

# Formal Models SS 2020: Assignment 8

Based on Video “Lecture 15. May 2014” on our webpage.  
Institute for Formal Models and Verification, JKU Linz

due 28.05.2020

Guideline:

- To indicate that you solved an exercise, tick it off in our MOODLE course until **10am on the day of the exercise (28.05.2020)**. Unmarking and marking exercises later is **not** possible.
- **Upload your solved exercises in the Moodle course. Generate a single PDF file, which contains all solved exercises, your name, and your matriculation number. Upload the PDF file - do not generate a ZIP!** Not following the format leads to deduction of points!
- We will randomly select and correct solved exercises.
- A sample solution will be provided.

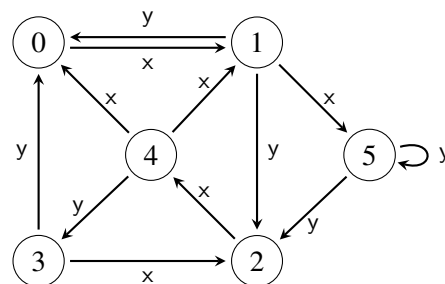
## Exercise 29

- Reformulate  $\forall x. (\phi \leftrightarrow \psi)$  using only  $\exists$  and operators  $\neg$  and  $\wedge$ .
- Define the semantics of the boolean operators  $\neg$ ,  $\wedge$ ,  $\vee$ ,  $\rightarrow$ , and  $\leftrightarrow$  in Simplified HML analogously to the definitions of the modal operators and boolean constants (see slide 53).
- Referring to the semantical rules of Simplified HML on slide 53, explain in detail why formula  $[a] 1$  is always true in a state  $s$  and why formula  $\langle a \rangle 0$  is always false.

## Exercise 30

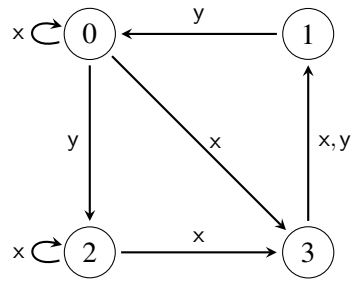
Given LTS  $L$  and Simplified HML formulae 1 to 5 as shown below.

1.  $\langle y \rangle 1$
2.  $[x] 0$
3.  $[y] [y] 0$
4.  $[y] \langle x \rangle 1$
5.  $\langle x \rangle ([y] 0 \wedge \langle x \rangle 1)$



- For each state  $s$  of  $L$ , determine which of the formulae 1 to 5 hold in  $s$ .
- Give the *nesting depth* of modal operators for each formula. What is the significance of the *nesting depth* of a formula in relation to the length of a *trace*.

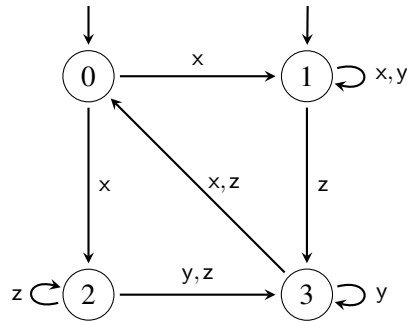
**Exercise 31**



Given an LTS  $L$  as above with  $\Sigma = \{x, y, z\}$ . Calculate  $[[\langle y \rangle 1 \rightarrow ([x]1 \wedge [y]0)]]$ .

**Exercise 32**

Given the LTS  $L$  shown in the figure below.



Decide for which states of  $L$  the following Simplified HML expressions hold. Put a cross into each cell of the table to indicate that the corresponding formula holds in the corresponding state. Otherwise leave the cell empty.

Simplified HML expression	State 0	State 1	State 2	State 3
$\langle x \rangle \langle x \rangle 1$				
$[x](\langle x \rangle 1 \wedge [y]0)$				
$([x] \langle y \rangle 1) \leftrightarrow ([y] \langle x \rangle 1)$				
$([x \vee y] \langle \neg y \rangle 1) \rightarrow ([y] \langle x \rangle 1)$				

Which of the formulae hold in  $L$ ?