INCREMENTAL INPROCESSING IN SAT SOLVING

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July 9, 2019, Lisbon

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INTRODUCTION

Boolean Satisfiability Problem (SAT)

- Propositional logic

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$$(a \vee \neg b) \wedge (a \vee b) \wedge (\neg a \vee \neg b)$$

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- NP-complete problem: Is this set of clauses satisfiable?

$$(a \lor \neg b) \land (a \lor b) \land (\neg a \lor \neg b)$$

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- NP-complete problem: Is this set of clauses satisfiable?
  - If yes: Provide satisfying truth assignment

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- NP-complete problem: Is this set of clauses satisfiable?
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$$\{a = \top, b = \bot\}$$

$$(a \vee \neg b) \wedge (a \vee b) \wedge (\neg a \vee \neg b)$$

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$$\{a = \top, b = \bot\} = \{a, \neg b\}$$

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Incremental SAT Solving

$C_1 \wedge \dots \wedge C_n$ $\qquad$ $SAT?$

Incremental SAT Solving

$\underbrace{C_1 \wedge \cdots \wedge C_n}_{\Delta_0}$ $\qquad$ $SAT?$

Incremental SAT Solving

$$\underbrace{\underbrace{C_1 \wedge \cdots \wedge C_n}_{\Delta_0}}_{F^0}$$

$SAT?$

Incremental SAT Solving

$$\underbrace{\underbrace{C_1 \wedge \cdots \wedge C_n}_{\Delta_0}}_{F^0} \wedge C'_1 \wedge \cdots \wedge C'_{n'} \qquad SAT?$$

Incremental SAT Solving

$$\underbrace{\underbrace{C_1 \wedge \cdots \wedge C_n}_{\Delta_0}}_{F^0} \wedge \underbrace{C'_1 \wedge \cdots \wedge C'_{n'}}_{\Delta_1} \qquad SAT?$$

Incremental SAT Solving

$$\underbrace{\underbrace{\underbrace{C_1 \land \cdots \land C_n}_{\Delta_0}}_{F^0} \land \underbrace{C'_1 \land \cdots \land C'_{n'}}_{\Delta_1}}_{F^1}$$

$SAT?$

Incremental SAT Solving

$$\underbrace{\underbrace{\underbrace{C_1 \wedge \cdots \wedge C_n}_{\Delta_0}}_{F^0} \wedge \underbrace{C'_1 \wedge \cdots \wedge C'_{n'}}_{\Delta_1}}_{F^1} \wedge C''_1 \wedge \cdots \wedge C''_{n''} \quad SAT?$$

Incremental SAT Solving

$$\underbrace{\underbrace{\underbrace{C_1 \wedge \cdots \wedge C_n}_{\Delta_0}}_{F^0} \wedge \underbrace{C'_1 \wedge \cdots \wedge C'_{n'}}_{\Delta_1}}_{F^1} \wedge \underbrace{C''_1 \wedge \cdots \wedge C''_{n''}}_{\Delta_2} \quad SAT?$$

Incremental SAT Solving

$$\underbrace{\underbrace{\underbrace{\underbrace{C_1 \wedge \cdots \wedge C_n}_{\Delta_0}}_{F^0} \wedge \underbrace{C'_1 \wedge \cdots \wedge C'_{n'}}_{\Delta_1}}_{F^1} \wedge \underbrace{C''_1 \wedge \cdots \wedge C''_{n''}}_{\Delta_2}}_{F^2} \quad SAT?$$

Incremental SAT Solving

$$\underbrace{\underbrace{\underbrace{\underbrace{C_1 \wedge \cdots \wedge C_n}_{\Delta_0}}_{F^0} \wedge \underbrace{C'_1 \wedge \cdots \wedge C'_{n'}}_{\Delta_1}}_{F^1} \wedge \underbrace{C''_1 \wedge \cdots \wedge C''_{n''}}_{\Delta_2}}_{F^2} \quad SAT?$$

For each $i = 0 \ldots m$ is $F^i = \bigwedge_{d=0}^{i} \Delta_d$ satisfiable?

Incremental SAT Solving

$$\underbrace{\underbrace{\underbrace{\underbrace{C_1 \land \cdots \land C_n}_{\Delta_0}}_{F^0} \land \underbrace{C'_1 \land \cdots \land C'_{n'}}_{\Delta_1}}_{F^1} \land \underbrace{C''_1 \land \cdots \land C''_{n''}}_{\Delta_2}}_{F^2} \quad SAT?$$

For each $i = 0 \dots m$ is $F^i = \bigwedge_{d=0}^{i} \Delta_d$ satisfiable?

- Extend formula with new clauses

Incremental SAT Solving

$$\underbrace{\underbrace{\underbrace{\underbrace{C_1 \wedge \cdots \wedge C_n}_{\Delta_0}}_{F^0} \wedge \underbrace{C'_1 \wedge \cdots \wedge C'_{n'}}_{\Delta_1}}_{F^1} \wedge \underbrace{C''_1 \wedge \cdots \wedge C''_{n''}}_{\Delta_2}}_{F^2} \quad SAT?$$

For each $i = 0 \dots m$ is $F^i = \bigwedge_{d=0}^{i} \Delta_d$ satisfiable?

- Extend formula with new clauses
- Avoid repeated work
  - Keep gathered information (e.g. scores, search state variables)
  - Keep learned clauses

Inprocessing in SAT Solving [JärvisaloHeuleBiere-IJCAR'12]

- Satisfiability preserving clause addition or removal

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- Satisfiability preserving clause addition or removal
- Abstract framework that captures generally inprocessing
- Deduction rules applied on abstract states

Inprocessing in SAT Solving [JärvisaloHeuleBiere-IJCAR'12]

φ: Irredundant clauses

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φ: Irredundant clauses ρ: Redundant clauses

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φ: Irredundant clauses ρ: Redundant clauses σ: Eliminated clauses

Inprocessing in SAT Solving [JärvisaloHeuleBiere-IJCAR'12]

φ: Irredundant clauses	ρ: Redundant clauses	σ: Eliminated clauses
$a \vee b$		
$\neg a \vee b$		
a		

Inprocessing in SAT Solving [JärvisaloHeuleBiere-IJCAR'12]

φ: Irredundant clauses	ρ: Redundant clauses	σ: Eliminated clauses
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a		

$$\frac{\varphi\ [\,\rho\,]\ \sigma}{\varphi\ [\,\rho \wedge C\,]\ \sigma}\ \boxed{\sharp}$$

LEARN

where $\boxed{\sharp}$ is $\varphi \wedge \rho \equiv_{sat} \varphi \wedge \rho \wedge C$

Inprocessing in SAT Solving [JärvisaloHeuleBiere-IJCAR'12]

φ: Irredundant clauses	ρ: Redundant clauses	σ: Eliminated clauses
$a \lor b$	b	
$\neg a \lor b$		
a		

$$\frac{\varphi\ [\,\rho\,]\ \sigma}{\varphi\ [\,\rho \land C\,]\ \sigma}\ \boxed{\sharp}$$

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where $\boxed{\sharp}$ is $\varphi \land \rho \equiv_{sat} \varphi \land \rho \land C$

Inprocessing in SAT Solving [JärvisaloHeuleBiere-IJCAR'12]

φ: Irredundant clauses	ρ: Redundant clauses	$\boldsymbol{\sigma}$: Eliminated clauses
$\boldsymbol{a \vee b}$	$\boldsymbol{b}$	
$\boldsymbol{\neg a \vee b}$	$\boldsymbol{a \vee c}$	
$\boldsymbol{a}$		

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φ: Irredundant clauses	ρ: Redundant clauses	σ: Eliminated clauses
$a \vee b$	b	
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a		

$$\frac{\varphi\ [\,\rho \wedge C\,]\ \sigma}{\varphi \wedge C\ [\,\rho\,]\ \sigma}$$

STRENGTHEN

Inprocessing in SAT Solving [JärvisaloHeuleBiere-IJCAR'12]

φ: Irredundant clauses ρ: Redundant clauses $\boldsymbol{\sigma}$: Eliminated clauses

φ	ρ	σ
$a \lor b$	$\cancel{b}$	
$\neg a \lor b$	$a \lor c$	
a		
b		

$$\frac{\varphi\ [\,\rho \land C\,]\ \sigma}{\varphi \land C\ [\,\rho\,]\ \sigma}$$

STRENGTHEN

Inprocessing in SAT Solving [JärvisaloHeuleBiere-IJCAR'12]

$\boldsymbol{\varphi}$: Irredundant clauses $\boldsymbol{\rho}$: Redundant clauses $\boldsymbol{\sigma}$: Eliminated clauses

$\boldsymbol{a \lor b}$	$\cancel{\boldsymbol{b}}$	
$\boldsymbol{\neg a \lor b}$	$\boldsymbol{a \lor c}$	
$\boldsymbol{a}$		
$\boldsymbol{b}$		

$$\frac{\varphi\ [\,\rho \land C\,]\ \sigma}{\varphi\ [\,\rho\,]\ \sigma}$$

FORGET

Inprocessing in SAT Solving [JärvisaloHeuleBiere-IJCAR'12]

φ: Irredundant clauses ρ: Redundant clauses σ: Eliminated clauses

φ	ρ	σ
$a \vee b$	$\cancel{b}$	
$\neg a \vee b$	$\cancel{a \vee c}$	
a		
b		

$$\frac{\varphi\ [\,\rho \wedge C\,]\ \sigma}{\varphi\ [\,\rho\,]\ \sigma}$$

FORGET

Inprocessing in SAT Solving [JärvisaloHeuleBiere-IJCAR'12]

φ: Irredundant clauses	ρ: Redundant clauses	σ: Eliminated clauses
$a \lor b$	$\cancel{b}$	
$\neg a \lor b$	$\cancel{a \lor c}$	
a		
b		

$$\frac{\varphi \land C \; [\, \rho \,] \; \sigma}{\varphi \; [\, \rho \land C \,] \; \sigma \cdot (l : C)} \; \boxed{\flat}$$

WEAKEN

where $\boxed{\flat}$ is $\varphi \land C \equiv^{l}_{sat} \varphi$

Inprocessing in SAT Solving [JärvisaloHeuleBiere-IJCAR'12]

φ: Irredundant clauses ρ: Redundant clauses σ: Eliminated clauses

φ	ρ	σ
$\cancel{a \vee b}$	$\cancel{b}$	$b : (a \vee b)$
$\neg a \vee b$	$\cancel{a \vee c}$	
a	$a \vee b$	
b		

$$\frac{\varphi \wedge C \; [\, \rho \,] \; \sigma}{\varphi \; [\, \rho \wedge C \,] \; \sigma \cdot (l : C)} \; \boxed{\flat}$$

WEAKEN

where $\boxed{\flat}$ is $\varphi \wedge C \equiv^{l}_{sat} \varphi$

Inprocessing in SAT Solving [JärvisaloHeuleBiere-IJCAR'12]

φ: Irredundant clauses ρ: Redundant clauses σ: Eliminated clauses

φ	ρ	σ
$\cancel{a \vee b}$	$\cancel{b}$	$b : (a \vee b)$
$\neg a \vee b$	$\cancel{a \vee c}$	
a	$a \vee b$	
b		

$$\frac{\varphi\ [\,\rho\,]\ \sigma}{\varphi\ [\,\rho \wedge C\,]\ \sigma}\ \boxed{\sharp} \qquad \frac{\varphi\ [\,\rho \wedge C\,]\ \sigma}{\varphi\ [\,\rho\,]\ \sigma} \qquad \frac{\varphi\ [\,\rho \wedge C\,]\ \sigma}{\varphi \wedge C\ [\,\rho\,]\ \sigma} \qquad \frac{\varphi \wedge C\ [\,\rho\,]\ \sigma}{\varphi\ [\,\rho \wedge C\,]\ \sigma \cdot (l : C)}\ \boxed{\flat}$$

LEARN FORGET STRENGTHEN WEAKEN

where $\boxed{\sharp}$ is $\varphi \wedge \rho \equiv_{sat} \varphi \wedge \rho \wedge C$ and $\boxed{\flat}$ is $\varphi \wedge C \equiv^{\ell}_{sat} \varphi$

Solution Reconstruction [Sörensson 2009, JHB-IJCAR'12]

- Inprocessing is satisfiability but not model preserving

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- Inprocessing is satisfiability but not model preserving
- Solution reconstruction is necessary

Solution Reconstruction [Sörensson 2009, JHB-IJCAR'12]

$$\mathcal{F}^0 = (\boldsymbol{a} \vee \boldsymbol{b}) \wedge (\neg \boldsymbol{a} \vee \boldsymbol{b}) \wedge (\boldsymbol{a})$$

$\boldsymbol{a} \vee \boldsymbol{b}$

$\neg \boldsymbol{a} \vee \boldsymbol{b}$

$\boldsymbol{a}$

Solution Reconstruction [Sörensson 2009, JHB-IJCAR'12]

$$\mathcal{F}^0 = (a \lor b) \land (\neg a \lor b) \land (a)$$

$\cancel{a \lor b}$	$a \lor b$	$b : (a \lor b)$
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a		

Solution Reconstruction [Sörensson 2009, JHB-IJCAR'12]

$$\mathcal{F}^0 = (a \lor b) \land (\neg a \lor b) \land (a)$$

$\cancel{a \lor b}$	$a \lor b$	$b : (a \lor b)$
$\cancel{\neg a \lor b}$	$\neg a \lor b$	$b : (\neg a \lor b)$
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$$\mathcal{F}^0 = (\boxed{a} \lor b) \land (\neg a \lor b) \land (\boxed{a})$$

$\cancel{a \lor b}$	$a \lor b$	$b : (a \lor b)$
$\cancel{\neg a \lor b}$	$\neg a \lor b$	$b : (\neg a \lor b)$
a		

$$\tau = \{a = \top, b = \bot\}$$

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$\cancel{\boldsymbol{a} \vee \boldsymbol{b}}$	$\boldsymbol{a} \vee \boldsymbol{b}$	$\boldsymbol{b} : (\boldsymbol{a} \vee \boldsymbol{b})$
$\cancel{\neg \boldsymbol{a} \vee \boldsymbol{b}}$	$\neg \boldsymbol{a} \vee \boldsymbol{b}$	$\boldsymbol{b} : (\neg \boldsymbol{a} \vee \boldsymbol{b})$
$\boldsymbol{a}$		

$$\boldsymbol{\tau} = \{\boldsymbol{a} = \top, \boldsymbol{b} = \bot\}$$

$$\mathcal{R}(\tau, \varepsilon) = \tau, \qquad \mathcal{R}(\tau, \sigma \cdot (\omega : D)) = \begin{cases} \mathcal{R}(\tau, \sigma) & \text{if } \tau(D) = \top \\ \mathcal{R}((\tau \circ \omega), \sigma) & \text{otherwise} \end{cases}$$

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$$\mathcal{F}^0 = (a \vee b) \wedge (\neg a \vee b) \wedge (a)$$

$\cancel{a \vee b}$	$a \vee b$	$b : (a \vee b)$
$\cancel{\neg a \vee b}$	$\neg a \vee b$	$b : (\neg a \vee b)$
a		

$$\tau = \{a = \top, b = \bot\}$$

$$\mathcal{R}(\{a = \top, b = \bot\}, (b : (a \vee b)) \cdot (b : (\neg a \vee b)))$$

Solution Reconstruction [Sörensson 2009, JHB-IJCAR'12]

$$\mathcal{F}^0 = (a \vee b) \wedge (\neg a \vee b) \wedge (a)$$

$\cancel{a \vee b}$	$a \vee b$	$b : (a \vee b)$
$\cancel{\neg a \vee b}$	$\neg a \vee b$	$b : (\neg a \vee b)$
a		

$$\tau = \{a = \top, b = \bot\}$$

$$\mathcal{R}(\{a = \top, b = \bot\}, (b : (a \vee b)) \cdot (b : (\neg a \vee b))) =$$
$$\mathcal{R}(\{a = \top, b = \top\}, (b : (a \vee b)))$$

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$$\mathcal{F}^0 = (a \vee b) \wedge (\neg a \vee b) \wedge (a)$$

$\cancel{a \vee b}$	$a \vee b$	$b : (a \vee b)$
$\cancel{\neg a \vee b}$	$\neg a \vee b$	$b : (\neg a \vee b)$
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$$\tau = \{a = \top, b = \bot\}$$

$$\mathcal{R}(\{a = \top, b = \bot\}, (b : (a \vee b)) \cdot (b : (\neg a \vee b))) =$$
$$\mathcal{R}(\{a = \top, b = \top\}, (b : (a \vee b)))$$

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$$\mathcal{F}^0 = (\boldsymbol{a} \vee \boldsymbol{b}) \wedge (\neg \boldsymbol{a} \vee \boldsymbol{b}) \wedge (\boldsymbol{a})$$

$\cancel{\boldsymbol{a} \vee \boldsymbol{b}}$	$\boldsymbol{a} \vee \boldsymbol{b}$	$\boldsymbol{b} : (\boldsymbol{a} \vee \boldsymbol{b})$
$\cancel{\neg \boldsymbol{a} \vee \boldsymbol{b}}$	$\neg \boldsymbol{a} \vee \boldsymbol{b}$	$\boldsymbol{b} : (\neg \boldsymbol{a} \vee \boldsymbol{b})$
$\boldsymbol{a}$		

$$\boldsymbol{\tau} = \{\boldsymbol{a} = \top, \boldsymbol{b} = \bot\}$$

$$\begin{aligned}
&\boldsymbol{\mathcal{R}}(\{\boldsymbol{a} = \top, \boldsymbol{b} = \bot\}, (\boldsymbol{b} : (\boldsymbol{a} \vee \boldsymbol{b})) \cdot (\boldsymbol{b} : (\neg \boldsymbol{a} \vee \boldsymbol{b}))) = \\
&\boldsymbol{\mathcal{R}}(\{\boldsymbol{a} = \top, \boldsymbol{b} = \top\}, (\boldsymbol{b} : (\boldsymbol{a} \vee \boldsymbol{b}))) = \\
&\boldsymbol{\mathcal{R}}(\{\boldsymbol{a} = \top, \boldsymbol{b} = \top\}, \varepsilon) = \{\boldsymbol{a} = \top, \boldsymbol{b} = \top\}.
\end{aligned}$$

Solution Reconstruction [Sörensson 2009, JHB-IJCAR'12]

$$\mathcal{F}^0 = (a \lor b) \land (\neg a \lor b) \land (a)$$

$$\begin{array}{c|c|c}
\cancel{a \lor b} & a \lor b & b : (a \lor b) \\
\cancel{\neg a \lor b} & \neg a \lor b & b : (\neg a \lor b) \\
a & &
\end{array}$$

$$\tau = \{a = \top, b = \bot\}$$

$$\begin{aligned}
&\mathcal{R}(\{a = \top, b = \bot\}, (b : (a \lor b)) \cdot (b : (\neg a \lor b))) = \\
&\mathcal{R}(\{a = \top, b = \top\}, (b : (a \lor b))) = \\
&\mathcal{R}(\{a = \top, b = \top\}, \varepsilon) = \{a = \top, b = \top\}.
\end{aligned}$$

INCREMENTAL INPROCESSING

Incremental Inprocessing – Problem

$$\mathcal{F}^0 = (\boldsymbol{a} \vee \boldsymbol{b}) \wedge (\neg\boldsymbol{a} \vee \neg\boldsymbol{b})$$

$\boldsymbol{a} \vee \boldsymbol{b}$		
$\neg\boldsymbol{a} \vee \neg\boldsymbol{b}$		

Incremental Inprocessing – Problem

$$\mathcal{F}^0 = (\boldsymbol{a} \lor \boldsymbol{b}) \land (\neg \boldsymbol{a} \lor \neg \boldsymbol{b})$$

$\cancel{\boldsymbol{a} \lor \boldsymbol{b}}$	$\boldsymbol{a} \lor \boldsymbol{b}$	$\boldsymbol{a} : (\boldsymbol{a} \lor \boldsymbol{b})$
$\neg \boldsymbol{a} \lor \neg \boldsymbol{b}$		

Incremental Inprocessing – Problem

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$\cancel{\boldsymbol{a} \vee \boldsymbol{b}}$	$\boldsymbol{a} \vee \boldsymbol{b}$	$\boldsymbol{a} : (\boldsymbol{a} \vee \boldsymbol{b})$
$\cancel{\neg \boldsymbol{a} \vee \neg \boldsymbol{b}}$	$\neg \boldsymbol{a} \vee \neg \boldsymbol{b}$	$\neg \boldsymbol{b} : (\neg \boldsymbol{a} \vee \neg \boldsymbol{b})$

Incremental Inprocessing – Problem

$$\mathcal{F}^0 = (\boldsymbol{a} \vee \boldsymbol{b}) \wedge (\neg\boldsymbol{a} \vee \neg\boldsymbol{b})$$

$\cancel{\boldsymbol{a} \vee \boldsymbol{b}}$	$\boldsymbol{a} \vee \boldsymbol{b}$	$\boldsymbol{a} : (\boldsymbol{a} \vee \boldsymbol{b})$
$\cancel{\neg\boldsymbol{a} \vee \neg\boldsymbol{b}}$	$\neg\boldsymbol{a} \vee \neg\boldsymbol{b}$	$\neg\boldsymbol{b} : (\neg\boldsymbol{a} \vee \neg\boldsymbol{b})$

$$\boldsymbol{\tau} = \{\boldsymbol{a} = \bot, \boldsymbol{b} = \bot\}$$

Incremental Inprocessing – Problem

$$\mathcal{F}^0 = (\boldsymbol{a} \vee \boldsymbol{b}) \wedge (\neg \boldsymbol{a} \vee \neg \boldsymbol{b})$$

$\cancel{\boldsymbol{a} \vee \boldsymbol{b}}$	$\boldsymbol{a} \vee \boldsymbol{b}$	$\boldsymbol{a} : (\boldsymbol{a} \vee \boldsymbol{b})$
$\cancel{\neg \boldsymbol{a} \vee \neg \boldsymbol{b}}$	$\neg \boldsymbol{a} \vee \neg \boldsymbol{b}$	$\neg \boldsymbol{b} : (\neg \boldsymbol{a} \vee \neg \boldsymbol{b})$

$$\boldsymbol{\tau} = \{\boldsymbol{a} = \bot, \boldsymbol{b} = \bot\}$$

$$\mathcal{R}(\{\boldsymbol{a} = \bot, \boldsymbol{b} = \bot\}, (\boldsymbol{a} : (\boldsymbol{a} \vee \boldsymbol{b})) \cdot (\neg \boldsymbol{b} : (\neg \boldsymbol{a} \vee \neg \boldsymbol{b}))) = \{\boldsymbol{a} = \top, \boldsymbol{b} = \bot\}$$

Incremental Inprocessing – Problem

$$\mathcal{F}^0 = (\boldsymbol{a} \vee \boldsymbol{b}) \wedge (\neg \boldsymbol{a} \vee \neg \boldsymbol{b}) \quad \mathcal{F}^1 = \mathcal{F}^0 \wedge (\neg \boldsymbol{a}) \wedge (\neg \boldsymbol{b})$$

$\cancel{\boldsymbol{a} \vee \boldsymbol{b}}$	$\boldsymbol{a} \vee \boldsymbol{b}$	$\boldsymbol{a} : (\boldsymbol{a} \vee \boldsymbol{b})$
$\cancel{\neg \boldsymbol{a} \vee \neg \boldsymbol{b}}$	$\neg \boldsymbol{a} \vee \neg \boldsymbol{b}$	$\neg \boldsymbol{b} : (\neg \boldsymbol{a} \vee \neg \boldsymbol{b})$
$\neg \boldsymbol{a}$		
$\neg \boldsymbol{b}$		

Incremental Inprocessing – Problem

$$\mathcal{F}^0 = (\boldsymbol{a} \lor \boldsymbol{b}) \land (\neg \boldsymbol{a} \lor \neg \boldsymbol{b}) \quad \mathcal{F}^1 = \mathcal{F}^0 \land (\neg \boldsymbol{a}) \land (\neg \boldsymbol{b})$$

$\cancel{\boldsymbol{a} \lor \boldsymbol{b}}$	$\boldsymbol{a} \lor \boldsymbol{b}$	$\boldsymbol{a} : (\boldsymbol{a} \lor \boldsymbol{b})$
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$\cancel{\neg \boldsymbol{a} \vee \neg \boldsymbol{b}}$	$\neg \boldsymbol{a} \vee \neg \boldsymbol{b}$	$\neg \boldsymbol{b} : (\neg \boldsymbol{a} \vee \neg \boldsymbol{b})$
$\neg \boldsymbol{a}$		
$\neg \boldsymbol{b}$		

$$\boldsymbol{\tau} = \{\boldsymbol{a} = \bot, \boldsymbol{b} = \bot\}$$

$$\mathcal{R}(\{\boldsymbol{a} = \bot, \boldsymbol{b} = \bot\}, (\boldsymbol{a} : (\boldsymbol{a} \vee \boldsymbol{b})) \cdot (\neg \boldsymbol{b} : (\neg \boldsymbol{a} \vee \neg \boldsymbol{b}))) = \{\boldsymbol{a} = \top, \boldsymbol{b} = \bot\}$$

Incremental Inprocessing – Problem

$$\mathcal{F}^0 = (\boldsymbol{a} \lor \boldsymbol{b}) \land (\neg \boldsymbol{a} \lor \neg \boldsymbol{b}) \quad \mathcal{F}^1 = \mathcal{F}^0 \land (\neg \boldsymbol{a}) \land (\neg \boldsymbol{b})$$

$\cancel{\boldsymbol{a} \lor \boldsymbol{b}}$	$\boldsymbol{a} \lor \boldsymbol{b}$	$\boldsymbol{a} : (\boldsymbol{a} \lor \boldsymbol{b})$
$\cancel{\neg \boldsymbol{a} \lor \neg \boldsymbol{b}}$	$\neg \boldsymbol{a} \lor \neg \boldsymbol{b}$	$\neg \boldsymbol{b} : (\neg \boldsymbol{a} \lor \neg \boldsymbol{b})$
$\neg \boldsymbol{a}$		
$\neg \boldsymbol{b}$		

$$\boldsymbol{\tau} = \{\boldsymbol{a} = \bot, \boldsymbol{b} = \bot\}$$

$$\mathcal{R}(\{\boldsymbol{a} = \bot, \boldsymbol{b} = \bot\}, (\boldsymbol{a} : (\boldsymbol{a} \lor \boldsymbol{b})) \cdot (\neg \boldsymbol{b} : (\neg \boldsymbol{a} \lor \neg \boldsymbol{b}))) = \{\boldsymbol{a} = \top, \boldsymbol{b} = \bot\}$$

Incremental Inprocessing – Possible solutions

- Forbid inprocessing partially:
  - Freeze & Melt – 'Don't touch' variables
    [EénSörensson-ENTCS'03, KupferschmidLewisSchubertBecker-FMSD'11]

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- Preprocessing in incremental SAT [NadelRyvchinStrichman-SAT'12]
  - Variable elimination, (self-)subsumption

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- Preprocessing in incremental SAT [NadelRyvchinStrichman-SAT'12]
  - Variable elimination, (self-)subsumption
- General solution: **Incremental Inprocessing**
  - Adapt and extend inprocessing rules

Constrained Learning

$\boldsymbol{a} \vee \boldsymbol{b}$

$$\frac{\varphi\ [\,\rho\,]\ \sigma}{\varphi\ [\,\rho \wedge C\,]\ \sigma}\ \boxed{\sharp}$$

LEARN

where $\boxed{\sharp}$ is $\varphi \wedge \rho \equiv_{sat} \varphi \wedge \rho \wedge C$

Constrained Learning

$\boldsymbol{a} \vee \boldsymbol{b}$ | $\neg \boldsymbol{a} \vee \neg \boldsymbol{b}$ |

$$\frac{\varphi\ [\,\rho\,]\ \sigma}{\varphi\ [\,\rho \wedge C\,]\ \sigma}\ \boxed{\sharp}$$

LEARN

where $\boxed{\sharp}$ is $\varphi \wedge \rho \equiv_{sat} \varphi \wedge \rho \wedge C$

Constrained Learning

$\boldsymbol{a \vee b}$	$\boldsymbol{\neg a \vee \neg b}$	
$\boldsymbol{a}$		
$\boldsymbol{b}$		

$$\frac{\varphi\,[\,\rho\,]\;\sigma}{\varphi\,[\,\rho \wedge C\,]\;\sigma}\;\boxed{\sharp}$$

LEARN

where $\boxed{\sharp}$ is $\varphi \wedge \rho \equiv_{sat} \varphi \wedge \rho \wedge C$

Constrained Learning

$a \lor b$	$\neg a \lor \neg b$	
a		
b		

$$\frac{\varphi\ [\,\rho\,]\ \sigma}{\varphi\ [\,\rho \land C\,]\ \sigma}\ \boxed{\sharp}$$

LEARN

where $\boxed{\sharp}$ is $\varphi \land \rho \equiv_{sat} \varphi \land \rho \land C$

$\Longrightarrow$

$$\frac{\varphi\ [\,\rho\,]\ \sigma}{\varphi\ [\,\rho \land C\,]\ \sigma}\ \boxed{\sharp}$$

LEARN^{-}

where $\boxed{\sharp}$ is $\varphi \land \rho \models C$

Constrained Learning

$a \vee b$	$\neg a \vee \neg b$	
a		
b		

$$\frac{\varphi\ [\,\rho\,]\ \sigma}{\varphi\ [\,\rho \wedge C\,]\ \sigma}\ \boxed{\sharp}$$

LEARN

where $\boxed{\sharp}$ is $\varphi \wedge \rho \equiv_{sat} \varphi \wedge \rho \wedge C$

$$\Longrightarrow$$

$$\frac{\varphi\ [\,\rho\,]\ \sigma}{\varphi\ [\,\rho \wedge C\,]\ \sigma}\ \boxed{\sharp}$$

LEARN^{-}

where $\boxed{\sharp}$ is $\varphi \wedge \rho \models C$

- Weaker than original LEARN
  - No extended resolution (e.g. blocked clause addition)

Stronger Weakening

$$\frac{\varphi \wedge C \; [\, \rho \,] \; \sigma}{\varphi \; [\, \rho \wedge C \,] \; \sigma \cdot (l : C)} \; \boxed{\flat}$$

WEAKEN

where $\boxed{\flat}$ is $\varphi \wedge C \equiv^{l}_{sat} \varphi$

Stronger Weakening

$$\frac{\varphi \wedge C\ [\,\rho\,]\ \sigma}{\varphi\ [\,\rho \wedge C\,]\ \sigma \cdot (l : C)}\ \boxed{\flat}$$

WEAKEN

where $\boxed{\flat}$ is $\varphi \wedge C \equiv^{l}_{sat} \varphi$

$\Nearrow$

$$\frac{\varphi \wedge C\ [\,\rho\,]\ \sigma}{\varphi\ [\,\rho\,]\ \sigma}\ \boxed{\varnothing}$$

DROP

where $\boxed{\varnothing}$ is $\varphi \models C$

Stronger Weakening

$$\frac{\varphi \wedge C\ [\,\rho\,]\ \sigma}{\varphi\ [\,\rho \wedge C\,]\ \sigma \cdot (l : C)}\ \boxed{\flat}$$

Weaken

where $\boxed{\flat}$ is $\varphi \wedge C \equiv^{l}_{sat} \varphi$

$$\frac{\varphi \wedge C\ [\,\rho\,]\ \sigma}{\varphi\ [\,\rho\,]\ \sigma}\ \boxed{\varnothing}$$

Drop

where $\boxed{\varnothing}$ is $\varphi \models C$

$$\frac{\varphi \wedge C\ [\,\rho\,]\ \sigma}{\varphi\ [\,\rho\,]\ \sigma \cdot (\omega : C)}\ \boxed{\flat}$$

Weaken^{+}

where $\boxed{\flat}$ is $\varphi \wedge C \equiv^{\omega}_{sat} \varphi$

Stronger Weakening

$$\frac{\varphi \wedge C\ [\rho]\ \sigma}{\varphi\ [\rho]\ \sigma \cdot (\omega : C)}\ \boxed{\flat}$$

$$\textsc{Weaken}^{+}$$

where $\boxed{\flat}$ is $\varphi \wedge C \equiv^{\omega}_{sat} \varphi$

Stronger Weakening

$$\dfrac{\varphi \wedge C\ [\rho]\ \sigma}{\varphi\ [\rho]\ \sigma \cdot (\omega : C)}\ \boxed{\flat}$$

$$\textsc{Weaken}^{+}$$

where $\boxed{\flat}$ is $\varphi \wedge C \equiv^{\omega}_{sat} \varphi$

- Syntax: ω is set of literals s.t. $\omega \cap C \neq \emptyset$

Stronger Weakening

$$\frac{\varphi \wedge C \; [\rho] \; \sigma}{\varphi \; [\rho] \; \sigma \cdot (\omega : C)} \; \boxed{\flat} \qquad \textsc{Weaken}^{+}$$

where $\boxed{\flat}$ is $\varphi \wedge C \equiv^{\omega}_{sat} \varphi$

- Syntax: ω is set of literals s.t. $\omega \cap C \neq \emptyset$
- Semantics: Propagation Redundancy [HeuleKieslBiere-CADE'17]

Stronger Weakening

$$\frac{\varphi \wedge C\ [\,\rho\,]\ \sigma}{\varphi\ [\,\rho\,]\ \sigma \cdot (\omega : C)}\ \boxed{\flat}$$

WEAKEN^{+}

where $\boxed{\flat}$ is $\varphi \wedge C \equiv_{sat}^{\omega} \varphi$

- Syntax: ω is set of literals s.t. $\omega \cap C \neq \emptyset$
- Semantics: Propagation Redundancy [HeuleKiesIBiere-CADE'17]
  - Most general **reconstructive** redundancy property

Stronger Weakening

$$\frac{\varphi \wedge C \; [\,\rho\,] \; \sigma}{\varphi \; [\,\rho\,] \; \sigma \cdot (\omega : C)} \; \boxed{\flat} \quad \textsc{Weaken}^{+}$$

where $\boxed{\flat}$ is $\varphi \wedge C \equiv^{\omega}_{sat} \varphi$

- Syntax: ω is set of literals s.t. $\omega \cap C \neq \emptyset$
- Semantics: Propagation Redundancy [HeuleKiesIBiere-CADE'17]
  - Most general **reconstructive** redundancy property
  - Polynomially reconstructible via witness ω:

Proposition: If $\tau(\varphi) = \top$ and $\tau(C) \neq \top$ then $(\tau \circ \omega)(\varphi \wedge C) = \top$

Incremental Clause Addition

$$\frac{\varphi\ [\,\rho\,]\ \sigma}{\varphi \wedge \Delta\ [\,\rho\,]\ \sigma}\ \boxed{\mathcal{I}}$$

ADDCLAUSES

where $\boxed{\mathcal{I}}$ is that each clause of Δ is clean w.r.t. σ

Incremental Clause Addition

$$\frac{\varphi\ [\,\rho\,]\ \sigma}{\varphi \wedge \Delta\ [\,\rho\,]\ \sigma}\ \boxed{\mathcal{I}}$$

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where $\boxed{\mathcal{I}}$ is that each clause of Δ is **clean** w.r.t. σ

A clause C is **clean** w.r.t. a sequence of witness labelled clauses σ if and only if for all $(\omega : D) \in \sigma$ we have that $\neg C \cap \omega = \emptyset$.

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where $\boxed{\mathcal{I}}$ is that each clause of Δ is **clean** w.r.t. σ

A clause C is **clean** w.r.t. a sequence of witness labelled clauses σ if and only if for all $(\omega : D) \in \sigma$ we have that $\neg C \cap \omega = \emptyset$.

Lemma: If a clause C is clean on a sequence of witness labelled clauses σ, then for all truth assignments τ with $\tau(C) = \top$ we have that $\mathcal{R}(\tau, \sigma)(C) = \top$.

Reversing Weakenings

$$\frac{\varphi\ [\,\rho\,]\ \sigma \cdot (\omega : C) \cdot \sigma'}{\varphi \wedge C\ [\,\rho\,]\ \sigma \cdot \sigma'}\ \boxed{\partial}$$

RESTORE

where $\boxed{\partial}$ is C is clean w.r.t. σ'

Incremental Inprocessing Rules

$$\frac{\varphi\ [\rho]\ \sigma}{\varphi\ [\rho \wedge C]\ \sigma}\ \boxed{\sharp}$$

LEARN$^-$

$$\frac{\varphi\ [\rho \wedge C]\ \sigma}{\varphi\ [\rho]\ \sigma}$$

FORGET

$$\frac{\varphi\ [\rho \wedge C]\ \sigma}{\varphi \wedge C\ [\rho]\ \sigma}$$

STRENGTHEN

$$\frac{\varphi\ [\rho]\ \sigma}{\varphi \wedge \Delta\ [\rho]\ \sigma}\ \boxed{\mathcal{I}}$$

ADDCLAUSES

$$\frac{\varphi \wedge C\ [\rho]\ \sigma}{\varphi\ [\rho]\ \sigma \cdot (\omega : C)}\ \boxed{\flat}$$

WEAKEN$^+$

$$\frac{\varphi \wedge C\ [\rho]\ \sigma}{\varphi\ [\rho]\ \sigma}\ \boxed{\emptyset}$$

DROP

$$\frac{\varphi\ [\rho]\ \sigma \cdot (\omega : C) \cdot \sigma'}{\varphi \wedge C\ [\rho]\ \sigma \cdot \sigma'}\ \boxed{\partial}$$

RESTORE

where $\boxed{\sharp}$ is $\varphi \wedge \rho \models C$, $\boxed{\flat}$ is $\varphi \wedge C \equiv^{\omega}_{sat} \varphi$, $\boxed{\emptyset}$ is $\varphi \models C$,

$\boxed{\partial}$ is C is clean w.r.t. σ' and $\boxed{\mathcal{I}}$ is that each clause in Δ is clean w.r.t. σ

Incremental Inprocessing Rules

$$\frac{\varphi\;[\rho]\;\sigma}{\varphi\;[\rho \wedge C]\;\sigma}\;\boxed{\sharp}$$

LEARN^{-}

$$\frac{\varphi\;[\rho \wedge C]\;\sigma}{\varphi\;[\rho]\;\sigma}$$

FORGET

$$\frac{\varphi\;[\rho \wedge C]\;\sigma}{\varphi \wedge C\;[\rho]\;\sigma}$$

STRENGTHEN

$$\frac{\varphi\;[\rho]\;\sigma}{\varphi \wedge \Delta\;[\rho]\;\sigma}\;\boxed{\mathcal{I}}$$

ADDCLAUSES

$$\frac{\varphi \wedge C\;[\rho]\;\sigma}{\varphi\;[\rho]\;\sigma \cdot (\omega : C)}\;\boxed{\flat}$$

WEAKEN^{+}

$$\frac{\varphi \wedge C\;[\rho]\;\sigma}{\varphi\;[\rho]\;\sigma}\;\boxed{\varnothing}$$

DROP

$$\frac{\varphi\;[\rho]\;\sigma \cdot (\omega : C) \cdot \sigma'}{\varphi \wedge C\;[\rho]\;\sigma \cdot \sigma'}\;\boxed{\partial}$$

RESTORE

where $\boxed{\sharp}$ is $\varphi \wedge \rho \models C$, $\boxed{\flat}$ is $\varphi \wedge C \equiv^{\omega}_{sat} \varphi$, $\boxed{\varnothing}$ is $\varphi \models C$,

$\boxed{\partial}$ is C is clean w.r.t. σ' and $\boxed{\mathcal{I}}$ is that each clause in Δ is clean w.r.t. σ

Incremental Inprocessing – Formal correctness

- Gathered information (including learned clauses) can be kept

$\boldsymbol{F^{i+1} \models \rho^{i}_{k_i}}$ where $\rho^i_{k_i}$ is ρ at the end of the evaluation of F^i

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- Satisfiability preserving derivation continuation

$$\boldsymbol{F^{i} \equiv_{sat} \varphi^{i}_{j} \wedge \rho^{i}_{j}} \text{ for all } j \text{ with } 0 \leq j \leq k_i$$

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- Satisfiability preserving derivation continuation

$$\boldsymbol{F^i \equiv_{sat} \varphi^i_j \wedge \rho^i_j} \text{ for all } j \text{ with } 0 \leq j \leq k_i$$

- Solution reconstruction in any satisfiable state

$$\boldsymbol{\tau(\varphi^i_{k_i}) = \top \implies \mathcal{R}(\tau, \sigma^i_{k_i})(F^i) = \top} \text{ for all } i \text{ with } 0 \leq i \leq m$$

IMPLEMENTATION

Algorithm to Restore and Add Clauses

AlgorithmRestoreAddClauses (new clauses Δ, reconstruction stack σ)

$(\omega_1 : C_1) \cdots (\omega_n : C_n) \;:=\; \sigma$

for i **from** 1 **to** n

    if exists $\ell \in \omega_i$ where $\neg\ell$ occurs in Δ **then**

        $\Delta \;:=\; \Delta \cup C_i\,, \quad \sigma \;:=\; \sigma \setminus (\omega_i : C_i)$

return $\langle \Delta, \sigma \rangle$

EXPERIMENTS

Experiments

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- safety property track of HWMCC'17: 300 AIGER models

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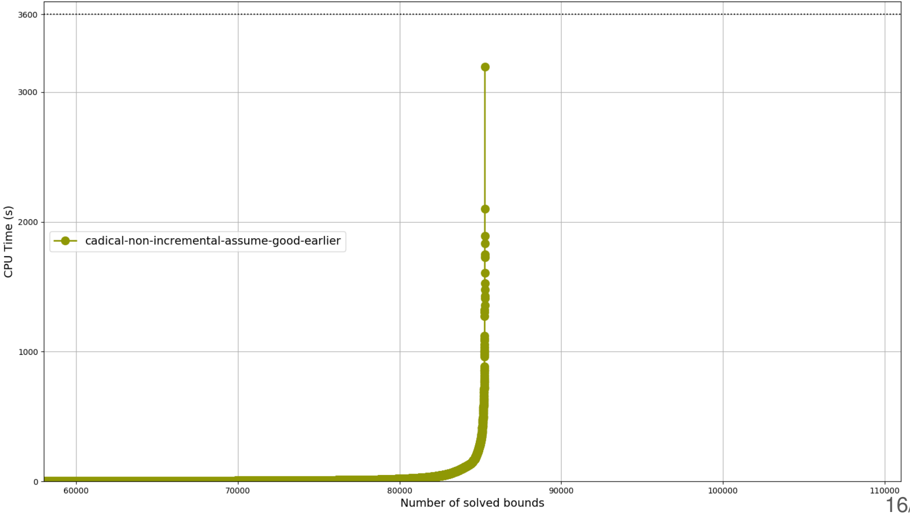
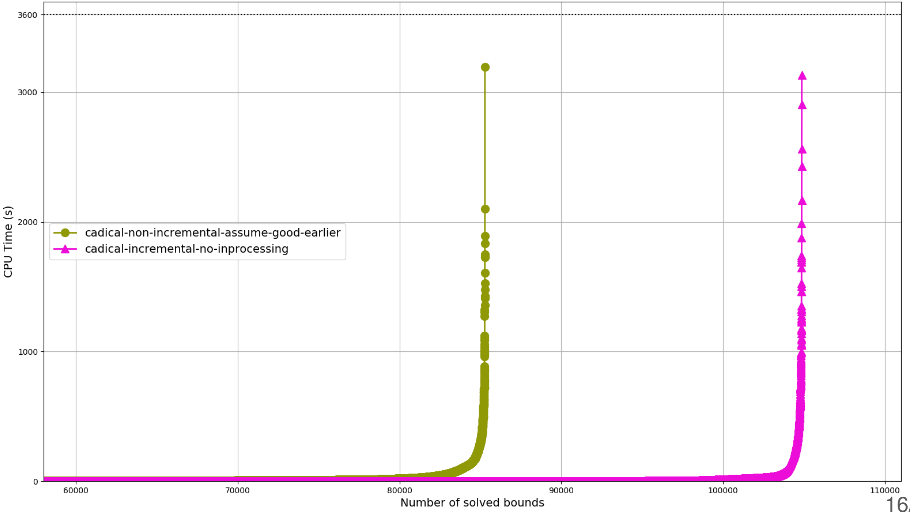
CaMiCaL: bounded model checker with CaDiCaL as back-end

- safety property track of HWMCC'17: 300 AIGER models
- solving a bound = (incremental) SAT call

Experimental Results



Experimental Results



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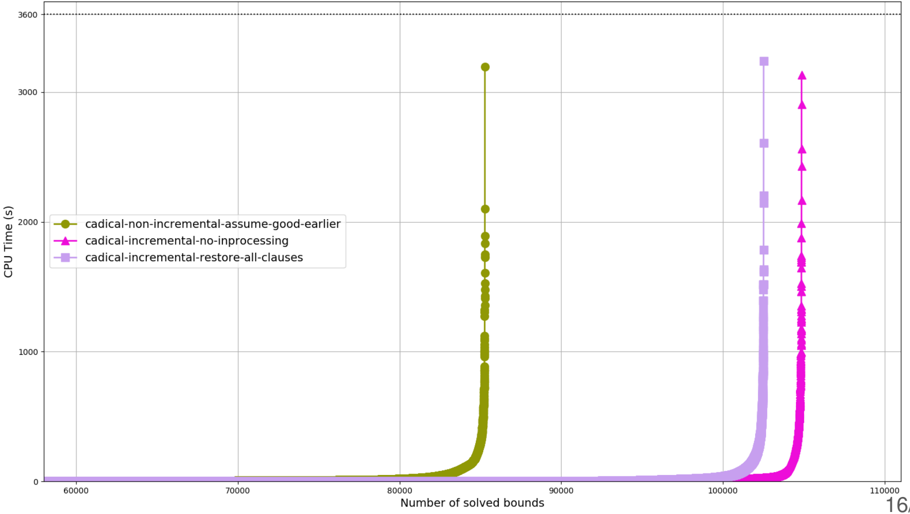

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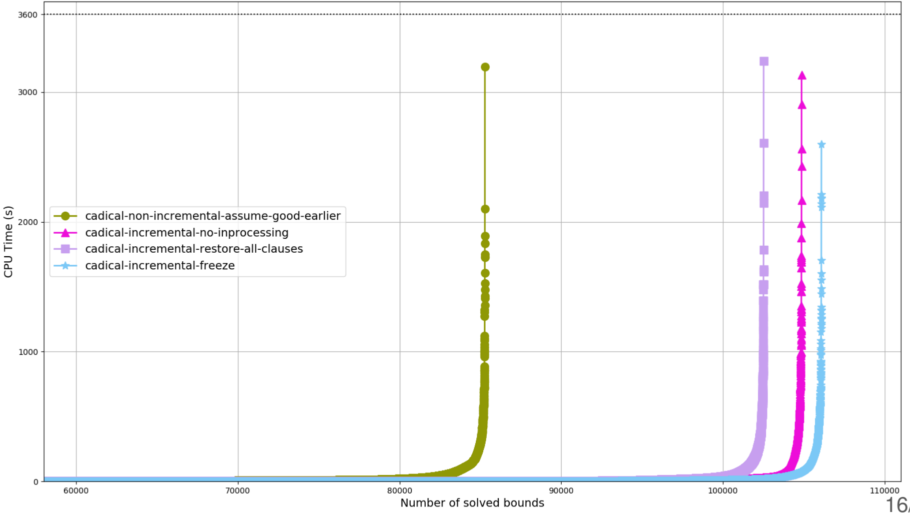

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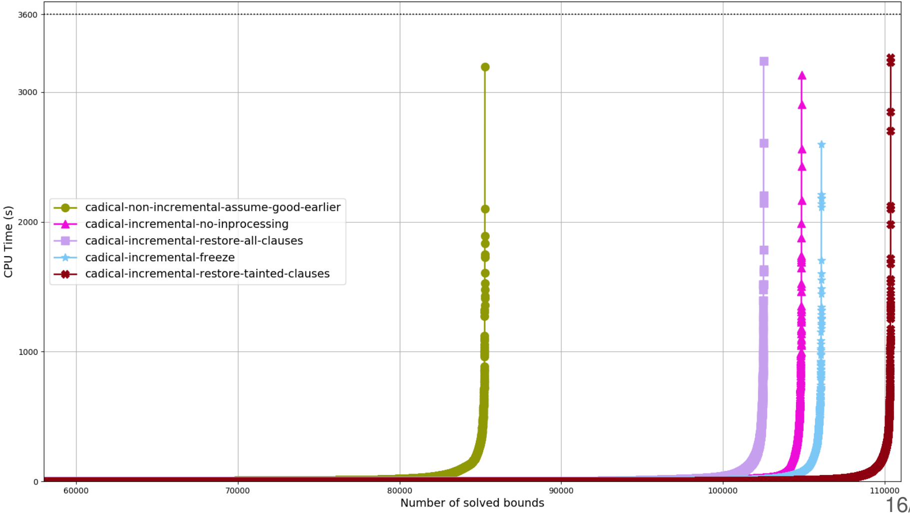

Conclusion & Future Work

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  - LEARN^{-} (for e.g. blocked clause addition)

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- Incremental inprocessing in SAT
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  - No need for variable freezing
- Simple and efficient implementation

Future Work:

- Limitations
  - Learn^- (for e.g. blocked clause addition)
  - Clean clause definition
- Inprocessing under assumptions

Thank you for your attention!

INCREMENTAL INPROCESSING IN SAT SOLVING

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Experimental Results