PlaistedBiereZhu] or simply use BDDs