Exercise 33 Draw a computation tree for each of the following CTL formulae (see also lecture slides 63-65 and lecture video "Lecture 12.June 2014" (min 29:00-32:30)).

1. \( \text{EF } p \)
2. \( \text{EX } p \)
3. \( \text{EG } p \)
4. \( \text{AX } p \)
5. \( \text{A}[p \ U \ q] \)
6. \( \text{E}[p \ U \ q] \)

Exercise 34

Given Kripke structure \( K \) as shown below. Which of the following CTL formulas hold in \( K \)?

a) \( \text{AG } (\neg a \rightarrow c) \)

b) \( \text{E } ((c \lor d) \ U \ b) \)

c) \( \text{AG } ((c \land d) \rightarrow \text{EX } a) \)

d) \( \text{EF } ((a \land \neg c) \rightarrow \text{EX } c) \)
Exercise 35

Given trace $\pi$ in Kripke structure on the right and LTL formula $f$. Decide if $f$ holds in $\pi$, i.e., $\pi \models f$.

<table>
<thead>
<tr>
<th>Trace $\pi$</th>
<th>Formula $f$</th>
<th>yes</th>
<th>no</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(A,B,D)^{a_2}$</td>
<td>$GF(x)$</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>$A,B,(C)^{a_2}$</td>
<td>$GF(y)$</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>$A,(B)^{a_3}$</td>
<td>$FG(x)$</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>$(A,B,D)^{a_1}$</td>
<td>$(\chi U y)$</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>$(A,B,D)^{a_1}$</td>
<td>$G(y \lor Xx)$</td>
<td>✔</td>
<td>✔</td>
</tr>
</tbody>
</table>

Exercise 36

Given two Kripke structures $K_1, K_2$.

We call a temporal logic $L$ compositional if $K_1 \models f$ and $K_2 \models g$ implies that $K_1 \times K_2 \models f \land g$ where $f,g \in L$ and $K_1 \times K_2$ is the product of $K_1$ and $K_2$.

We assume that the two Kripke structures are defined over the same set of atomic propositions. The definition of the product of Kripke structures is similar as for automata plus $L((s_1,s_2)) = L_1(s_1) \cup L_2(s_2)$. Note that the union of the labels of two states can result in a contradiction, e.g., $a$ and $\neg a$ hold. In such a case, the state is not included in the product.

Is CTL compositional? If yes, prove that it is compositional, if no, give an example showing that CTL is not compositional.