Exercise 21

Let $N$ be the PTN shown below.

- Specify $N$ formally as a 5-tuple $N = (P, I, T, G, C)$. How many markings for $N$ are possible theoretically?
- Now let $M$ and $M'$ be two markings of $N$, with $M(r) = 0, M(s) = 2, M(t) = 0$ and $M'(r) = 1, M'(s) = 3, M'(t) = 1$, respectively. Which are the transitions that can fire in $M$ and $M'$, respectively? What are the possible new markings obtained from this?
- Draw the LTS corresponding to $N$.

Exercise 22

a) Reformulate $\forall x. (\phi \leftrightarrow \psi)$ using only $\exists$ and operators $\neg$ and $\land$. Specify all intermediate steps.

b) Explain in your own words the effects of reordering quantifiers. More precisely, explain the semantical difference between $\forall x \exists y. \phi$ and $\exists y \forall x. \phi$ in general.
**Exercise 23**

a) List the unit literals of the following QBF and simplify it with unit propagation.

\[ \exists a \forall x \forall y \forall z \exists b \exists c \exists d. ((y \lor z \lor c) \land (\neg x \lor \neg c) \land (a \lor z) \land (b \lor x \lor \neg d) \land (c \lor \neg x) \land (\neg z) \land (d)) \]

b) Explain by an example why unit propagation is not sound for a clause of size one containing only a universal literal.

c) Apply pure literal elimination on the following QBF.

\[ \forall x \exists a \forall y \exists b \exists c \forall z ((\neg y \lor \neg c \lor a) \land (\neg y \lor x \lor z) \land (\neg x \lor z) \land (x \lor \neg c) \land (\neg y \lor b) \land (\neg b \lor c \lor a)) \]

d) Consider the QBF below. List the clauses in which universal reduction is possible.

\[ \forall a \forall b \forall c \exists x \exists y \exists z ((\neg x \lor \neg c \lor a) \land (\neg y \lor \neg x \lor a) \land (\neg x) \land (x \lor \neg b \lor c) \land (\neg y \lor \neg b) \land (\neg y \lor a)) \]

**Exercise 24**

What are the truth values of the following two QBFs?

a) \[ \forall a \forall b \forall c \exists x \exists y \exists z ((y \lor z \lor c \lor \neg b) \land (\neg z \lor \neg a) \land (a \lor b \lor c \lor y \lor \neg z) \land (\neg c \lor \neg b \lor \neg y) \land (\neg b \lor c)) \]

b) \[ \forall x \exists y \forall z. ((z \leftrightarrow x) \land (z \leftrightarrow y)) \]