Exercise 29

a) Reformulate $\forall x. (\phi \leftrightarrow \psi)$ using only $\exists$ and operators $\neg$ and $\land$. Specify all intermediate steps.

b) Referring to the semantical rules of Simplified HML on slide 53, explain in detail why formula $[a] 1$ is always true in a state $s$ and why formula $\langle a \rangle 0$ is always false.

c) Explain the relation between $\neg [a] 1$ and $\langle a \rangle 0$.

Exercise 30

Given LTS $L$ as shown on the right.

a) List at least 10 different infinite traces in $L$, using $\omega$-notation, e.g. $ababab\ldots = (ab)\omega$.

b) Find 6 equivalences between traces from part a), using notation $\pi'$, e.g. $\pi_2 = \pi_1'$ for $\pi_1 = xyz$ and $\pi_2 = yz$. 
Exercise 31

Given the LTS $L$ shown in the figure below.

Decide for which states of $L$ the following HML expressions hold.

- $\langle x \rangle 1$
- $\langle x \rangle ([y] 0 \lor \langle z \rangle 1)$
- $([y] [x] 1) \lor (\langle x \rangle \langle y \rangle 0)$
- $\langle z \rangle 1 \land \langle z \rangle [y] 0$
- $([y] \langle x \rangle 1) \lor ([x] \langle y \rangle 1)$

Exercise 32

Given LTS $L$ and CTL/HML formulae 1 to 6 as shown below. For each state $s$ of $L$, determine which of formulae 1 to 6 hold in $s$.

1. $\text{EX}(\langle x \rangle 1)$  
2. $\text{AX}([y] 0)$  
3. $\text{AF}(\langle z \rangle 1 \rightarrow \langle y \rangle 1)$  
4. $\text{E}[[\langle x \rangle 1 \cup \langle z \rangle 1]$  
5. $\text{EG}([y] 1)$  
6. $\text{E}[[\langle y \rangle 1 \cup \langle z \rangle 1]$