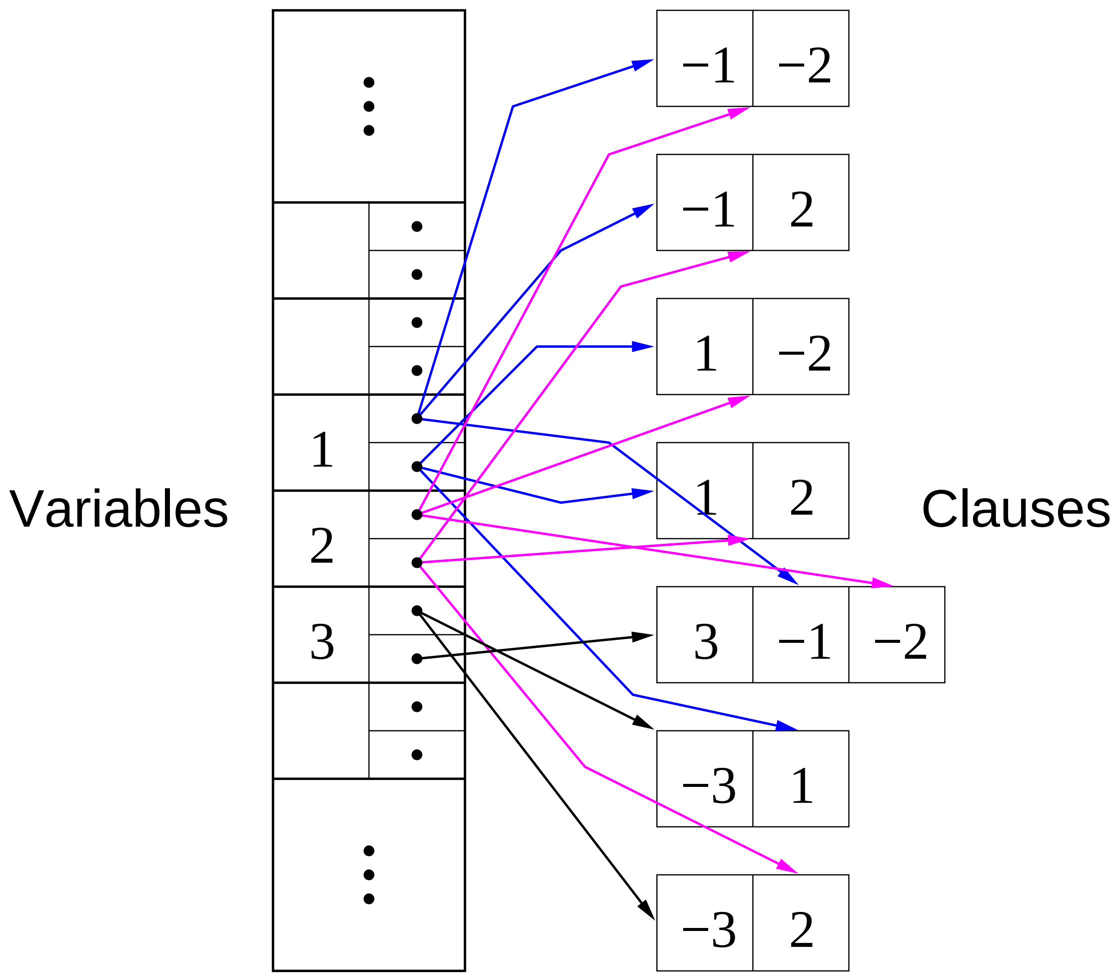
Simple Data Structures in DP Implementation



BCP Implementation Details

- each variable is marked as *unassigned*, *false*, or *true* ($\{X, 0, 1\}$)

- no explicit resolution:
  - when a literal is assigned visit all clauses where its negation occurs
  - find those clauses which have all but one literal assigned to false
  - assign remaining non false literal to *true* and continue

- decision:
  - heuristically find a variable that is still unassigned
  - heuristically determine phase for assignment of this variable

More Implementation Details

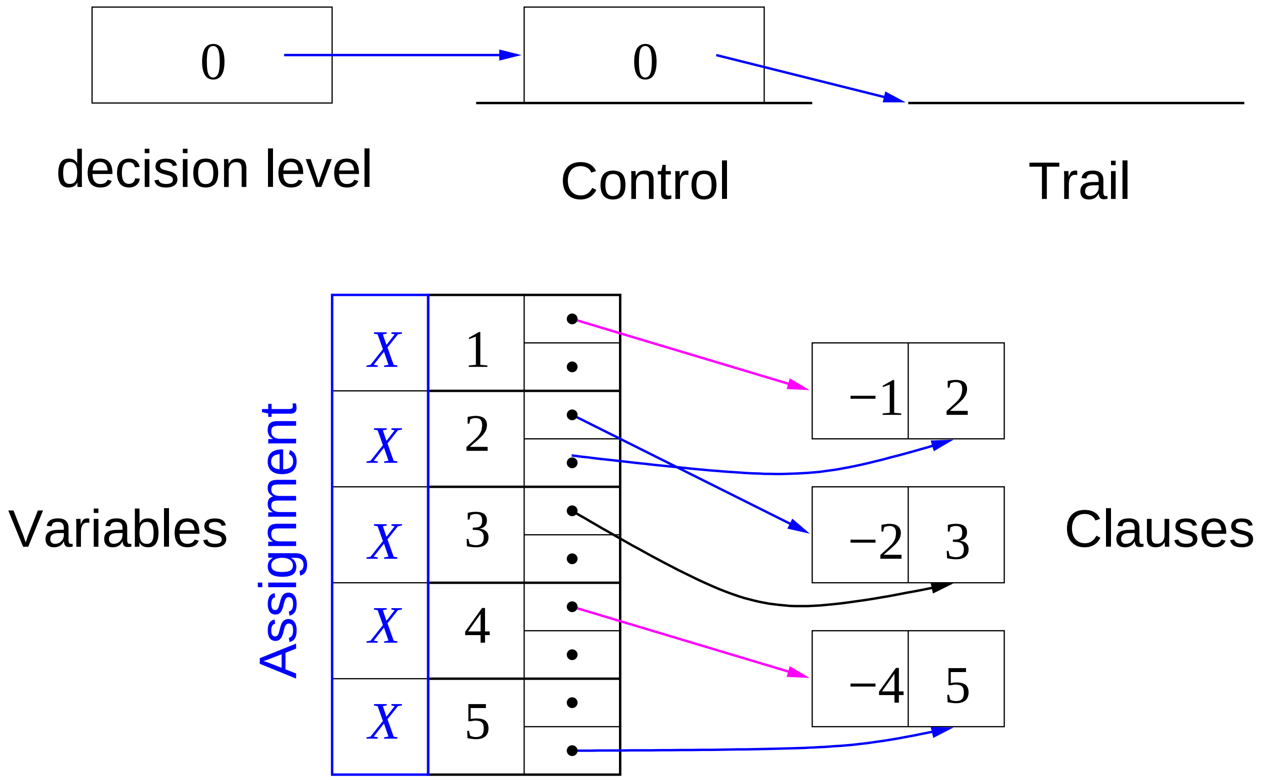
- *decision level* is the depth of recursive calls (= #nested decisions)

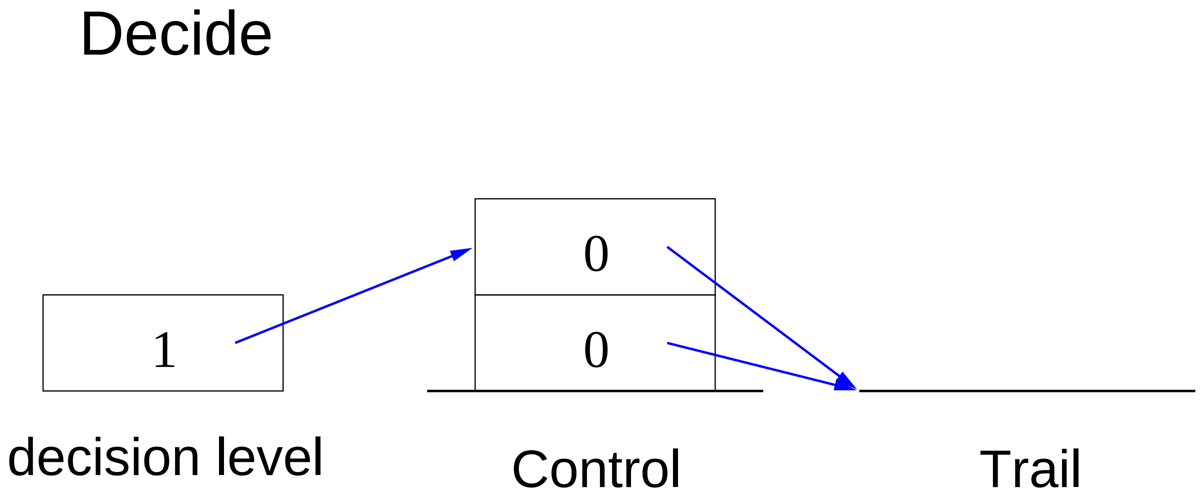
- the *trail* is a stack to remember order in which variables are assigned

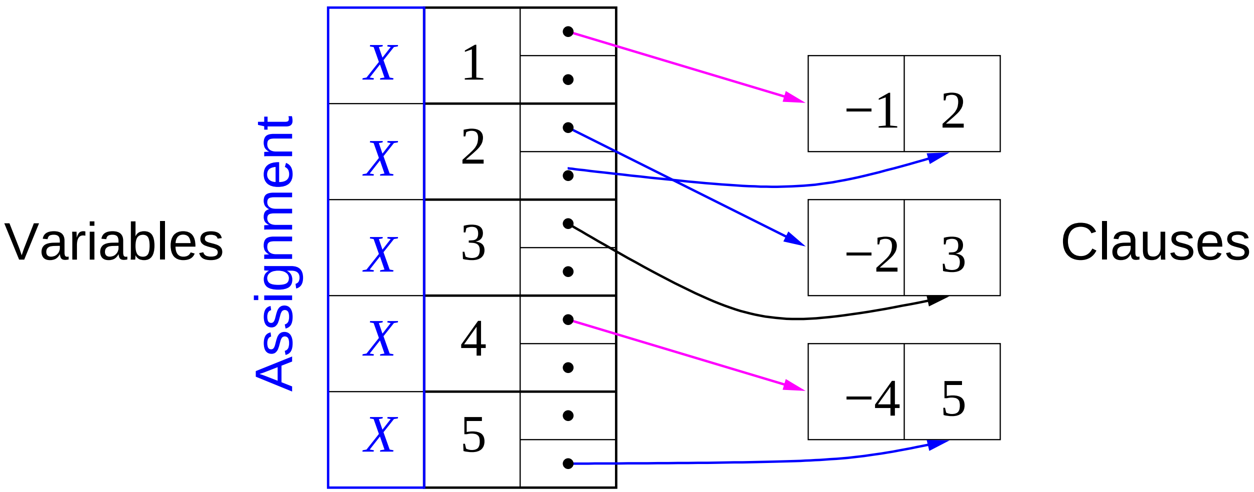
- for each decision level the old trail height is saved on the *control stack*

- undoing assignments in backtracking:
  - get old trail height from control stack
  - unassign all variables up to the old trail height

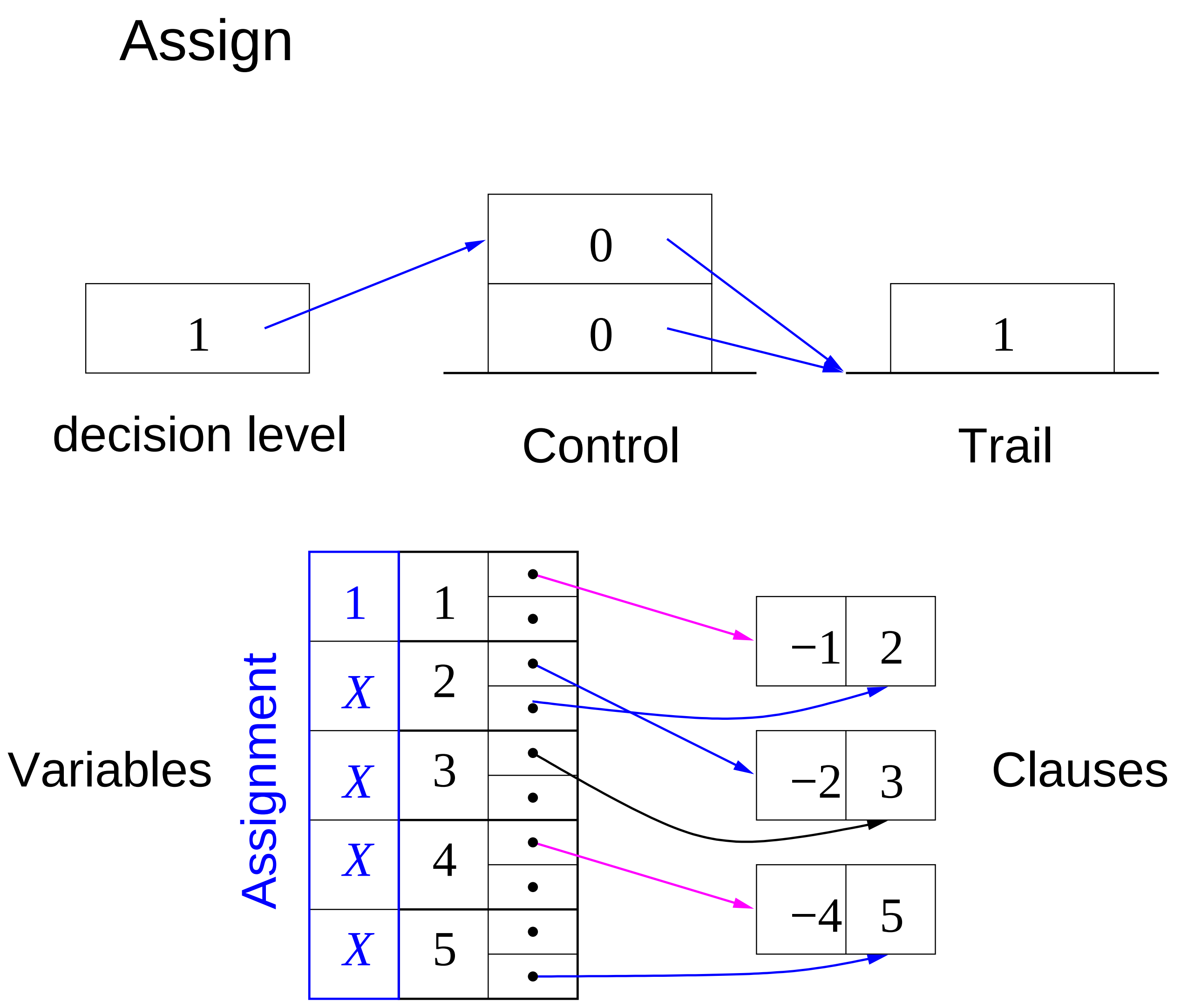
BCP Example


0
0
decision level
Control
Trail
Variables
Assignment
X
1
X
2
X
3
X
4
X
5
−1
2
−2
3
−4
5
Clauses

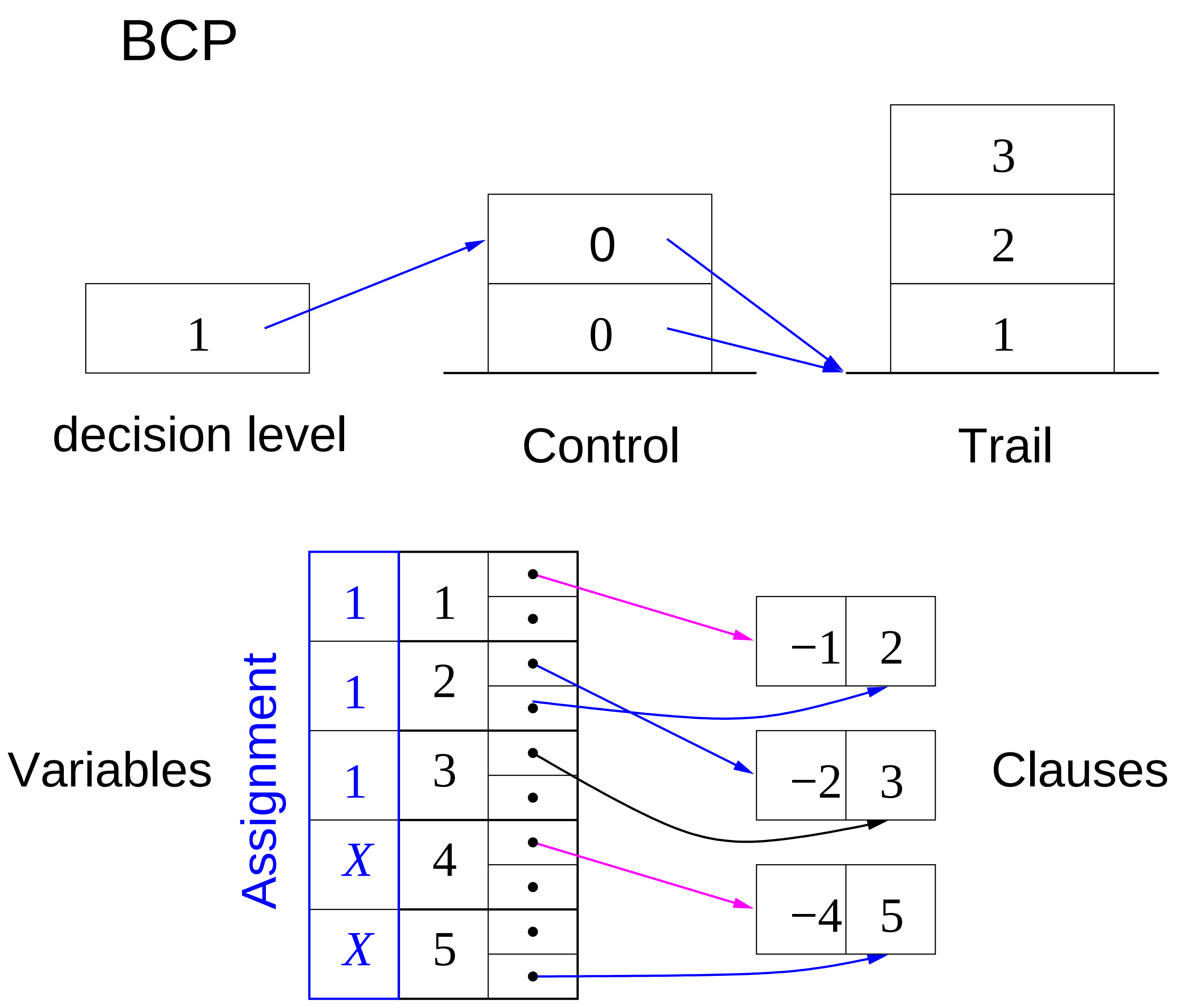

Decide
1
0
0
decision level
Control
Trail


Variables
Assignment
X 1
X 2
X 3
X 4
X 5
−1 2
−2 3
−4 5
Clauses

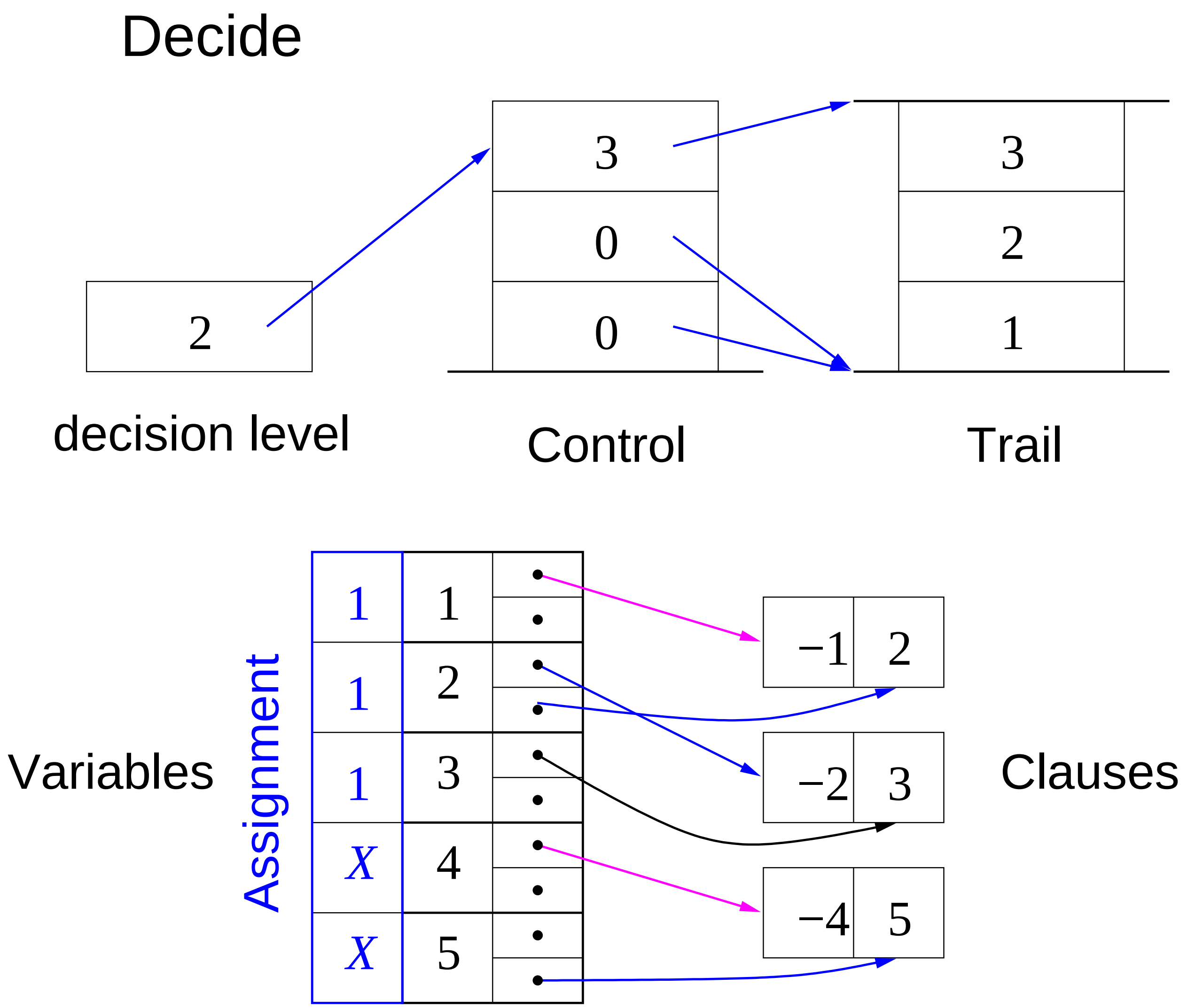
Example cont.



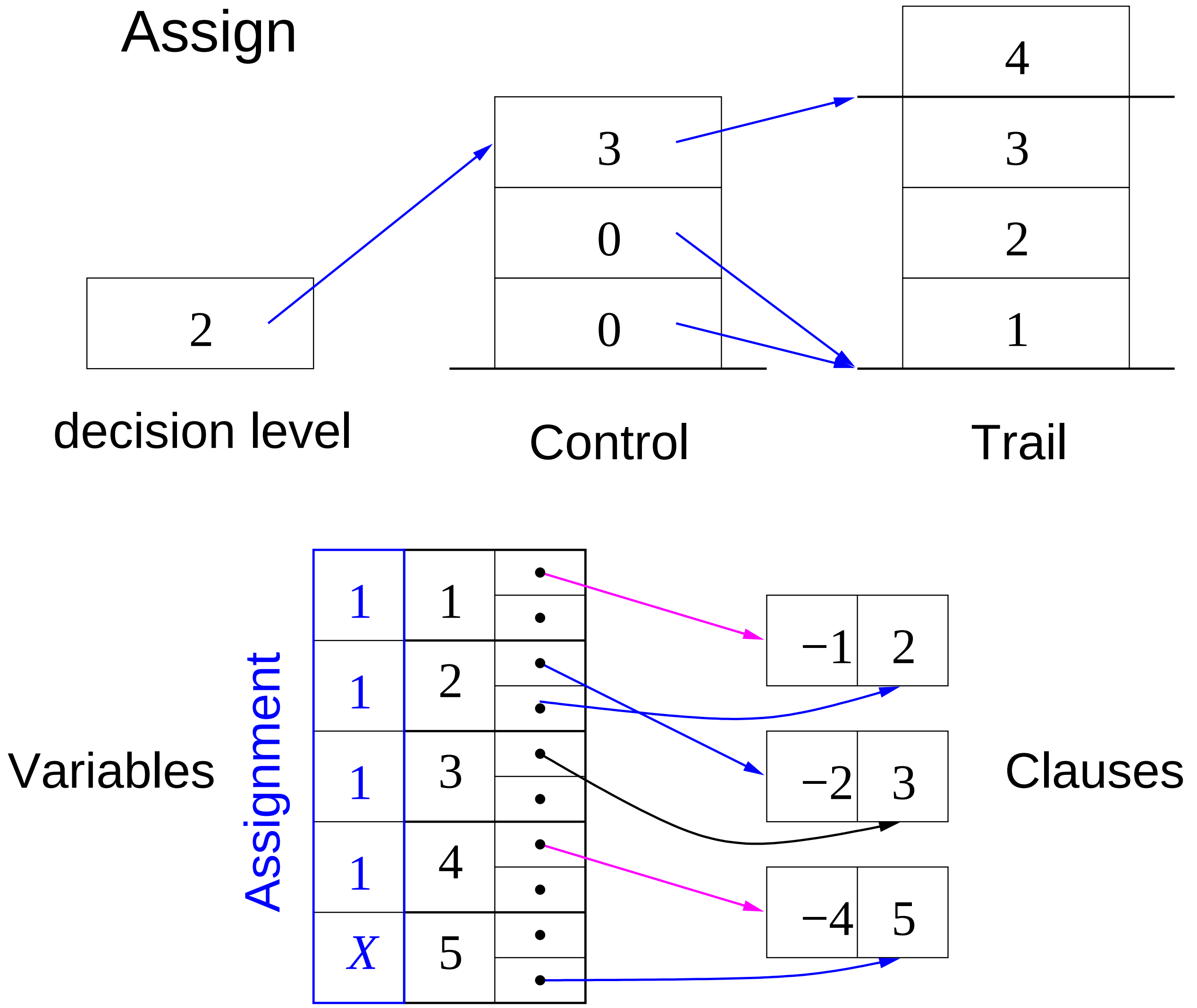
Example cont.



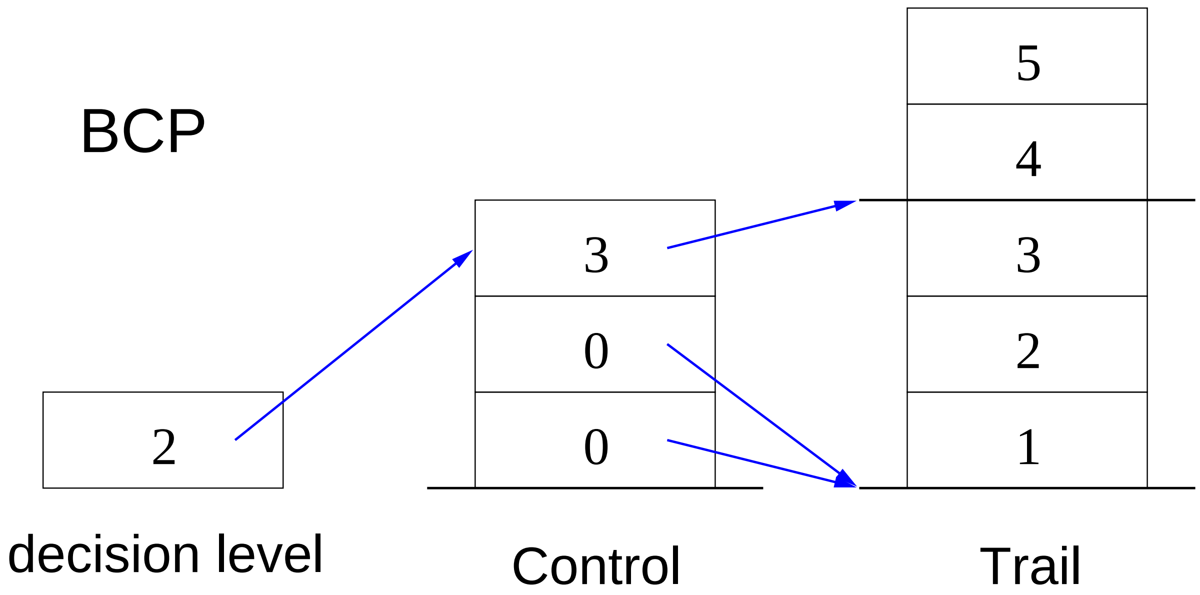
Example cont.

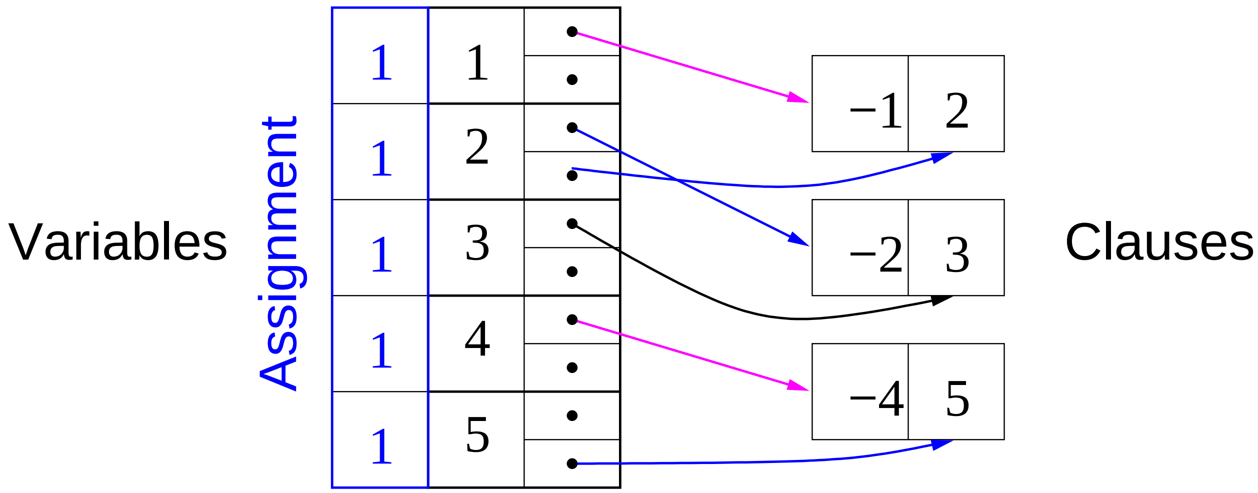


Example cont.



Example cont.





Decision Heuristics

- **static heuristics:**
  - one *linear* order determined before solver is started
  - usually quite fast, since only calculated once
  - can also use more expensive algorithms

- **dynamic heuristics**
  - typically calculated from number of occurences of literals
    (in unsatisfied clauses)
  - rather expensive, since it requires traversal of all clauses
    (or more expensive updates in BCP)
  - recently, *second order* dynamic heuristics (Chaff)

Cut Width Heuristics

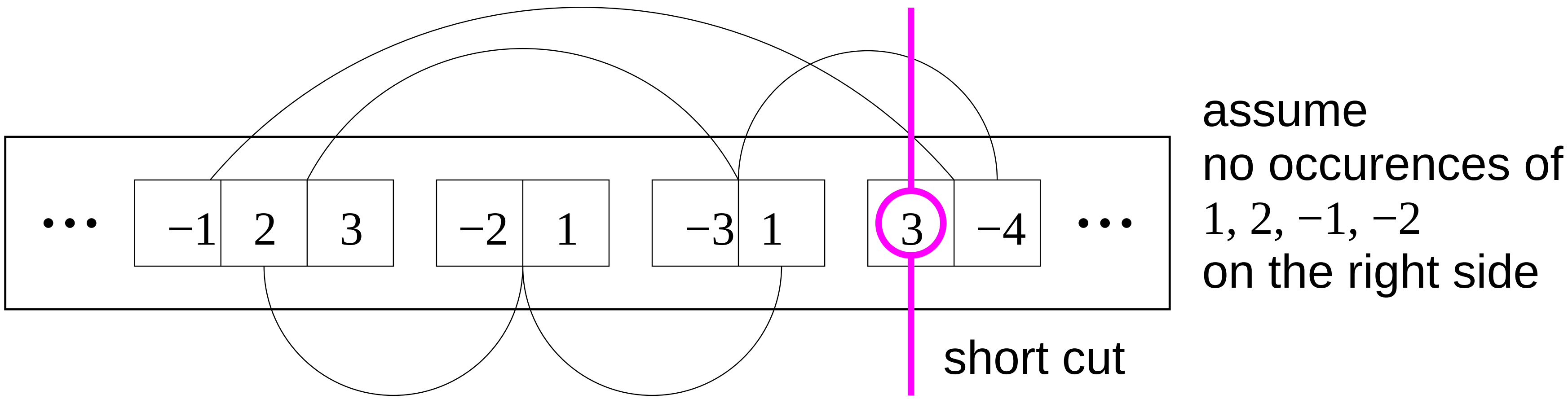
- view CNF as a graph:

  clauses as nodes, edges between clauses with same variable

- a *cut* is a set of variables that splits the graph in two parts

- recursively find short cuts that cut of parts of the graph

- static or dynamically order variables according to the cuts



Cut Width Algorithm

```
int
sat (CNF cnf)
{
  SetOfVariables cut = generate_good_cut (cnf);
  CNF assignment, left, right;

  left = cut_off_left_part (cut, cnf);
  right = cut_off_right_part (cut, cnf);

  forall_assignments (assignment, cut)
  {
    if (sat (apply (assignment, left)) && sat (apply (assignment, right)))
      return 1;
  }

  return 0;
}
```

Cut Width Heuristics cont.

- resembles cuts in circuits when CNF is generated with Tseitin transformation

- ideally cuts have constant or logarithmic size . . .

  - for instance in tree like circuits

  - so the problem is *reconvergence*:
    the same signal / variable is used multiple times

- . . . then satisfiability actually becomes polynomial (see exercise)

A clause is called *positive* if it contains a positive literal.

A clause is called *negative* if all its literals are negative.

A clause is a *Horn* clause if contains at most one positive literal.

CNF is in *Horn Form* iff all clauses are Horn clause (Prolog without negation)

Order assignments point-wise: $\sigma \leq \sigma'$ iff $\sigma(x) \leq \sigma'(x)$ for all $x \in V$

Horn Form with only positive clauses has minimal satisfying assignment.

Minimal satisfying assignment is obtained by BCP (polynomial).

A Horn Form is satisfiable iff the minimal assignments of its positive part satisfies all its negative clauses as well.

DP and Horn Form

- CNF in Horn Form: use above specialized fast algorithm
- non Horn: split on literals which occurs positive in non Horn clauses
  - actually choose variable which occurs most often in such clauses
- this gradually transforms non Horn CNF into Horn Form
- main heuristic in SAT solver SATO
- **Note:** In general, BCP in DP prunes search space by avoiding assignments incompatible to minimal satisfying assingment for the Horn part of the CNF.

non Horn part of CNF	Horn part of CNF

Other popular Decision Heuristics

- Dynamic Largest Individual Sum (DLIS)
  - fastest dynamic first order heuristic (eg GRASP solver)
  - choose literal (variable + phase) which occurs most often
  - ignore satisfied clauses
  - requires explicit traversal of CNF (or more expensive BCP)

- look-forward heuristics (eg SATZ solver)
  - do trial assignments and BCP for all unassigned variables (both phases)
  - if BCP leads to conflict, force toggled assignment of current trial decision
  - skip trial assignments implied by previous trial assignments
    (removes a factor of $|V|$ from the runtime of one decision search)