Exercise 29

Draw the LTS for PTN $N$ shown on the right with the initial marking as given in the figure.

Exercise 30

a) Reformulate $\forall x. (\phi \leftrightarrow \psi)$ using only $\exists$ and operators $\neg$ and $\land$. Specify all intermediate steps.

b) Explain in your own words the effects of reordering quantifiers. More precisely, explain the semantical difference between $\forall x \exists y. \phi$ and $\exists y \forall x. \phi$ in general.

c) Define the semantics of the boolean operators $\neg$, $\land$, $\lor$, $\rightarrow$, and $\leftrightarrow$ in Simplified HML analogously to the definitions of the modal operators and boolean constants (see slide 53).

d) Referring to the semantical rules of Simplified HML on slide 53, explain in detail why formula $[a] 1$ is always true in a state $s$ and why formula $\langle a \rangle 0$ is always false.
Exercise 31

Given LTS $L$ and Simplified HML formulae 1 to 5 as shown below.

1. $\langle y \rangle 1$
2. $[x] 0$
3. $[y] [y] 0$
4. $[y] \langle x \rangle 1$
5. $\langle x \rangle ([y] 0 \land \langle x \rangle 1)$

a) For each state $s$ of $L$, determine which of formulae 1 to 5 hold in $s$.

b) Given formula $f := [y] [y] 0$. Explain in detail how $f$ is evaluated recursively in states 1 and 5 of LTS $L$. That is, check if $1 \models f$ and if $5 \models f$, and show recursive applications of $\models$.

Exercise 32

Given an LTS $L$ as above with $\Sigma = \{x, y, z\}$. Do the following formulas hold in states 0, 1, 2, 3?

a) $[y] \langle x \rangle 1 \leftrightarrow [x] \langle y \rangle 0$

b) $(\neg [x] 0) \land \langle y \rangle 1$

c) $\langle x \rangle [y] \langle x \rangle 0$

d) $[\neg z] [y] \langle x \rangle 1$

e) $\langle y \rangle 1 \rightarrow ([x] 1 \land [y] 0)$