Formal Models SS 2015: Assignment 8

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Due 28.05.2015

Exercise 29

a) Reformulate $\forall x.\ (\phi \leftrightarrow \psi)$ using only $\exists$ and operators $\neg$ and $\wedge$. Specify all intermediate steps.

b) Explain in your own words the effects of reordering quantifiers. More precisely, explain the semantical difference between $\forall x \exists y.\ \phi$ and $\exists y \forall x.\ \phi$ in general.

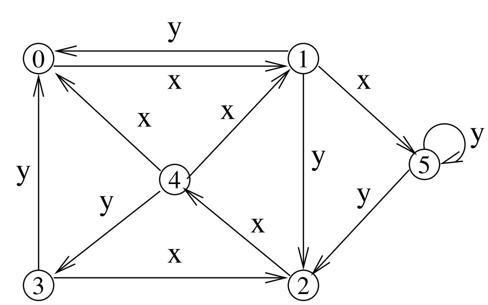
c) Define the semantics of the boolean operators $\neg$, $\wedge$, $\vee$, $\rightarrow$, and $\leftrightarrow$ in Simplified HML analogously to the definitions of the modal operators and boolean constants (see slide 53).

d) Referring to the semantical rules of Simplified HML on slide 53, explain in detail why formula $[a]\,1$ is always true in a state s and why formula $\langle a \rangle\,0$ is always false.

Exercise 30

Given LTS L and Simplified HML formulae 1 to 5 as shown below.

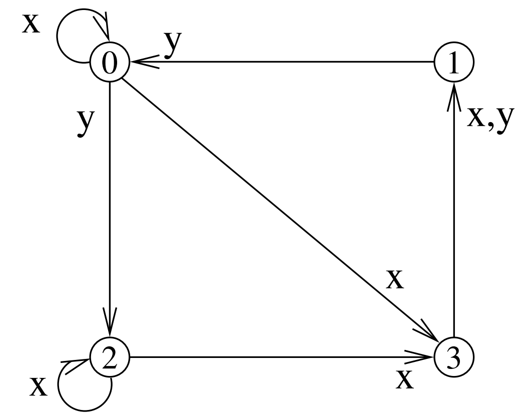
1. $\langle y \rangle\, 1$
2. $[x]\, 0$
3. $[y]\,[y]\, 0$
4. $[y]\,\langle x \rangle\, 1$
5. $\langle x \rangle ([y]\, 0 \wedge \langle x \rangle\, 1)$



a) For each state s of L, determine which of formulae 1 to 5 hold in s.

b) Given formula $f := [y]\,[y]\,0$. Explain in detail how f is evaluated recursively in states 1 and 5 of LTS L. That is, check if $1 \models f$ and if $5 \models f$, and show recursive applications of $\models$.

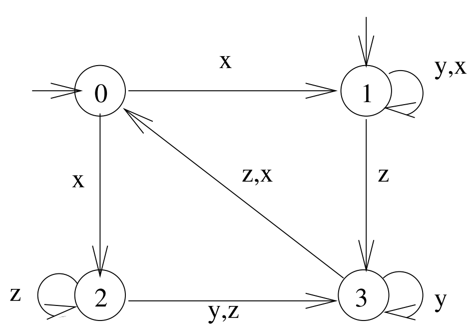
Exercise 31



Given an LTS L as above with $\Sigma = \{x, y, z\}$. Calculate $[[\langle y \rangle 1 \to ([x]1 \wedge [y]0)]]$, i.e., the set of all states in which the formula holds.

Exercise 32

Given the LTS L shown in the figure below.



Decide for which states of L the following HML expressions hold. Each correct row is awarded one point. Put a cross into each cell of the table to indicate that the corresponding formula holds in the corresponding state. Otherwise leave the cell empty.

HML	State 0	State 1	State 2	State 3
$\langle x \rangle \langle z \rangle 1$				
$[x](\langle x \rangle 1 \wedge [y] 0)$				
$([x] \langle y \rangle 1) \leftrightarrow ([y] \langle x \rangle 1)$				
$([x \vee y] \langle \neg y \rangle 1) \to ([y] \langle x \rangle 1)$				

Which of the formulas hold in L? Note: A formula holds in L iff it holds in all initial states.