Lemmas on Demand for the Extensional Theory of Arrays

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Overview

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- Introduction
- Arrays
- Verification example
- Lemmas on demand
- Consistency checking
- Lemma encoding
- Handling equalities on arrays
- Conclusion

- Bit-Vectors
  - Bit-precise modelling of computation
  - Modular arithmetic
- Arrays
  - Modelling of memories
  - Main memory in software
  - Memory components in hardware, e.g. registers

Combination of both: bit-precise modelling and reasoning

- Scenarios
  - Pipelined vs. non-pipelined hardware
  - Equivalence-check memory semantics of software
- Let both algorithm start on the same array
- Symbolically execute both algorithms
- Finally, check whether the resulting arrays are equal or not

- Theory of Arrays [McCarthy62]

$$\begin{array}{lllll} \text{(A1)} & a = b \wedge i = j & \Rightarrow & read(a,i) = read(b,j) \\ \text{(A2)} & i = j & \Rightarrow & read(write(a,i,e),j) = e \\ \text{(A3)} & i \neq j & \Rightarrow & read(write(a,i,e),j) = read(a,j) \end{array}$$

- With (A1) to (A3) we cannot handle array inequalities

- We need additional axiom of extensionality (A4) resp. (A4′)

$$\text{(A4)} \quad a = b \quad \Leftarrow \quad \forall i\,(read(a,i) = read(b,i))$$

$$\text{(A4′)} \quad a \neq b \quad \Rightarrow \quad \exists \lambda\,(read(a,\lambda) \neq read(b,\lambda))$$

Verification of Selection Sort for bit-width = 32 and size = 4

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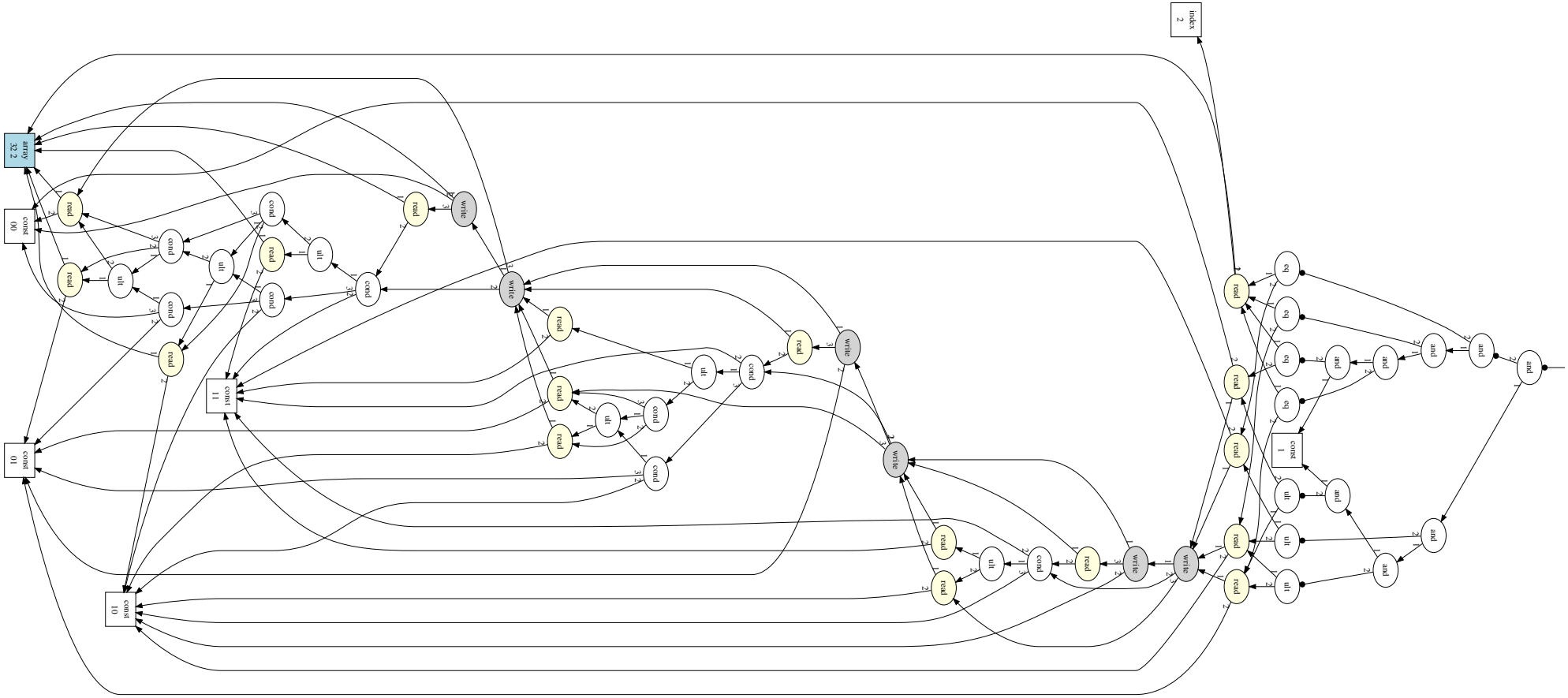

Lemmas On Demand (LOD) [DeMoura02]

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procedure lemmas-on-demand (ϕ)
  encode-to-sat (ϕ)
  loop
    $(r, \sigma) \leftarrow$ sat(ϕ)
    if ($r = unsatisfiable$) **return** *unsatisfiable*
    if (consistent (ϕ, σ)) **return** *satisfiable*
    add-lemma (ϕ, σ)

- Do not eagerly encode all constraints up front
- Explicitly check constraints that have not been encoded
  - Use SAT solver assignment σ
- Incrementally refine formula

- Translate bit-vector but not array part of the formula
- Let SAT solver “guess” solution
  - If SAT solver cannot find a solution, terminate with *unsatisfiable*
- Explicitly check Array axioms A1 to A3
- If check succeeds then terminate with *satisfiable*
- If check fails
  - Add lemma to refine formula
  - Let SAT solver “guess” a new solution

Checking Array Axioms

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- Propagation-based algorithm
  - Direct application of (A1) to (A3)
- Annotate every array expression with its set of reads ρ
- For every read $read(b, i) \in \rho(write(a, j, e))$
  - (A2): If current assignment $\sigma(i) = \sigma(j)$, check if $\sigma(read(b, i)) = \sigma(e)$
  - (A3): If current assignment $\sigma(i) \neq \sigma(j)$, add read to $\rho(a)$
- Check read congruence (A1) on all array expressions
- Propagation only downwards and can be implemented with DFS or BFS

Example 1: Inconsistent Assignment

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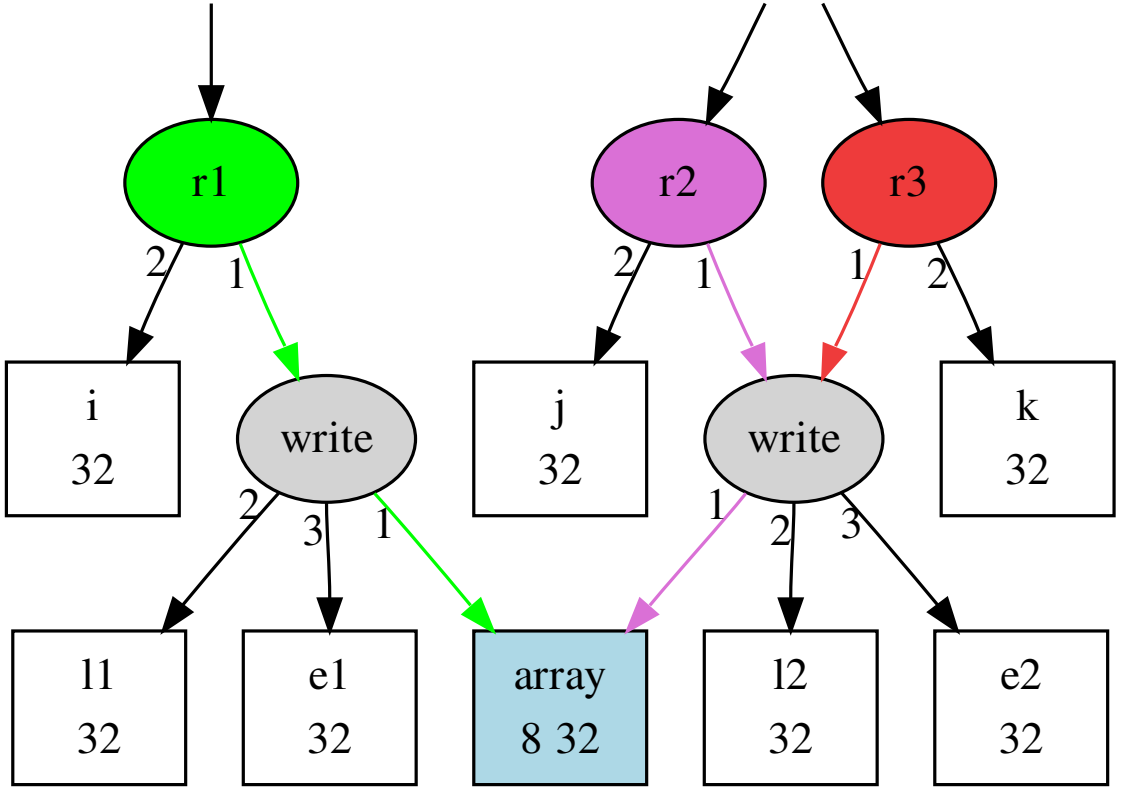
$$\sigma(i) = \sigma(j), \quad \sigma(k) = \sigma(l2)$$



$$\sigma(r1) \neq \sigma(r2), \quad \sigma(i) \neq \sigma(l1), \quad \sigma(j) \neq \sigma(l2), \quad \sigma(r3) \neq \sigma(e2)$$

Lemma Construction

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- We collect all assignments responsible for propagation
- We add a lemma of the following kind:
  - $i \neq l_1 \wedge j \neq l_2 \wedge \ldots \wedge i = j \quad \Rightarrow \quad r_1 = r_2$
- Lemma for inconsistency of r_1 and r_2 in example 1
  - $i \neq l_1 \wedge j \neq l_2 \wedge i = j \quad \Rightarrow \quad r_1 = r_2$
- Lemma for inconsistency of r_3 and right $write$ in example 1
  - $k = l_2 \quad \Rightarrow \quad r_3 = e_2$
- Lemmas can be directly encoded into CNF (no new expressions)

Adding Equalities on Arrays (1/2)

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- For every array equality a = b
  - Introduce fresh Tseitin variable $e_{a,b}$
  - Introduce two virtual reads $read(a,\lambda)$, $read(b,\lambda)$, for a fresh λ
  - Virtual reads are used as witness for array inequality
  - Encode $\bar{e}_{a,b} \quad \Rightarrow \quad read(a,\lambda) \neq read(b,\lambda)$

- If $\sigma(e_{a,b}) = 1$, propagate reads over array equalities
  - Ensures read congruence over equal arrays
  - Propagation can now be cyclic, e.g. $a = b \wedge b = c \wedge c = a$
  - We need fix-point algorithm

Adding Equalities on Arrays (2/2)

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- We must ensure congruence on write values for equal writes (only necessary in a setting without congruence closure)
- For example $write(a, i, e_1) = write(a, i, e_2)$ implies that $e_1 = e_2$
- We can treat every $write(a, i, e)$ as $read(a, i)$, where $read(a, i) = e$
- Propagate writes as reads
- In order to reach every array equality
  - We must also propagate upwards with respect to (A2) and (A3)
  - Only propagate upwards if value has not been overwritten

Example 2: Propagation upwards

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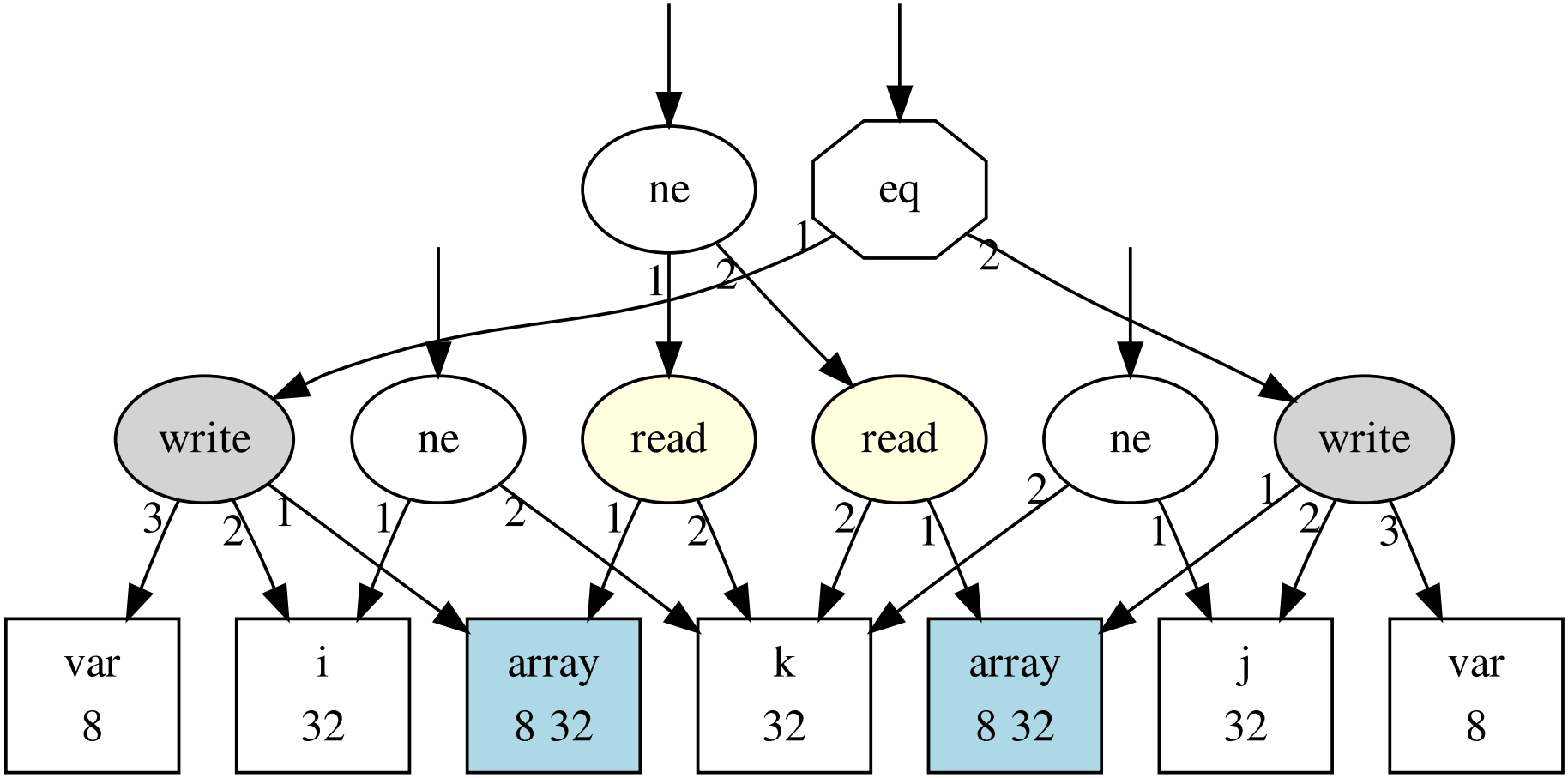

$$write(a,i,e_1) = write(b,j,e_2), \quad i \neq k, \quad j \neq k, \quad read(a,k) \neq read(b,k)$$

Extensional Experiments (1/2)

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			lemmas			*eager*		z3		cvc3	
benchmark	*S*	*P*	*R*	*T*	*M*	*T*	*M*	*T*	*M*	*T*	*M*
swapmem	s	2	16	0.1	0.6	**0.0**	**0.0**	0.1	5.0	554.1	18.2
swapmem	s	6	53	**0.4**	**1.2**	0.6	3.3	547.6	101.6	90.0	71.9
swapmem	s	8	37	**0.3**	**1.4**	1.5	5.4	TO	TO	333.8	175.8
swapmem	s	10	34	**0.5**	**1.7**	1.0	8.3	TO	TO	740.2	305.3
swapmem	s	12	64	**1.0**	**2.2**	3.5	11.2	TO	TO	TO	TO
swapmem	s	14	57	**1.6**	**2.4**	1.8	15.7	TO	TO	TO	TO
wchains	s	6	14	**0.0**	**0.0**	0.8	7.0	46.3	11.6	8.5	60.1
wchains	s	8	16	**0.0**	**0.0**	1.9	14.5	572.2	32.5	23.5	117.6
wchains	s	10	18	**0.1**	**1.3**	3.1	20.3	TO	TO	34.2	193.6
wchains	s	12	20	**0.1**	**1.4**	3.5	29.8	TO	TO	65.4	294.6
wchains	s	15	21	**0.1**	**1.5**	4.2	45.8	TO	TO	126.3	472.6
wchains	s	20	28	**0.3**	**2.0**	5.7	80.2	TO	TO	267.8	897.3
wchains	s	30	36	**0.6**	**2.8**	10.9	179.9	TO	TO	MO	MO
wchains	s	60	70	**2.3**	**4.8**	41.8	718.5	TO	TO	MO	MO
wchains	s	90	96	**4.8**	**6.8**	MO	MO	TO	TO	MO	MO
dubreva	u	2	19	0.1	0.6	**0.0**	**0.0**	0.5	5.0	TO	TO
dubreva	u	3	41	0.3	**0.7**	**0.1**	1.2	0.7	5.6	TO	TO
dubreva	u	4	239	**12.5**	**2.2**	**12.5**	6.5	TO	TO	TO	TO
dubreva	u	5	379	**27.4**	**3.7**	348.3	13.1	TO	TO	TO	TO
dubreva	u	6	1187	**322.2**	**12.4**	TO	TO	TO	TO	TO	TO
dubreva	u	7	1637	**663.1**	**18.2**	TO	TO	TO	TO	MO	MO

Extensional Experiments (2/2)

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			lemmas			*eager*		z3		cvc3	
benchmark	*S*	*P*	*R*	*T*	*M*	*T*	*M*	*T*	*M*	*T*	*M*
swapmem	u	2	13	**0.1**	**0.7**	**0.1**	0.8	1.2	5.5	6.1	9.0
swapmem	u	4	25	**1.1**	**1.0**	3.0	2.0	5.2	7.1	159.6	49.8
swapmem	u	6	37	**3.2**	**1.2**	27.9	4.2	16.1	8.6	TO	TO
swapmem	u	8	49	**9.0**	**1.5**	129.0	8.3	20.7	10.1	TO	TO
swapmem	u	10	61	**18.3**	**2.0**	411.2	15.4	30.9	11.0	TO	TO
swapmem	u	12	73	**34.7**	**2.4**	TO	TO	62.1	13.3	TO	TO
swapmem	u	14	85	**63.4**	**2.8**	TO	TO	102.7	19.4	TO	TO
wchains	u	2	17	**0.0**	**0.0**	**0.0**	**0.0**	TO	TO	4.1	5.8
wchains	u	4	33	**0.1**	**0.9**	1.7	3.8	10.5	16.4	TO	TO
wchains	u	6	49	**0.4**	**1.1**	13.5	8.2	1.9	5.5	MO	MO
wchains	u	8	65	**0.6**	**1.3**	54.8	15.0	848.0	725.1	MO	MO
wchains	u	10	81	**1.1**	**1.5**	158.2	23.3	59.7	14.6	MO	MO
wchains	u	12	97	**2.4**	**1.8**	333.5	34.3	93.7	22.4	MO	MO
wchains	u	14	113	**2.9**	**1.9**	588.2	46.7	19.3	6.9	MO	MO
wchains	u	16	129	**4.8**	**2.1**	TO	TO	35.7	8.1	MO	MO
wchains	u	18	145	**6.6**	**2.3**	TO	TO	209.8	26.2	MO	MO

Non-Extensional Experiments

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		lemmas			*eager*		z3		cvc3		stp	
benchmark	*S*	*R*	*T*	*M*	*T*	*M*	*T*	*M*	*T*	*M*	*T*	*M*
bubsort4	u	10	0.9	**0.8**	**0.2**	**0.8**	TO	TO	TO	TO	0.3	7.6
bubsort6	u	23	4.6	**1.1**	**2.3**	1.5	TO	TO	TO	TO	19.2	10.3
bubsort8	u	40	66.1	**1.8**	**15.8**	2.7	TO	TO	TO	TO	TO	TO
bubsort10	u	61	**461.4**	**3.1**	514.0	10.1	TO	TO	TO	TO	TO	TO
bubsort12	u	-	TO	TO	TO	TO	TO	TO	TO	TO	TO	TO
selsort2	u	8	0.2	**0.6**	**0.1**	0.7	TO	TO	TO	TO	0.2	7.3
selsort3	u	21	4.2	**0.8**	**0.8**	1.1	TO	TO	TO	TO	1.5	9.8
selsort4	u	42	35.6	**1.2**	**5.4**	1.7	TO	TO	TO	TO	14.6	16.9
selsort5	u	66	216.8	**2.3**	**34.7**	3.1	TO	TO	TO	TO	121.7	37.4
selsort6	u	-	TO	TO	**407.2**	**7.6**	TO	TO	TO	TO	TO	TO
selsort7	u	-	TO	TO	TO	TO	TO	TO	TO	TO	TO	TO
grep0084	s	191	31.1	**27.4**	23.7	142.7	81.1	73.2	TO	TO	**15.3**	124.4
grep0095	s	134	**24.6**	**26.3**	26.6	153.7	101.5	78.6	TO	TO	187.0	550.4
grep0106	s	300	41.6	**30.6**	**33.5**	164.9	103.2	88.0	TO	TO	186.2	510.2
grep0117	s	205	35.1	**29.9**	**34.7**	176.4	149.9	90.2	TO	TO	139.9	412.1
grep0777	s	474	**481.4**	**85.1**	MO	MO	TO	TO	TO	TO	MO	MO
testcase15	s	0	99.5	**457.9**	98.0	458.8	**32.4**	594.6	MO	MO	TO	TO
testcase16	s	0	73.4	**476.5**	73.4	**476.5**	**33.3**	627.4	MO	MO	TO	TO
testcase20	s	-	TO	TO	MO	MO	TO	TO	MO	MO	TO	TO
tbnail-sp1	s	0	**58.9**	**1173.7**	62.3	1178.2	MO	MO	MO	MO	TO	TO
610dd9dc	s	319	72.1	**12.5**	33.6	148.2	TO	TO	MO	MO	**21.6**	303.9

Conclusion

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- Lemmas on demand for Extensional Theory of Arrays
  - In our examples with Bit-vectors, but approach is more general
  - SAT solver is used offline as black box
- Algorithm based on propagation and direct application of array axioms
  - Non-extensional algorithm with DFS or BFS
  - Introducing equality on arrays requires fix-point algorithm
  - Virtual reads as witnesses for array inequalities
- Algorithm implemented in our SMT solver Boolector