# Evaluating CDCL Variable Scoring Schemes

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# Motivation



- VSIDS/EVSIDS important
  - empirically
  - no formal argument “why it works”
- reconsider simpler alternatives
  - particularly variable move to front schemes (VMTF)
  - requires careful data structure design
  - formalization of these heuristics
  - empirical evaluation
- so a step towards “Trying to Understand the Power of VSIDS”

# Decision Heuristics



- decision heuristics consist of
  - variable selection: which variable to assign next?
  - phase selection: assign variable to which phase (true or false)?
- *phase saving* [PipatsrisawatDarwiche'07]
  - select phase to which variable was assigned before
  - initialized by static one-side heuristics [JeroslowWang'90]
  - very effective and thus default in state-of-the-art solvers
- we consider only *variable selection* as decision heuristic for this study
  - clause based heuristics less effective (BerkMin, CMTF)
  - same applies to literal based heuristics (using literal scores)
- variable selection and **decision heuristic** boils down to
  - compute and maintain heuristic scores for variables
  - **select variable with highest score**

# Scores in CDCL Loop



```
for (;;) {
  if (bcp ()) {          // [X] reorder assigned variables
                         // eagerly is considered too costly
    if (restarting ())
      restart ();        // [E] reuse trail if possible thus
                         // need to know next decision variable
                         // [C] unassigning variables might
                         // lead to reordering to support (B)

    if (!decide ())      // [B] find next decision variable
      return SAT;        // (unassigned one with highest score)
  } else {
    if (!analyze ())     // [A] learn clause from conflict and bump
      return UNSAT;      // relevant variables  (increase score)
  }                      // [D] optionally rescore all variables
}                        // (explicitly or to avoid overflow)
                         // [C] backtrack and unassign variables
```

# Variable Scoring Schemes



- how to compute scores
  - static or dynamic
  - **bump** variables: when to *increase* scores and by how much
  - **rescore** variables: when to *decrease* scores and by how much
  - state-of-the-art: VSIDS (from Chaff)

    more precisely the exponential variant (EVSIDS) of MiniSAT

- data structures for finding decision variables
  - eager or lazy update of “order”
  - state-of-the-art: priority queue of variables ordered by score (MiniSAT)

- data structure depends on how scores are computed and vice versa

# Zero, First and Second Order Variable Scoring Schemes

- *zero* order scheme $=$ static scores
  - computed for instance once during preprocessing
  - still needs search for “best” unassinged variable
  - only total orders considered so far
- *first* order schemes $=$ dynamic but state less
  - for instance: score $=$ pos occs $\times$ neg occs
  - independent of how search reached current branch / search node
  - might be quite expensive to compute / update (linear in CNF size)
- *second* order schemes: variable score depends on history of search
  - first order $+$ learning $\Rightarrow$ second order
  - but can also be used to speed up search for “best” variable
  - goal is logarithmic or even constant algorithm for variable selection

# VSIDS



- VSIDS appeared in seminal Chaff paper from Princeton (2001)
- bump variables occurring in learned clauses
  - bumping means incrementing an integer VSIDS score

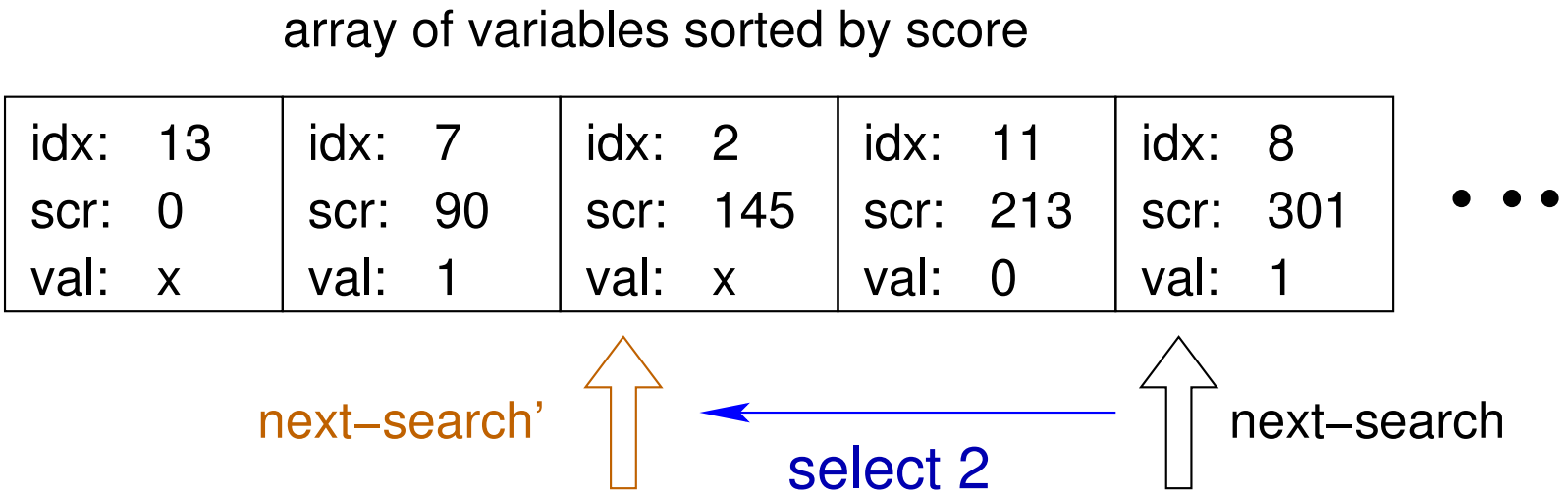
    current state-of-the-art: bump *all* variables used to derive learned clause
  - thus independent of state of clauses (satisfied or not)
- rescoring gives focus on recently used variables
  - scores are “decayed”, e.g., originally *divided by two* every 256th conflict
  - “low pass filter” on “use frequency” of variables

    `http://youtu.be/MOjhFywLre8`
- search for next unassigned variable with largest score
  - keeps an array of variables sorted by score
  - only re-sorts it w.r.t. score during rescoring (every 256th conflict)
  - uses right-most unassigned variable, thus original implementation **imprecise**

# ZChaff's Optimized Search for Next Decision Variable

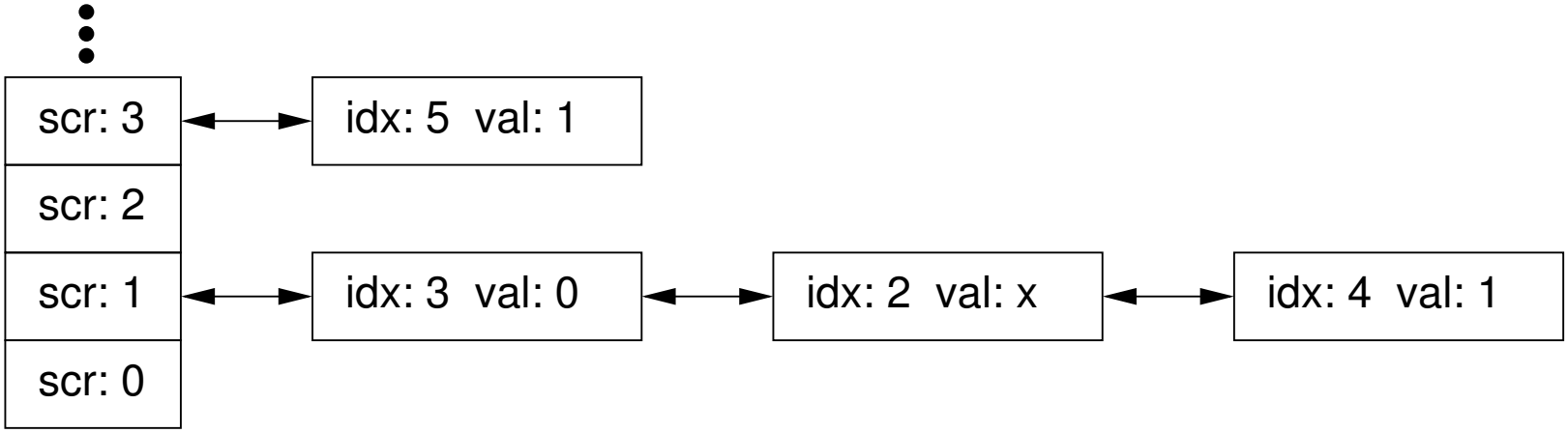


ZChaff's implementation actually sorted the other way around, largest score first




- basically start search at largest score variable
  - scan array towards smaller positions with smaller score
  - until *unassigned* variable is found, e.g., with idx 2 in the example
  - easily leads to accumulated quadratic search cost (for decisions)
- avoided by "caching" position of last found decision variable in "next-search"
  - next search can start from there
  - if variable right to next-search becomes assigned move next-search
  - no update of next-search during bumping (in ZChaff)

- original ZChaff implementation imprecise in three aspects
  - integer scores
  - effect of bumping not immediate
  - selected unassigned variable not necessarily the one with largest score
- JeruSAT [Nadel'02] partial fix was to
  - array of doubly linked lists of variables with same score
  - was still imprecise since it still used integer scores
  - costly eager update of data structure during (un)assigning, bumping and rescoring



# Variable Move To Front



- Siege SAT solver [Ryan'04] used *variable move to front* (VMTF)
  - bumped variables moved to head of *doubly linked list*
  - search for unassigned variable starts at head
  - variable selection is an online sorting algorithm of scores
  - classic "move-to-front" strategy achieves good amortized complexity
- original implementation severely restricted
  - only moved a subset of bumped variables
  - details about caching the search not described
    no source code published either
  - not exactly the same as VSIDS
- as consequence VMTF not used in state-of-the-art solvers

# MiniSAT's Exponential VSIDS (EVSIDS) from 2004



- floating point scores
  - allows fine grained rescore at every conflict
  - consider multiplying by $f = 0.9$ every score at each conflict
- actually, instead of updating scores of all variables (at every conflict)
  - only increase score of bumped variables by $g^i$
  - with $i$ the *conflict-index*, and $g = 1/f$
  - non-bumped variables not touched
- priority queue of variables ordered by score
  - implemented as binary heap with update (bubble up)
  - lazy assigned variable removal
    - remove largest score variable from heap until unassigned one found
    - put unassigned variables not on the heap back (logarithmic complexity)
- normalized VSIDS (NVSIDS) $\in [0, 1]$ as (theoretical) model [Biere'08]

# Summary Variable Scoring Schemes

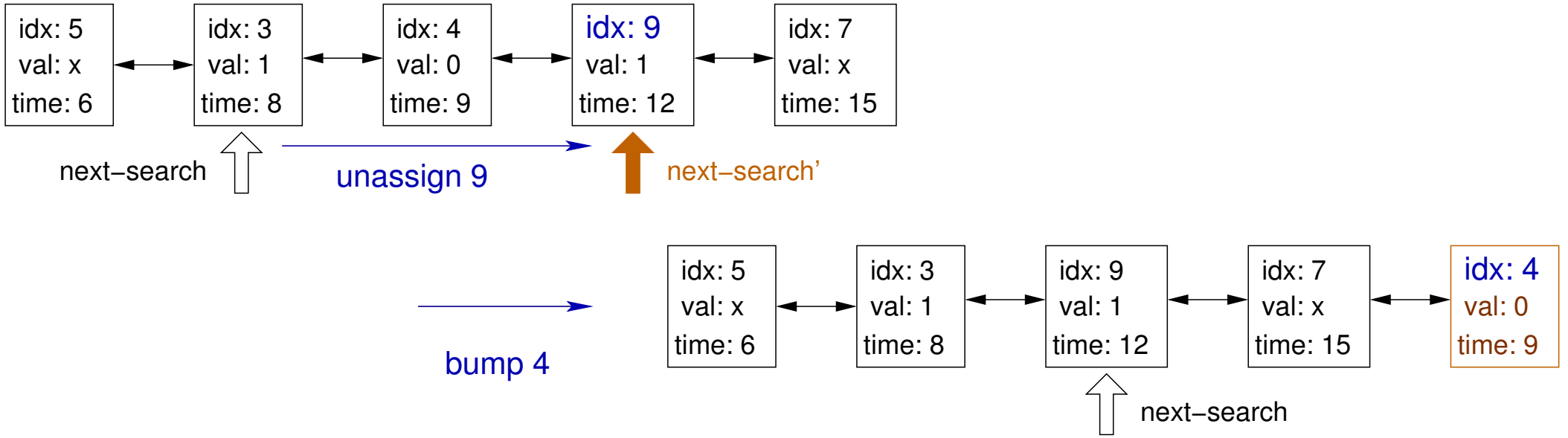
$s$ old score $s'$ new score

| | variable score $s'$ after $i$ conflicts | | |
|---|---|---|---|
| | bumped | not-bumped | |
| STATIC | $s$ | $s$ | static decision order |
| INC | $s+1$ | $s$ | increment scores |
| **SUM**[new] | $s+i$ | $s$ | sum of conflict-indices |
| VSIDS | $h_i^{256} \cdot s + 1$ | $h_i^{256} \cdot s$ | original implementation in Chaff |
| NVSIDS | $f \cdot s + (1-f)$ | $f \cdot s$ | normalized variant of VSIDS |
| EVSIDS | $s+g^i$ | $s$ | exponential MiniSAT dual of NVSIDS |
| **ACIDS**[new] | $(s+i)/2$ | $s$ | average conflict-index decision scheme |
| VMTF | $i$ | $s$ | variable move-to-front |

$0 < f < 1$ $\quad g = 1/f,$ $\quad h_i^m = 0.5$ if $m$ divides $i$, $\quad h_i^m = 1$ otherwise

# Fast VMTF Implementation

- fast simple implementation for caching searches in VMTF
  - doubly linked list does not have positions as an ordered array
  - bump = move-to-front = *dequeue* then *insertion* at the head
- time-stamp list entries with “insertion-time”
  - insertion-time same role as variable positions in ZChaff
  - maintained invariant: all variables right of next-search are assigned
  - requires (constant time) update to next-search while unassigning variables
  - occassionally (32-bit) time-stamps will overflow: update all time stamps

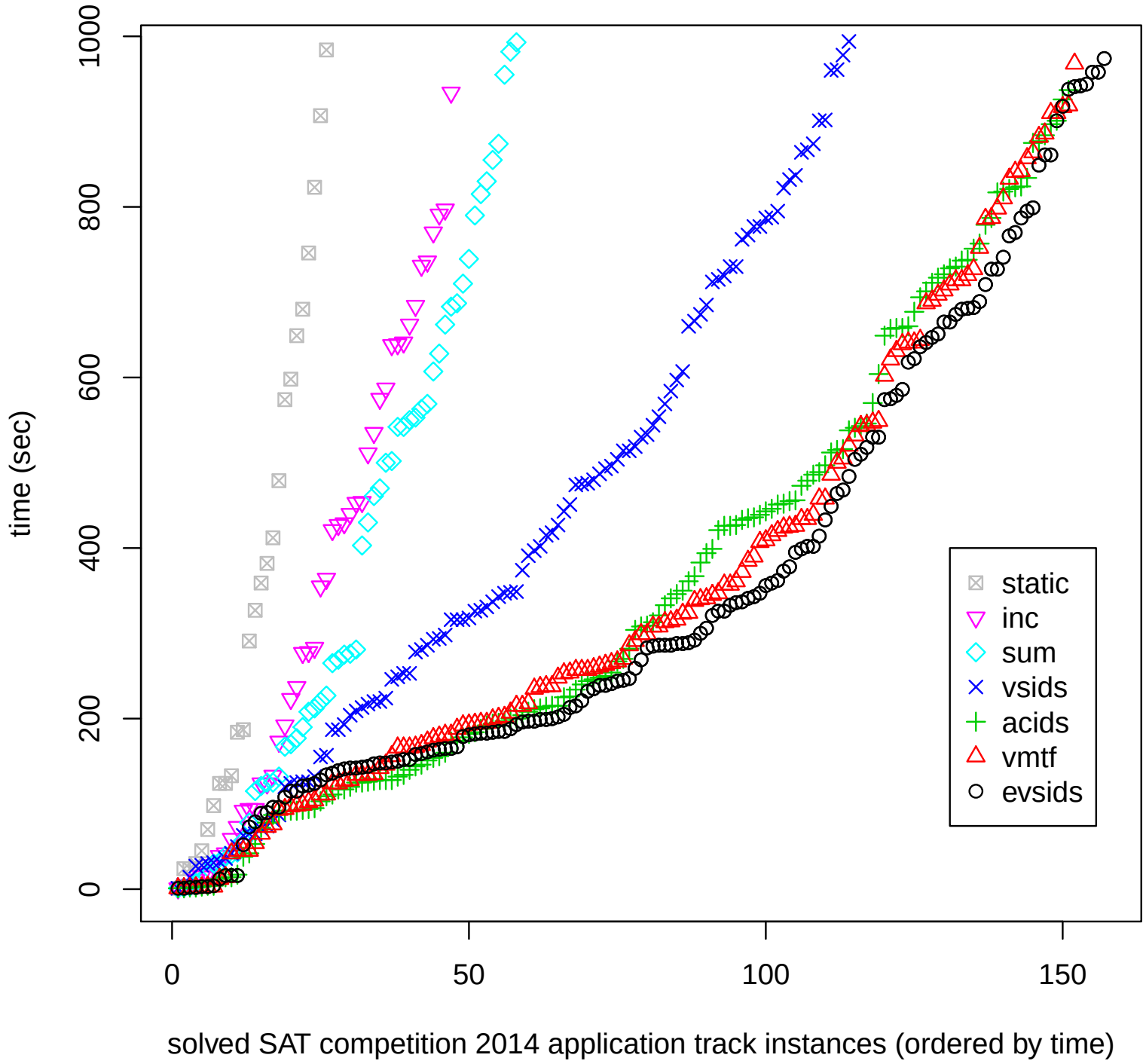


# Fast Generic Queue Implementation

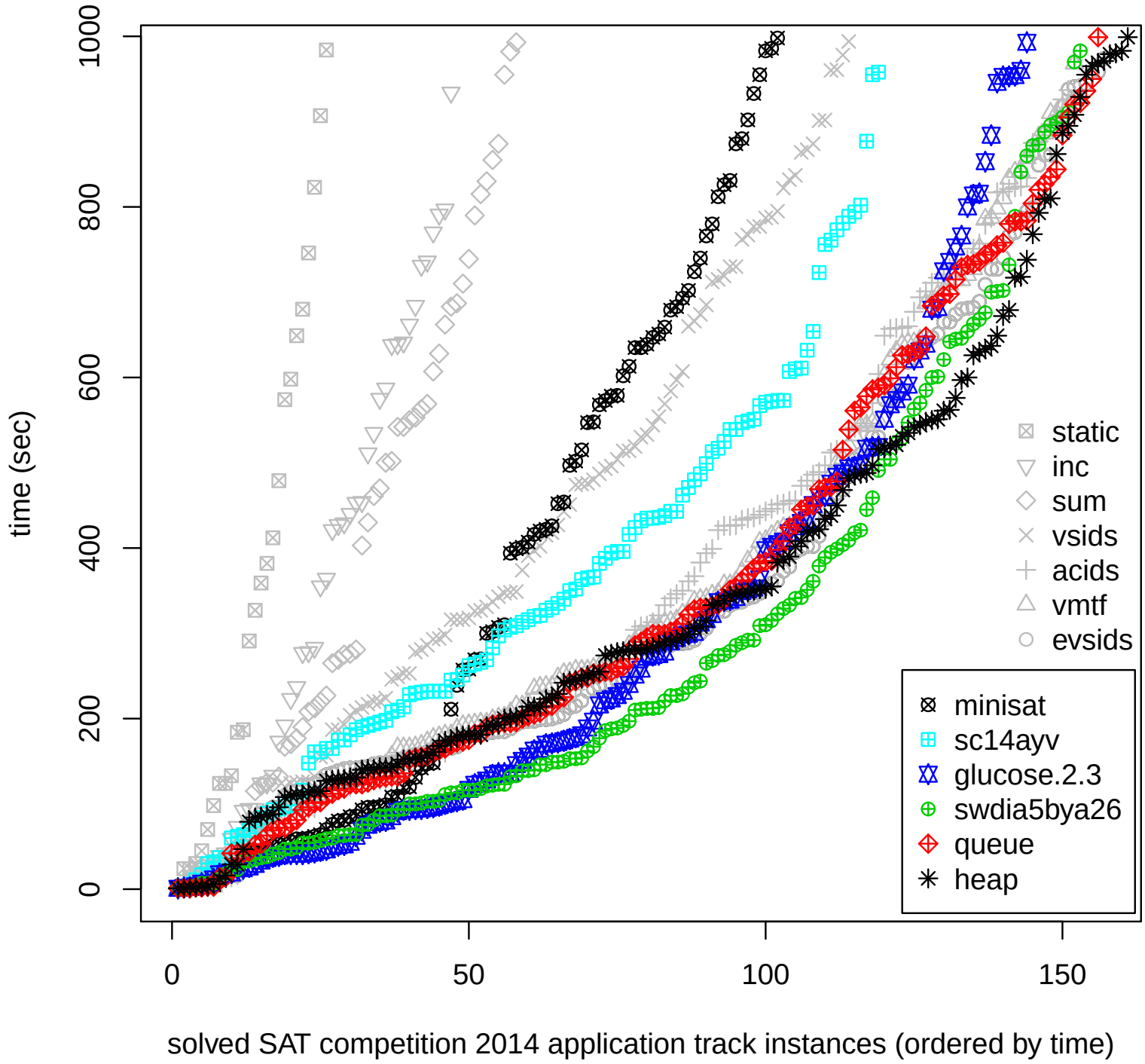


- one implementation working reasonably well for all schemes
- our preliminary results presented at BANFF'14 suggested VMTF not working
  - profiling revealed updating binary heap as a bottle neck
  - without time-stamping VMTF produces too much overhead
- array of variables split into doubly linked list of variables with same score **exponent**
  - similar to JeruSAT but for floating points
  - can be considered as a 1st level of **radix** / **bucket sort**
  - exponent-next-search position as in ZChaff
- each doubly linked list sorted by **mantissa** of scores
  - mantissa-next-search and insertion-times as in our fast VMTF implementation
  - key optimization is to add another **radix** / **bucket sort** level

    based on some bits of the mantissa
  - see paper for more optimization to avoid otherwise pathological cases
  - empirically not a bottle-neck anymore


time (sec)
0
200
400
600
800
1000
0
50
100
150
solved SAT competition 2014 application track instances (ordered by time)
static
inc
sum
vsids
acids
vmtf
evsids

| | evsids | vmtf | acids | vsids | sum | inc | static |
|---|---|---|---|---|---|---|---|
| solved | 157 | 152 | 151 | 114 | 58 | 47 | 26 |
| unsatisfiable | 87 | 85 | 82 | 51 | 22 | 17 | 9 |
| satisfiable | 70 | 67 | 69 | 63 | 36 | 30 | 17 |
| reductions (1e3 #) | 8 | 8 | 8 | 10 | 8 | 8 | 8 |
| restarts (1e3 #) | 5826 | 6000 | 5678 | 4491 | 2612 | 2387 | 5593 |
| rescored (1e3 #) | 253 | 0 | 0 | 2338 | 0 | 0 | 0 |
| conflicts (1e6 #) | 488 | 476 | 444 | 604 | 527 | 540 | 463 |
| decisions (1e6 #) | 3691 | 3581 | 3889 | 4263 | 2603 | 2567 | 21503 |
| simp (1e3 sec) | 29.7 | 30.0 | 29.4 | 32.6 | 34.5 | 34.1 | 31.2 |
| search (1e3 sec) | 143.1 | 146.4 | 147.9 | 174.9 | 203.9 | 209.7 | 226.7 |
| bump (1e3 sec) | 7.8 | 6.2 | 16.0 | 16.9 | 34.6 | 37.2 | 0.0 |
| decide (1e3 sec) | 2.3 | 2.5 | 2.6 | 2.8 | 1.7 | 1.7 | 12.9 |
| rescore (1e3 sec) | 0.2 | 0.0 | 0.0 | 2.6 | 0.0 | 0.0 | 0.0 |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evsids
minisat
sc14ayv
glucose.2.3
swdia5bya26
queue
heap

| | heap | queue | swd ia5by a26 | glu cose 2.3 | sc14 ayv | mini sat |
|---|---|---|---|---|---|---|
| solved | 161 | 156 | 153 | 144 | 119 | 101 |
| unsatisfiable | 90 | 86 | 81 | 79 | 60 | 41 |
| satisfiable | 71 | 70 | 72 | 65 | 59 | 60 |
| reductions (1e3 #) | 8 | 8 | 59 | 10 | 30 | — |
| restarts (1e3 #) | 5870 | 6003 | 3210 | 3846 | 7948 | 1782 |
| rescored (1e3 #) | 241 | 0 | — | — | 393 | — |
| conflicts (1e6 #) | 463 | 474 | 650 | 728 | 760 | 1090 |
| decisions (1e6 #) | 3874 | 3566 | 5868 | 6818 | 5002 | 8388 |
| simp (1e3 sec) | 29.2 | 29.7 | 0.8 | 0.8 | 32.4 | 2.2 |
| search (1e3 sec) | 141.8 | 144.6 | 165.4 | 172.5 | 164.4 | 206.5 |
| bump (1e3 sec) | 3.8 | 4.9 | — | — | 3.3 | — |
| decide (1e3 sec) | 4.9 | 2.5 | — | — | 6.4 | — |
| rescore (1e3 sec) | 0.1 | 0.0 | — | — | 0.0 | — |

# Conclusion



- surveyed and classified variable selection / scoring schemes
  - and came up with a new one ACIDS (as well as SUM)
  - EVSIDS, VMTF, ACIDS comparable in performance
  - with a generic fast queue implementation
- VMTF was considered to be obsolete
  - can be made effective (with less code than EVSIDS)
  - needs proper optimized implementation: time-stamping with insertion-time
  - VMTF might be easier to reason about in proof complexity
- threads to validity
  - unclear whether VMTF only works in combination with Glucose restarts
    see also our POS'15 paper and talk
  - benchmark selection in recent SAT competitions controversial
- future work: implement simple SAT solver only based on VMTF