Formal Models SS 2012: Assignment 10

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Due 21.06.2012

Exercise 37
Given Kripke structure K as shown below. For each of the following CTL formulae f, determine whether $K \models f$ or not. *Note* that there are two initial states. *Justify your answers* by referring to the semantical definitions, i.e. name paths and/or states sufficient to explain your answers.

For example, $K \not\models \mathbf{AX}\,(q \vee s)$ because for initial state 0, path $\pi = 0, 1, \ldots$ is a counterexample for that property: $\pi(1) = 1$ and $1 \not\models (q \vee s)$.

a) $\mathbf{AG}\,(q \rightarrow r)$

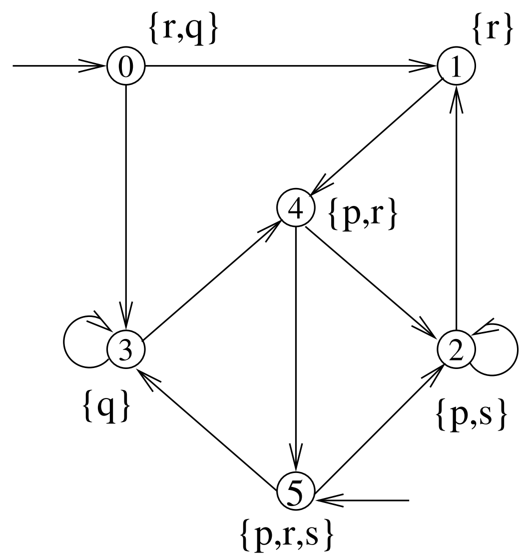
b) $\mathbf{E}\,(r\,\mathbf{U}\,s)$

c) $\mathbf{AG}\,((p \wedge r) \rightarrow \mathbf{EG}\,p)$

d) $\mathbf{A}\,((r \vee q)\,\mathbf{U}\,p)$

e) $\mathbf{AG}\,((p \wedge r) \rightarrow \mathbf{EG}\,s)$

f) $\mathbf{EF}\,(s \rightarrow \mathbf{EX}\,s)$



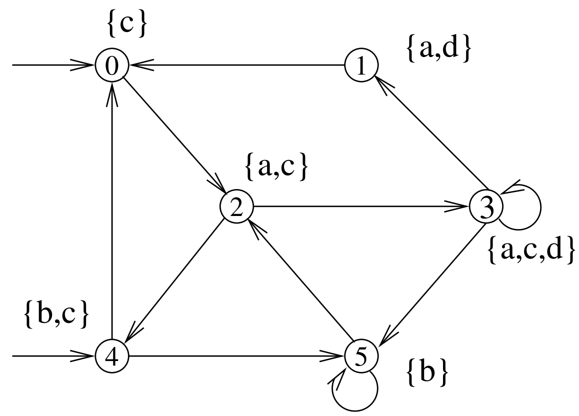
Exercise 38
Given Kripke structure K as shown below. For the following infinite traces π of K and LTL formulae f, determine whether $\pi \models f$ or not. Justify your answers in detail by referring to semantics of operators and to *concrete* states on paths π.

a) $\pi := (0,2,4)^\omega$ and $f := c\,\mathbf{U}\,b$

b) $\pi := (0,2,4)^\omega$ and $f := d\,\mathbf{U}\,c$

c) $\pi := (2,3,5)^\omega$ and $f := \mathbf{G}(b \rightarrow \mathbf{X}\neg b)$

d) $\pi := 0,2,(3)^\omega$ and $f := \mathbf{FG}\,d$



Exercise 39
For each of the following temporal formulae, check whether there is an equivalent formula in LTL^{det}. If so, then specify such an equivalent formula meeting the syntactic criteria for LTL^{det} as given on lecture slide 69. Note that subformulae p and q are atomic propositions, i.e. $p,q \in \mathcal{A}$.

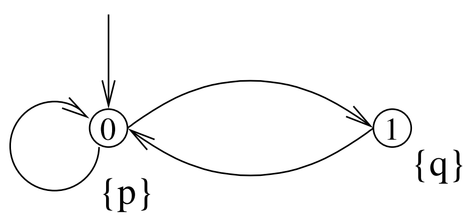
a) $p \rightarrow \mathbf{AX}\,q$

b) $(\mathbf{AF}\,p) \wedge \mathbf{AX}\,\neg p$

c) $(\mathbf{EX}\,\neg p) \wedge (\mathbf{AX}\,\neg p)$

d) $\neg((\mathbf{EX}\,\neg q) \vee (\mathbf{EF}\,\neg p))$

Exercise 40



- Given Kripke structure K as shown above. Justify your answers to the following questions.

  a) Does $K \models f$ hold for ACTL formula $f := \mathbf{AX}\,p \vee \mathbf{AX}\,q$?
  b) Let $g := f \setminus \mathbf{A}$. Does $K \models g$ hold?
  c) Based on the results of a) und b): are f and g equivalent?
  d) Based on the results of a), b) and c): is there an LTL formula which is equivalent to f?

- Given CTL formula $f := \mathbf{AF}\,(r \rightarrow \mathbf{AG}\,a)$, where r and a are atomic propositions, i.e. $r,a \in \mathcal{A}$. Draw a Kripke structure K with exactly one initial state such that $K \not\models f$ but $K \models f \setminus \mathbf{A}$ (Hint: there is such K with no more than 3 states). Is there an LTL formula which is equivalent to f? Justify your answers.