Exercise 25

a) Define the semantics of the boolean operators \( \neg, \land, \lor, \rightarrow, \) and \( \leftrightarrow \) in Simplified HML analogously to the definitions of the modal operators and boolean constants (see slide 53).

b) Referring to the semantical rules of Simplified HML on slide 53, explain in detail why formula \([a] 1\) is always true in a state \(s\) and why formula \(\langle a\rangle 0\) is always false.

c) Explain the relation between \(\neg [a] 1\) and \(\langle a\rangle 0\).

Exercise 26

Given LTS \(L\) and Simplified HML formulae 1 to 5 as shown below.

1. \(\langle y\rangle 1\)
2. \([x] 0\)
3. \([x] 1\)
4. \([x][y] 0\)
5. \([y]\langle x\rangle 0\)

a) For each state \(s\) of \(L\), determine which of formulae 1 to 5 hold in \(s\).

b) Given formula \(f := [y] [y] 0\). Explain in detail how \(f\) is evaluated recursively in states 1 and 5 of LTS \(L\). That is, check if \(1 \models f\) and if \(5 \models f\), and show recursive applications of \(\models\).
Exercise 27

Given the LTS $L$ shown in the figure below.

![Diagram of LTS](image)

Decide for which states of $L$ the following HML expressions hold.

- $\langle x \rangle 1 \lor [y] 1$
- $\langle x \rangle ([y] 0 \lor [z] 1)$
- $([y] [x] 1) \lor (\langle x \rangle \langle y \rangle 0)$
- $\langle z \rangle 1 \land [y] \langle y \rangle 0$
- $([y] \langle y \rangle 1) \lor ([x] \langle x \rangle 1)$

Exercise 28

Given LTS $L$ as shown below.

![Diagram of LTS](image)

a) List all different infinite traces in $L$, using $\omega$-notation, e.g. $abab \cdots = (ab)^\omega$.

b) Find 6 equivalences between traces from part a), using notation $\pi^i$, e.g. $\pi_2 = \pi_1^1$ for $\pi_1 = xyz$ and $\pi_2 = yz$. 