Given Kripke structure $K$ as shown below. For each of the following CTL formulae $f$, determine whether $K \models f$ or not. Note that there are two initial states. Justify your answers by referring to the semantical definitions, i.e. name paths and/or states sufficient to explain your answers.

For example, $K \not\models \text{AX} (q \lor s)$ because for initial state 0, path $\pi = 0, 1, \ldots$ is a counterexample for that property: $\pi(1) = 1$ and $1 \not\models (q \lor s)$.

a) $\text{AG} (q \rightarrow r)$
b) $\text{E} (r \text{ U } s)$
c) $\text{AG} ((p \land r) \rightarrow \text{EG } p)$
d) $\text{A} ((r \lor q) \text{ U } p)$
e) $\text{AG} ((p \land r) \rightarrow \text{EG } s)$
f) $\text{EF} (s \rightarrow \text{EX } s)$
Exercise 38
Given Kripke structure $K$ as shown below. For the following infinite traces $\pi$ of $K$ and LTL formulae $f$, determine whether $\pi \models f$ or not. Justify your answers in detail by referring to semantics of operators and to concrete states on paths $\pi$.

a) $\pi := (0, 2, 4)^\omega$ and $f := c U b$

b) $\pi := (0, 2, 4)^\omega$ and $f := d U c$

c) $\pi := (2, 3, 5)^\omega$ and $f := G(b \rightarrow X \neg b)$

d) $\pi := 0, 2, (3)^\omega$ and $f := FG d$

Exercise 39
For each of the following temporal formulae, check whether there is an equivalent formula in $LTL^{det}$. If so, then specify such an equivalent formula meeting the syntactic criteria for $LTL^{det}$ as given on lecture slide 69. Note that subformulae $p$ and $q$ are atomic propositions, i.e. $p, q \in \mathcal{A}$.

a) $p \rightarrow AX q$

b) $(AF p) \land AX \neg p$

c) $(EX \neg p) \land (AX \neg p)$

d) $\neg ((EX \neg q) \lor (EF \neg p))$

Exercise 40

- Given Kripke structure $K$ as shown above. Justify your answers to the following questions.

  a) Does $K \models f$ hold for ACTL formula $f := AX p \lor AX q$?

  b) Let $g := f \setminus A$. Does $K \models g$ hold?

  c) Based on the results of a) and b): are $f$ and $g$ equivalent?

  d) Based on the results of a), b) and c): is there an LTL formula which is equivalent to $f$?

- Given CTL formula $f := AF (r \rightarrow AG a)$, where $r$ and $a$ are atomic propositions, i.e. $r, a \in \mathcal{A}$. Draw a Kripke structure $K$ with exactly one initial state such that $K \not\models f$ but $K \models f \setminus A$ (Hint: there is such $K$ with no more than 3 states). Is there an LTL formula which is equivalent to $f$? Justify your answers.