About the SAT Solvers

Limmat, Compsat, Funex
and the QBF Solver

Quantor

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SAT’03, Santa Margherita Ligure, Portofino, Italy
Separate Clause Structure

 literals stack

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
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<td>-1</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>-2</td>
<td>0</td>
<td>1</td>
<td>-3</td>
<td>0</td>
</tr>
</tbody>
</table>

 clause stack

<table>
<thead>
<tr>
<th>idx</th>
<th>w0</th>
<th>w1</th>
<th>sz</th>
<th>idx</th>
<th>w0</th>
<th>w1</th>
<th>sz</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>5</td>
<td>6</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td></td>
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<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

 variable stack

<table>
<thead>
<tr>
<th>o0</th>
<th>o1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

idx of first literal
idx of 1st watched
idx of 2nd watched
size

o0/o1 are stacks of clause indices in which resp. literal is watched

variable index

SA T'03 – Santa Margherita Ligure – Portofino – Italy – May 2003
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Separate Clause Structure

- implemented in both Limmat and Funex but not in Compsat

- 1st **benefit**: direct access to other watched literal
  - no traversal when assigning literals where the other watched is satisfied

- 2nd **benefit**: clause information accessible in BCP
  - heuristics: shorter clauses as reasons in assignments preferred

- major **drawback**: additional indirection in occurrence lookup
  - Zchaff and Compsat: occurrence stacks store literal positions directly
Compsat: Watched-Literals Approach

eg [VanGelder]

\[\begin{array}{cccccccc}
X & X & 0 & 0 & 0 & 0 & X & X \\
1 & 7 & 8 & 5 & 4 & 2 & 9 & 3 \\
\end{array}\]

assignment

watched

\[\begin{array}{cccccccc}
X & 0 & 0 & 0 & 0 & 0 & X & X \\
1 & 7 & 8 & 5 & 4 & 2 & 9 & 3 \\
\end{array}\]

assignment

watched

traversal

\[\begin{array}{cccccccc}
X & X & 0 & 0 & 0 & 0 & 0 & X \\
1 & 9 & 8 & 5 & 4 & 2 & 7 & 3 \\
\end{array}\]

assignment

watched
van Gelder’s approach allows very simple data structures (integers and integer stacks)

- compact memory layout
  - as in BDD library ABCD: minimize time by minimizing space

- various space optimized integer stacks
  - compact stack with 8 Byte anchor (C++ STL requires 12 Byte anchor)
  - elements can be 16 bit or 32 bit (may change dynamically)
  - fully configurable

⇒ therefore very low memory footprint
• submitted version of Compsat had a serious last minute bug

• **BCP queue was not flushed after restart**

• showed up in large benchmarks only
  
  – at least one restart required

• bug escaped automated test suite

• one line bug fix, since flushing of BCP queue already implemented
  
  – added new test cases with restart intervals of length 1
• incorporated **Berkmin** style decision function (clause linking)
  – cache of satisfied clauses reduces time spent in decision function

• selection of decision functions is specified as **ω**-regular expression
  – decision functions: \( \text{dlis}, \text{horn}, \text{chaff}, \text{berkmin} \)
  – default selection: \( (\text{horn.berkmin}^3000)^\infty \)

• dedicated BCP for **binary**, **short** and **long** clauses respectively

• **fast restarts** initially; restarts slow down later
\[ \exists a, b \left[ \forall x \left[ \exists c, d \left[ f(a, b, c, d, x) \right] \right] \right] \]
\[ \equiv \exists a, b \left[ \exists c, d \left[ f(a, b, c, d, x) \right] <x/1> \land \exists c, d \left[ f(a, b, c, d, x) \right] <x/0> \right] \]
\[ \equiv \exists a, b \left[ \exists c, d, x \left[ x \land f(a, b, c, d, x) \right] \land \exists c, d \left[ f(a, b, c, d, 0) \right] \right] \]
\[ \equiv \exists a, b \left[ \exists c, d, x \left[ x \land f(a, b, c, d, x) \right] \land \exists c', d' \left[ f(a, b, c', d', 0) \right] \right] \]
\[ \equiv \exists a, b, c, d, c', d' \left[ x \land f(a, b, c, d, x) \land f(a, b, c', d', 0) \right] \]
\[\exists a, b \ [ \ \forall x, y \ [ \ \exists c, d \ [ f(a, b, c, d, x, y) ] ] \] \]

\[\equiv \exists a, b \ [ \ \forall y \ [ \ \exists c, d \ [ f(a, b, c, d, x, y) ] ] ] \]

\[\equiv \exists a, b \ [ \ \forall y \ [ \ \exists c, d \ [ f(a, b, c, d, x, y) ] <x/1> \land \exists c, d \ [ f(a, b, c, d, x, y) ] <x/0> ] ] \]

\[\equiv \exists a, b \ [ \ \forall y \ [ \ \exists c, d, x \ [ x \land f(a, b, c, d, x, y) ] \land \exists c, d \ [ f(a, b, c, d, 0, y) ] ] ] \]

\[\equiv \exists a, b \ [ \ \forall y \ [ \ \exists c, d, x \ [ x \land f(a, b, c, d, x, y) ] \land \exists c', d' \ [ f(a, b, c', d', 0, y) ] ] ] \]

\[\equiv \exists a, b \ [ \ \forall y \ [ \ \exists c, d, c', d' \ [ x \land f(a, b, c, d, x, y) \land f(a, b, c', d', 0, y) ] ] ] \]
• elimination of innermost universal variable with most occurrences (DLIS)
  – heuristically maximizes the number of removed clauses after expansion

• simplification of CNF matrix and quantifier prefix by
  – unit resolution
  – pruning of unates, zombies, and empty scopes
  – elimination of satisfied clauses
  – garbage collection of indices

• for existential problems use builtin DPLL style SAT solver
Future Work

- quantify out innermost existential variables by resolution (DP)
  - investigate when to eliminate universal or existential variables

- simplify CNF by subsumption tests

- apply look-forward strategies like learning
  - facts involving universal variables may lead to early conflicts
  - existential implications or equivalences may lead to elimination

- compare with other quantifier elimination algorithms
  - eg [PlaistedBiereZhu] or simply use BDDs