Stochastic Local Search for Satisfiability Modulo Theories

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Introduction

Satisfiability Modulo Theories (SMT) is essential for many practical applications, e.g., in hard- and software verification, and increasingly also in other scientific areas like computational biology. We present a novel **stochastic local search (SLS)** algorithm to solve SMT problems, especially those in the theory of **bit-vectors**, directly **on the theory level**.

SLS for SMT

SLS for SAT

+

Theory Information

↓

Industrial Applications

SLS solvers for SAT are known to work very well on random benchmarks, but usually perform bad on industrial problems, because they are not aware of structural information.

A user defined neighbourhood relation and a score function to guide the search are sufficient.

We **extend the classical SLS loop** in order to **exploit the structure** that is contained in bit-vector formulas. **Many techniques** used in modern SLS solvers for SAT **can be lifted** to the SMT level. Experimental results show that our approach **outperforms current SLS solvers for SAT** and can **compete with state-of-the-art bit-vector solvers** on many **industrial instances**.

Architecture

- Input formula F as a conjunction of *assertions* in ***Negation Normal Form*** *(NNF)*.
- Given an *assignment* α to all variables and a constant $c \in [0,1]$, we define a **score function for bit-vector expressions** (an extension to bit-vector formulas in NNF is natural):

$$s(t_1^{[n]} = t_2^{[n]}, \alpha) = \begin{cases} 1 & \text{if } t_1|_\alpha = t_2|_\alpha \\ c \cdot \left(1 - \frac{h(t_1|_\alpha, t_2|_\alpha)}{n}\right) & \text{otherwise} \end{cases}$$

$$s(t_1^{[n]} \leq t_2^{[n]}, \alpha) = \begin{cases} 1 & \text{if } t_1|_\alpha \leq t_2|_\alpha \\ c \cdot \left(1 - \frac{t_2|_\alpha - t_1|_\alpha}{2^n}\right) & \text{otherwise} \end{cases}$$

- **Possible moves**: Bit-flips, increment, decrement, negation.
- **Techniques lifted from SAT**: Neighbourhood restriction to pre-selected assertions (similar to *WalkSAT*), additive weighting scheme for assertions (similar to *PAWS*), random walks, restarts (similar to *Luby*).
- **Additional techniques**: *Upper Confidence Bounds (UCB)* selection scheme (as used for bandits), *Variable Neighbourhood Search (VNS)*.

Example

Consider the assertion a: $x + 3 = \neg x$, where x is a bit-vector of size $n = 6$ (in practice, n is often much larger), $\neg$ represents bitwise negation, and the $+$ operation is as usual, i.e., with overflow semantics. The equation has two solutions: $x = [0,1,1,1,1,0]$ and $x = [1,1,1,1,1,0]$. If we initialize the search at $x = [0,\dots,0]$ and use $c = 1$ for computing the score s, the trace of visited states could look as follows:

x: 000000 ✗
a: 000011 = 111111
s: 2/6

$\xrightarrow{\textit{negation}}$

x: 111111 ✗
a: 000010 = 000000
s: 5/6

$\xrightarrow{\textit{bit-flip [0]}}$

x: 111110 ✓
a: 000001 = 000001
s: 6/6

✓

↓ *bit-flip [0]*

x: 000001 ✗
a: 000110 = 111110
s: 2/6

[1] … [4] …

↓ … *[5]*

x: 100000 ✗
a: 100011 = 011111
s: 2/6

All states reachable by a single bit-flip from the initial one, have a score $s < 5/6$.

Experiments

We ran experiments on two different sets of benchmarks. The first benchmark family is the **QF_BV** benchmark set, which consists of **7498 instances** that can be found in the SMT-LIB and are also part of the SMT Competition. The second benchmark family is the **SAGE2** benchmark set, consisting of **8017 instances**. Those problems were generated as part of the SAGE project at Microsoft, describing some testcases for automated whitebox fuzz testing and are known to be hard for state-of-the-art SMT solvers.

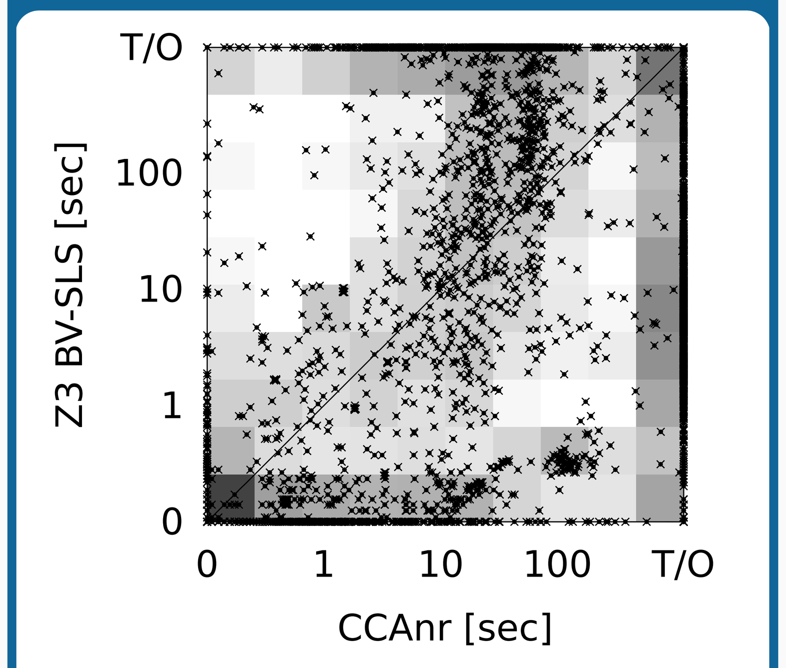
Number of solved instances

	QF_BV	SAGE2
CCAnr	**5409**	64
CCASat	4461	8
probSAT	3816	10
Sparrow	3806	12
VW2	2954	4
PAWS	3331	**143**
YalSAT	3756	142
Z3 (Default)	7173	5821
BV-SLS	**6172**	**3719**

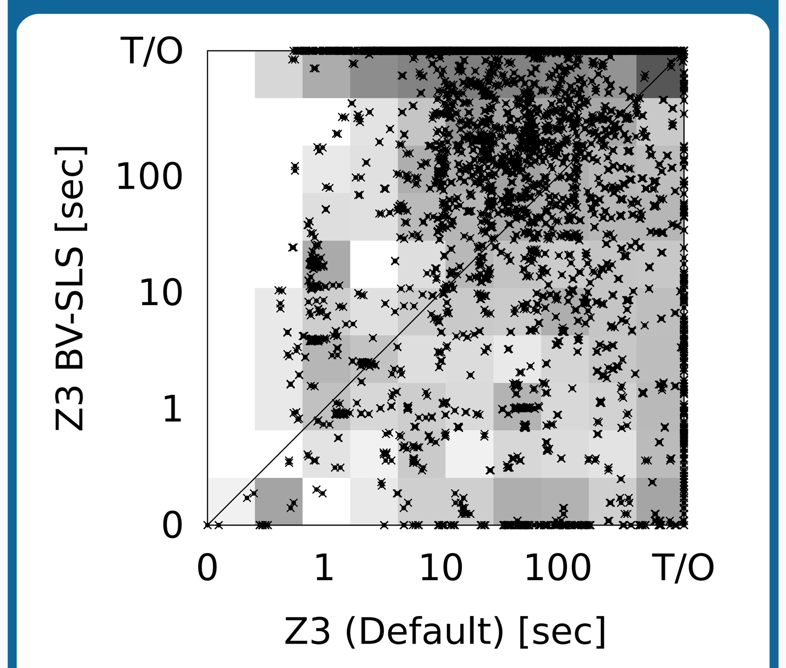
All experiments were run on a Windows HPC cluster of dual Quad-Xeon (E54xx) machines, 16 GB RAM, and used a time limit of 1200 seconds.

We compared our new solver BV-SLS to the most recent version of Microsoft's state-of-the-art SMT solver Z3 (which is based on *bit-blasting*) and also evaluated several SLS solvers for SAT on the propositional encodings of our benchmarks in *Conjunctive Normal Form (CNF)*.

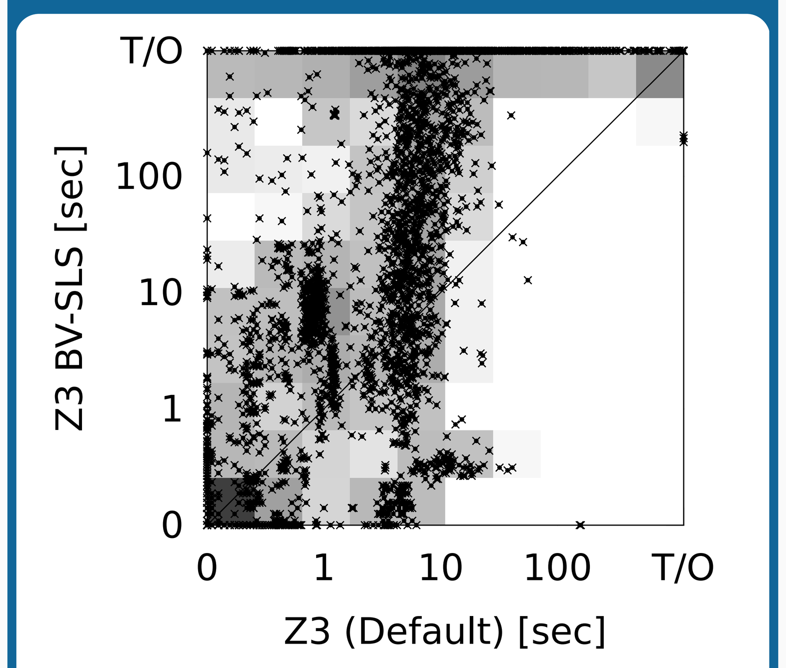
QF_BV, BV-SLS vs CCAnr



SAGE2, BV-SLS vs Z3 (Default)



QF_BV, BV-SLS vs Z3 (Default)



Conclusion

- Novel **SLS algorithm** directly **on the theory level**.
  → Bridging the gap between SMT and SLS.
- **Techniques** used for SAT **can be successfully lifted** to the SMT level.
- Solver BV-SLS **outperforms SLS for SAT** on the propositional encoding.
  → Benefit of using word-level information.
- Insights into the **importance of exploiting problem structure** also in SAT SLS solvers.
- Still a gap in performance compared to state-of-the-art bit-vector solvers in general, but outperforming Z3 on many **industrial instances of practical relevance**.
  → Interesting possibilities in combining our approach with existing techniques.
- Natural extension to **additional theories**.

Additional Information

All source code is available at http://z3.codeplex.com as part of the Z3 project. Contact:

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