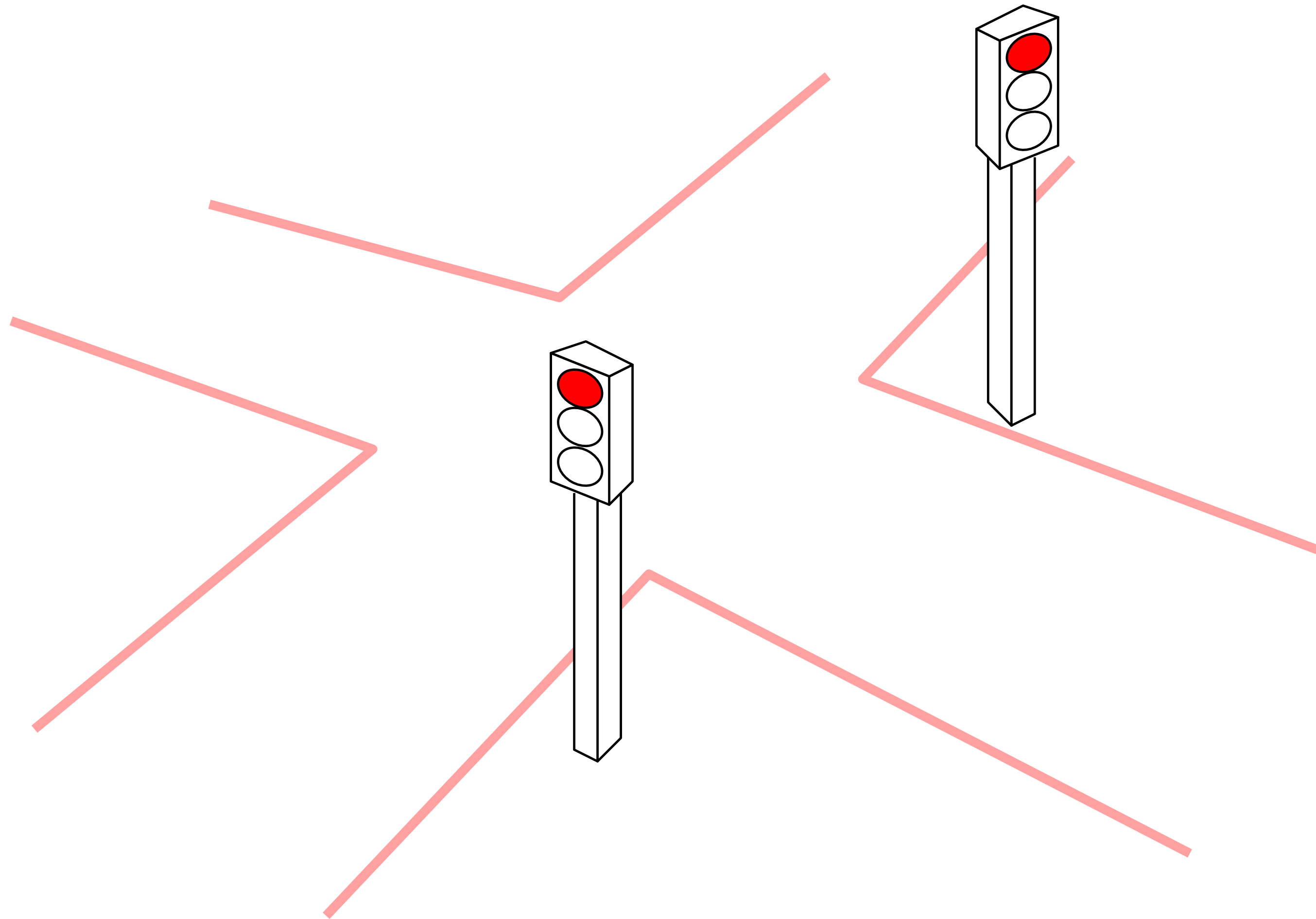
Model Checking

- check **algorithmically** temporal / sequential properties
  - systems are originally **finite state**
  - simple model: finite state automaton
- **comparison** of automata can be seen as model checking
  - check that the output streams of two finite state systems “match”
  - process algebra: simulation and bisimulation checking
- **temporal logics** as specification mechanism
  - safety, liveness and more general temporal operators, fairness

Model Checking for Infinite Systems

- fixpoint algorithms with symbolic representations:
  - timed automata (clocks)
  - hybrid automata (differential equations)
  - termination guaranteed if finite quotient structure exists
- simply run model checker *for some time*, e.g. Java Pathfinder
- run time verification
  1. example: add checker synthesized from temporal spec
  2. example: run all schedules for one test case
- check programs (incl. loops and recursion) over finite domains, e.g. SLAM

Traffic Light Controller (TLC)



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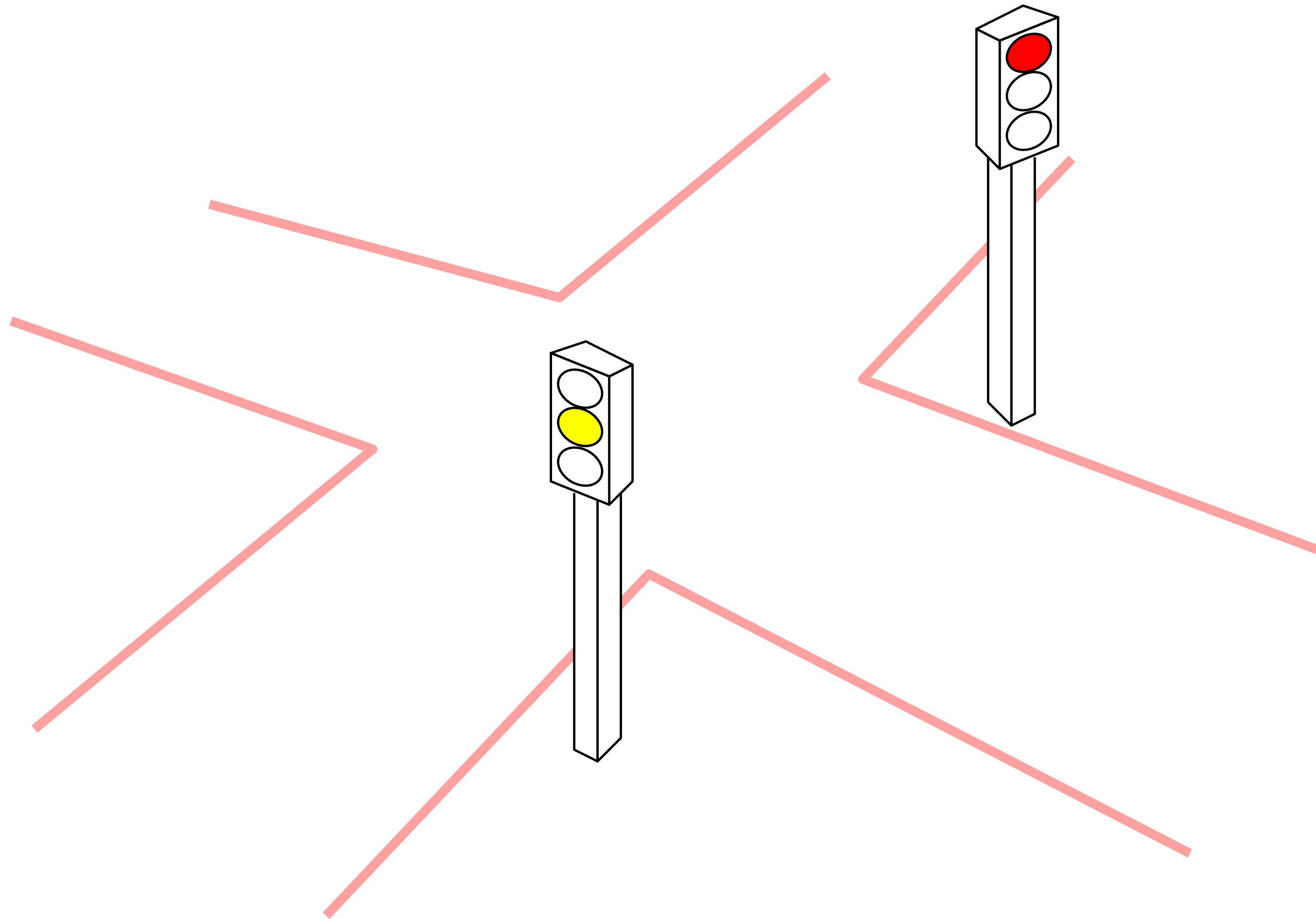

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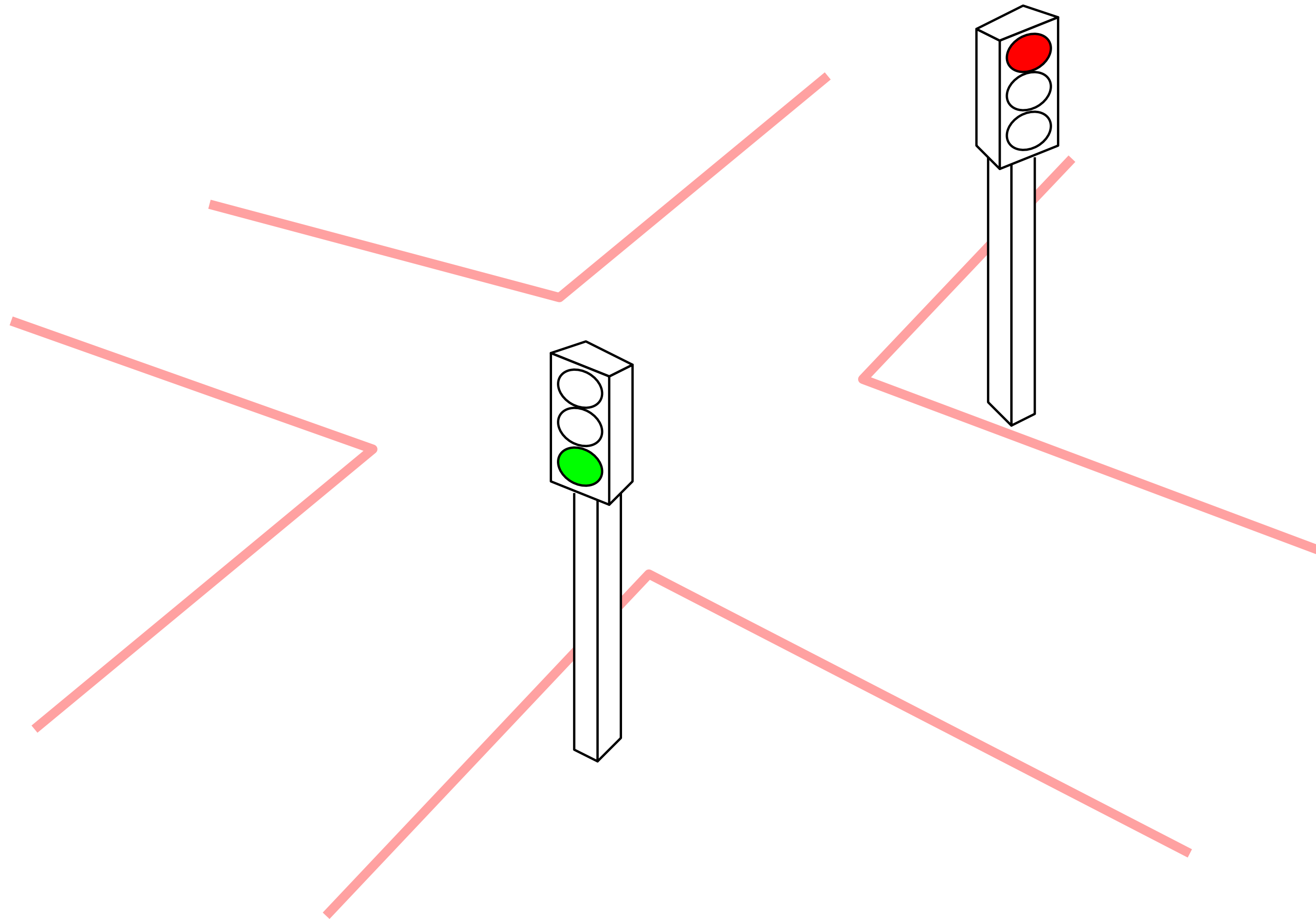

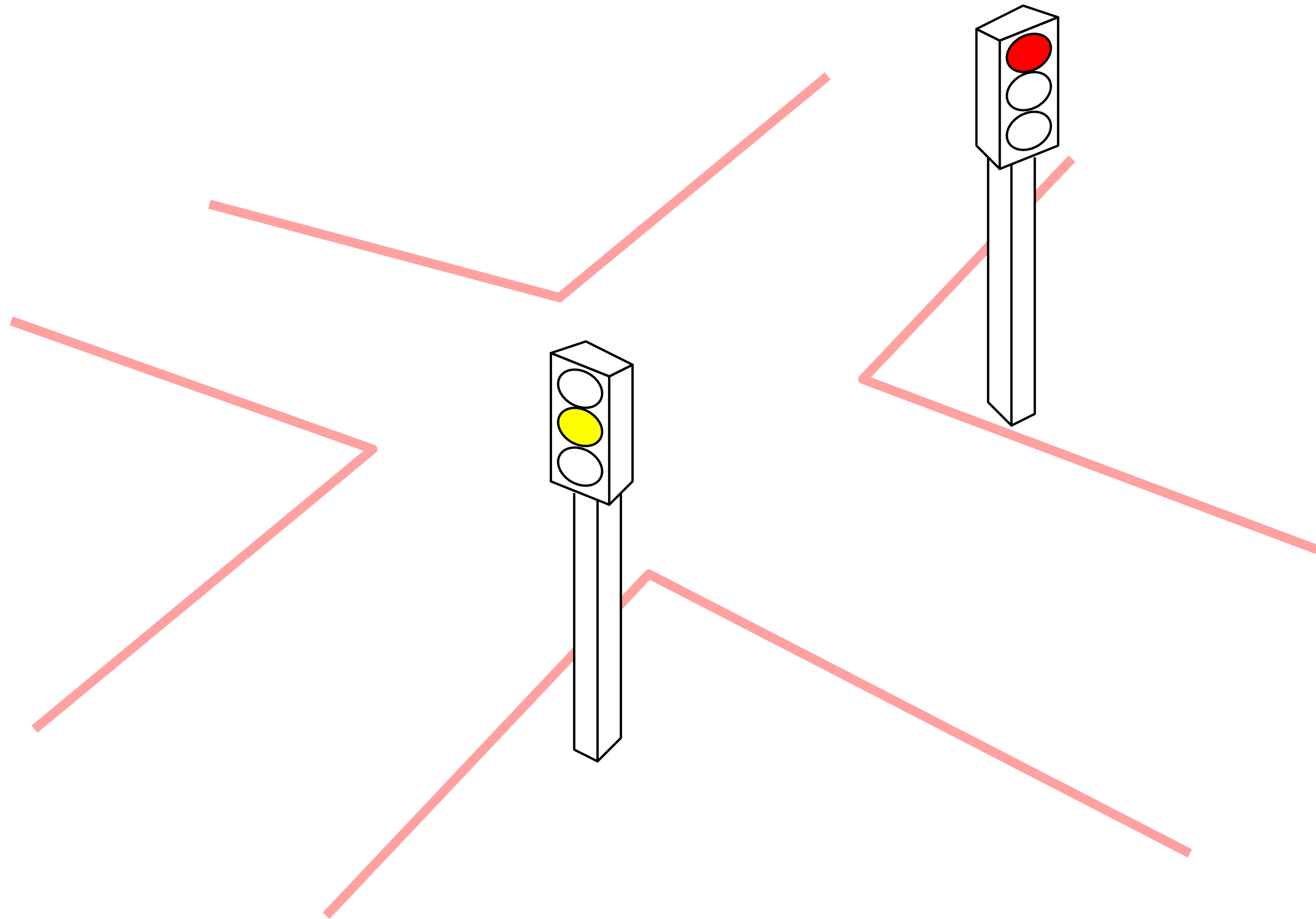

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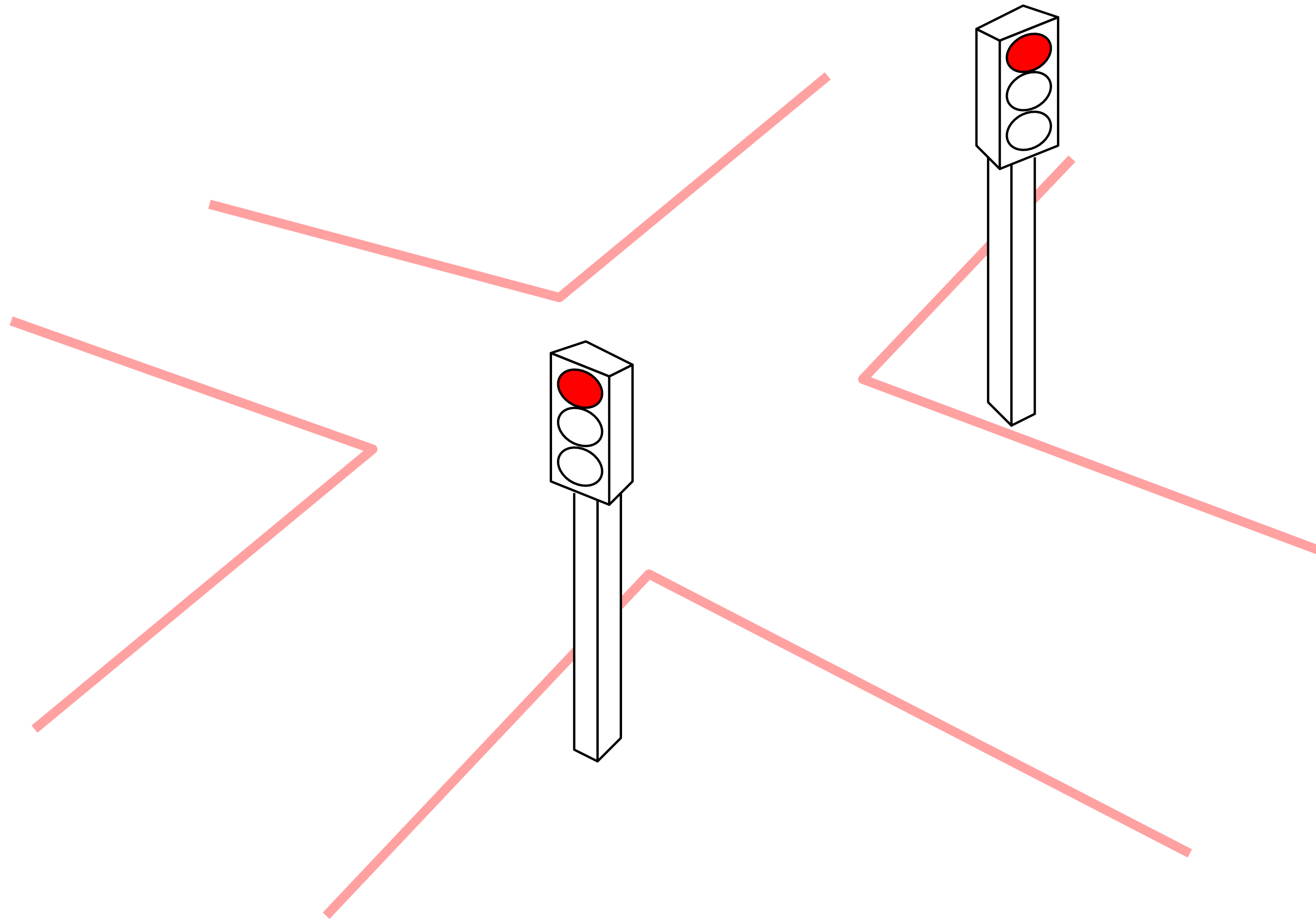

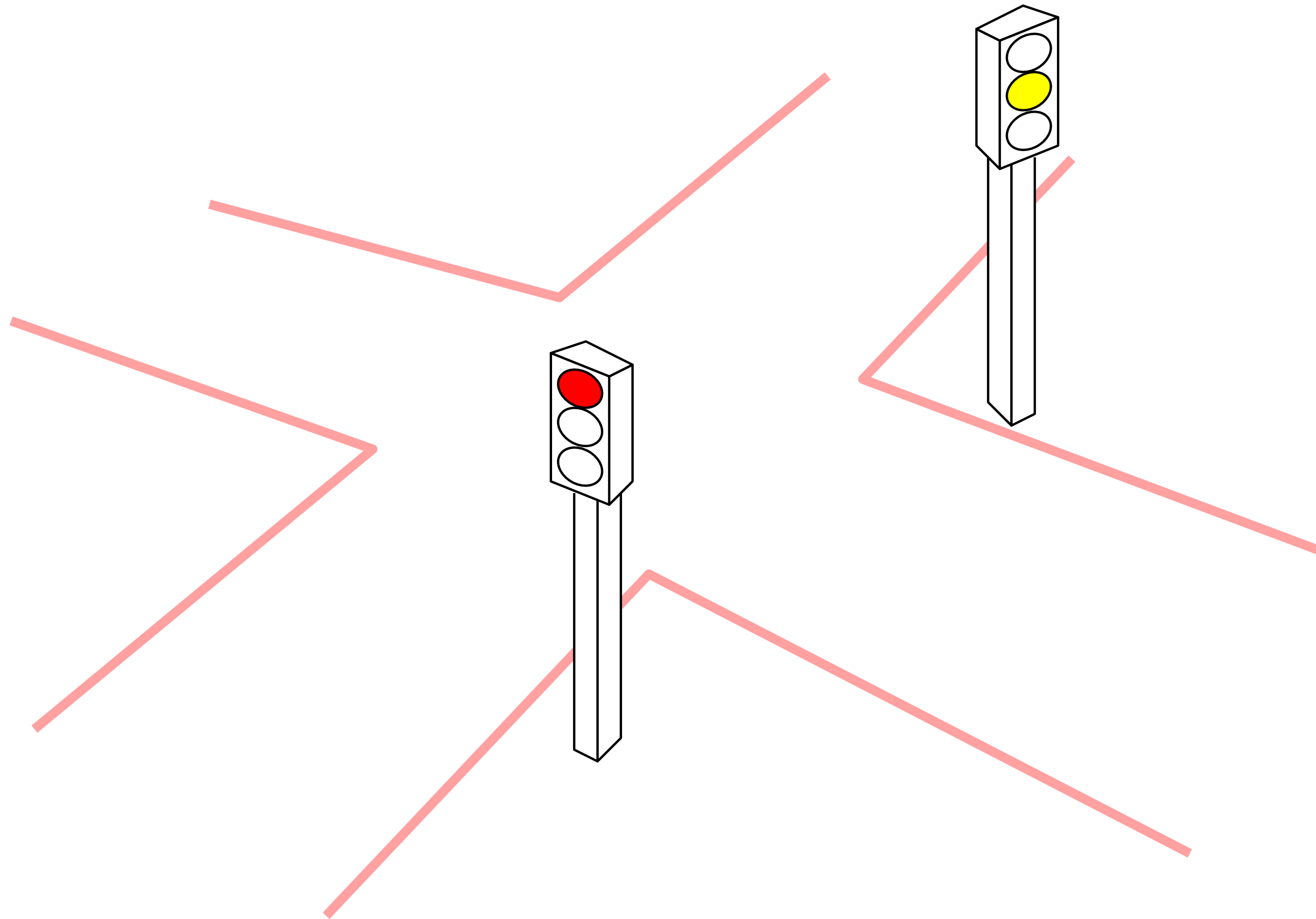

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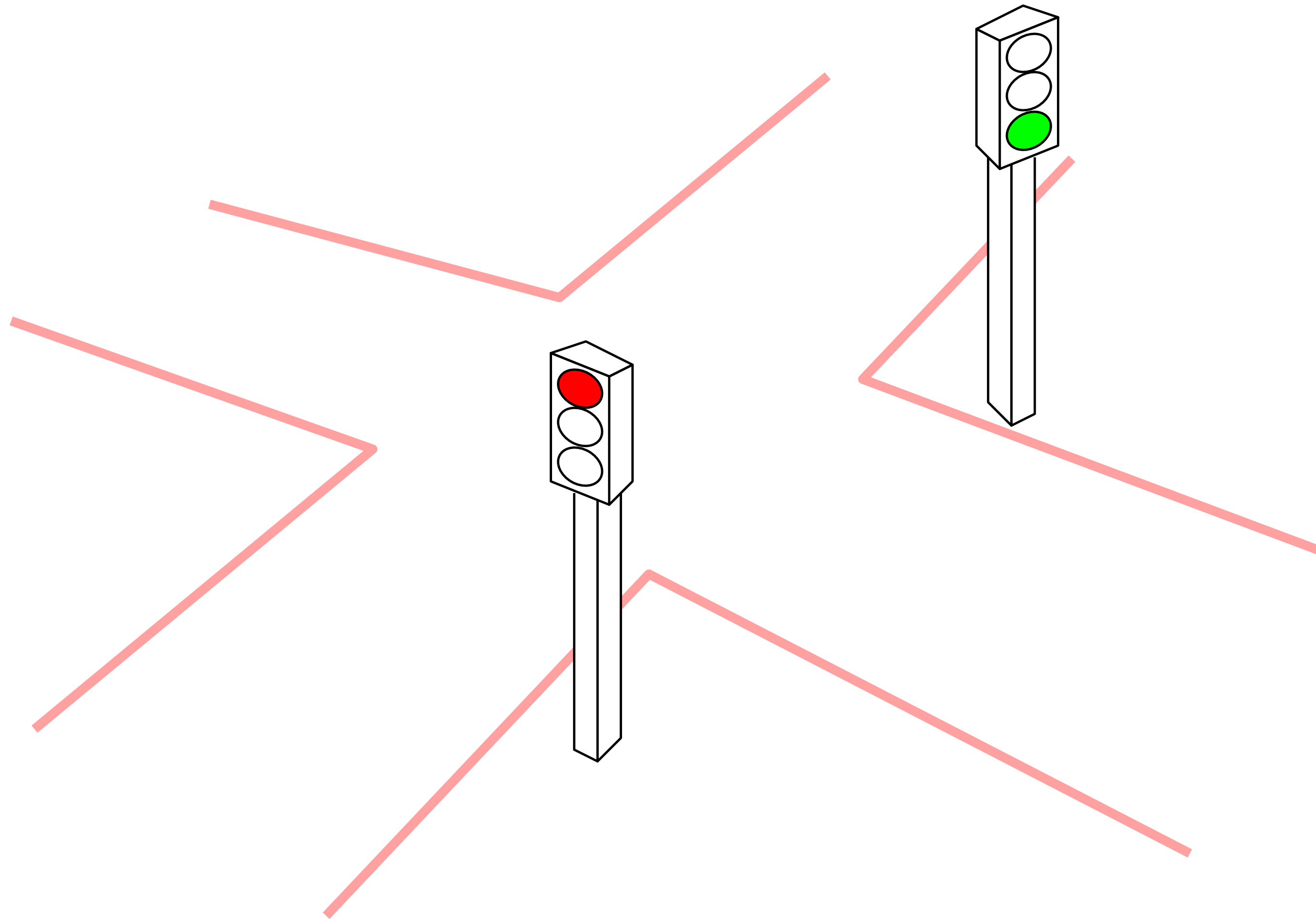

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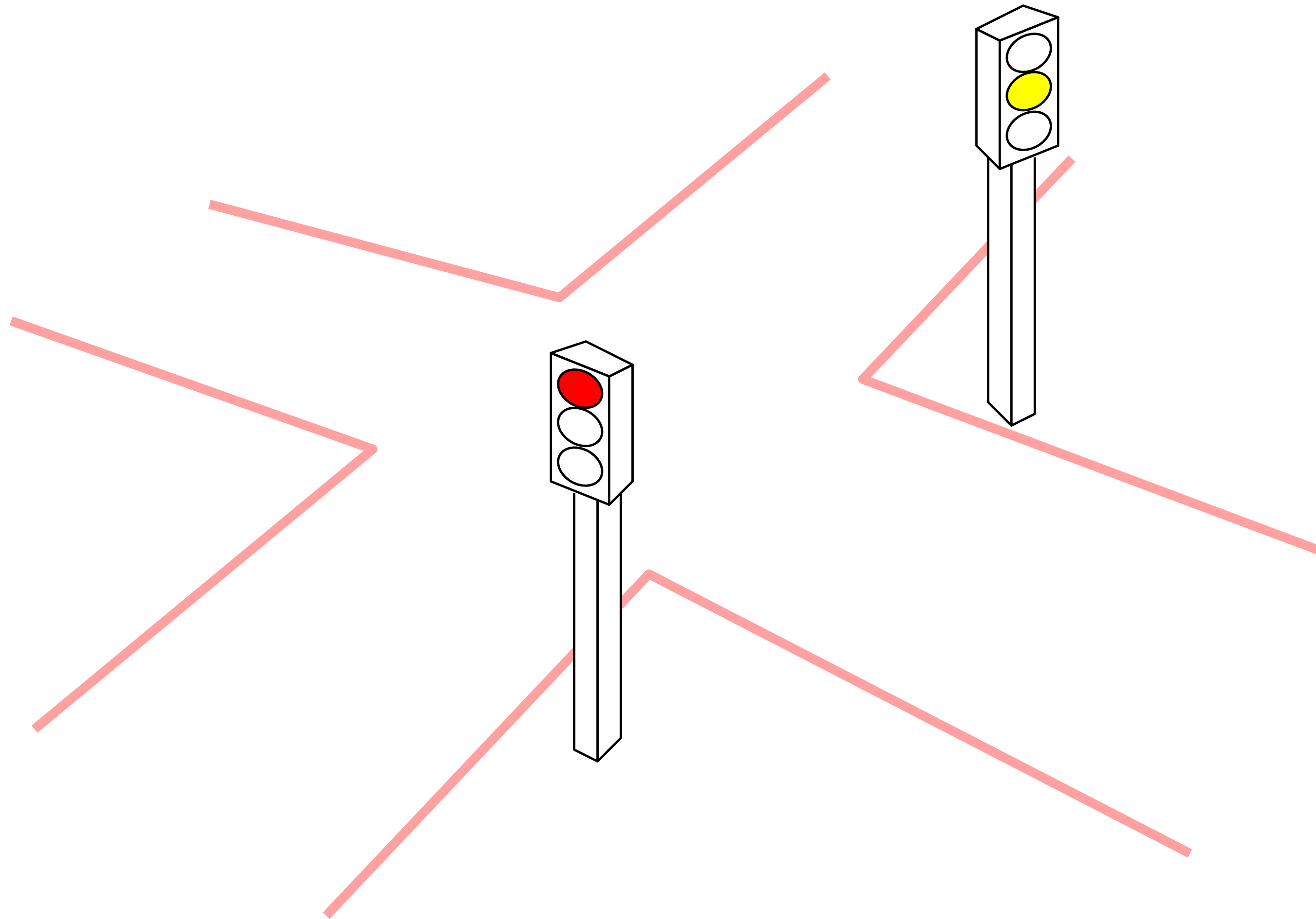

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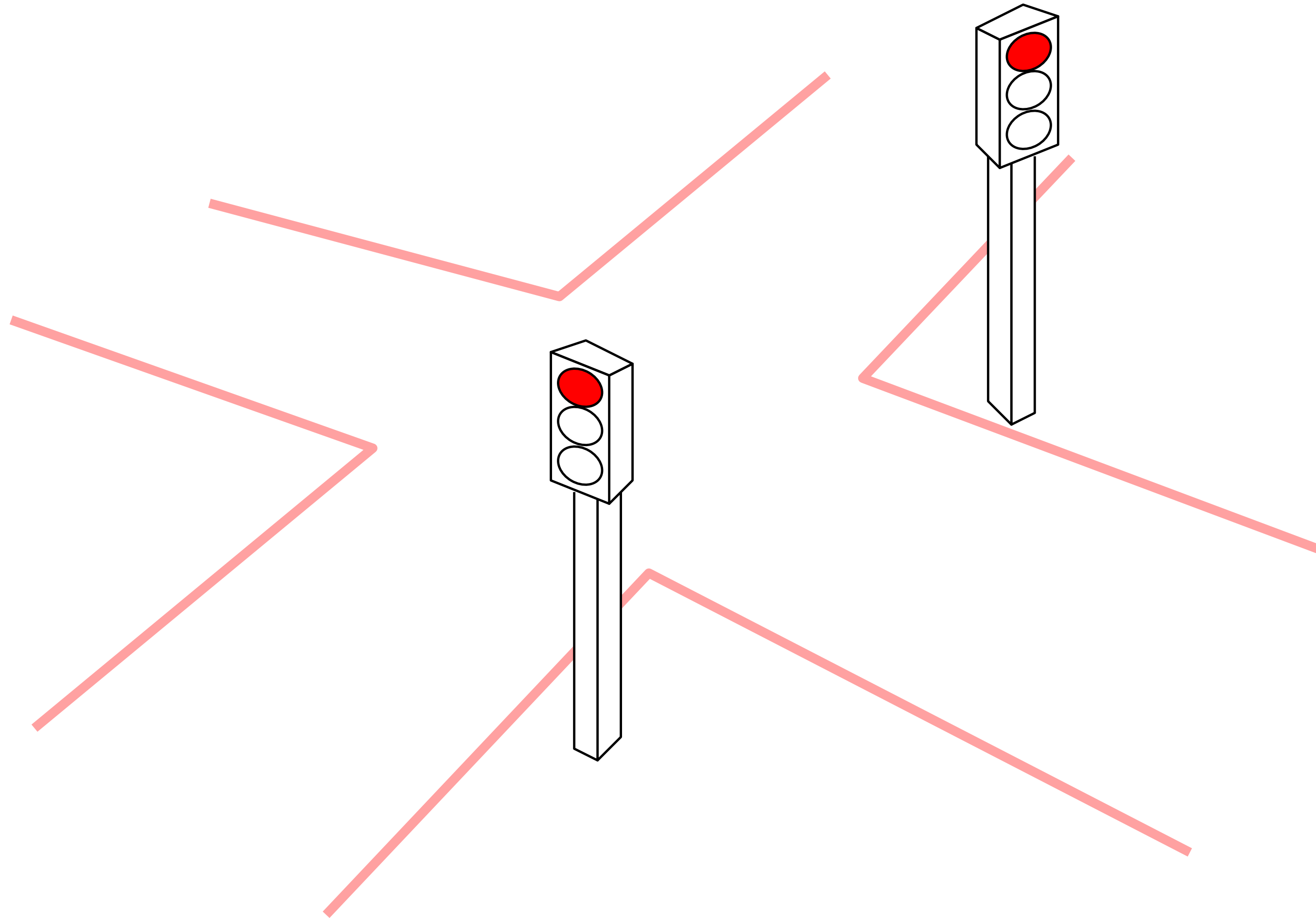
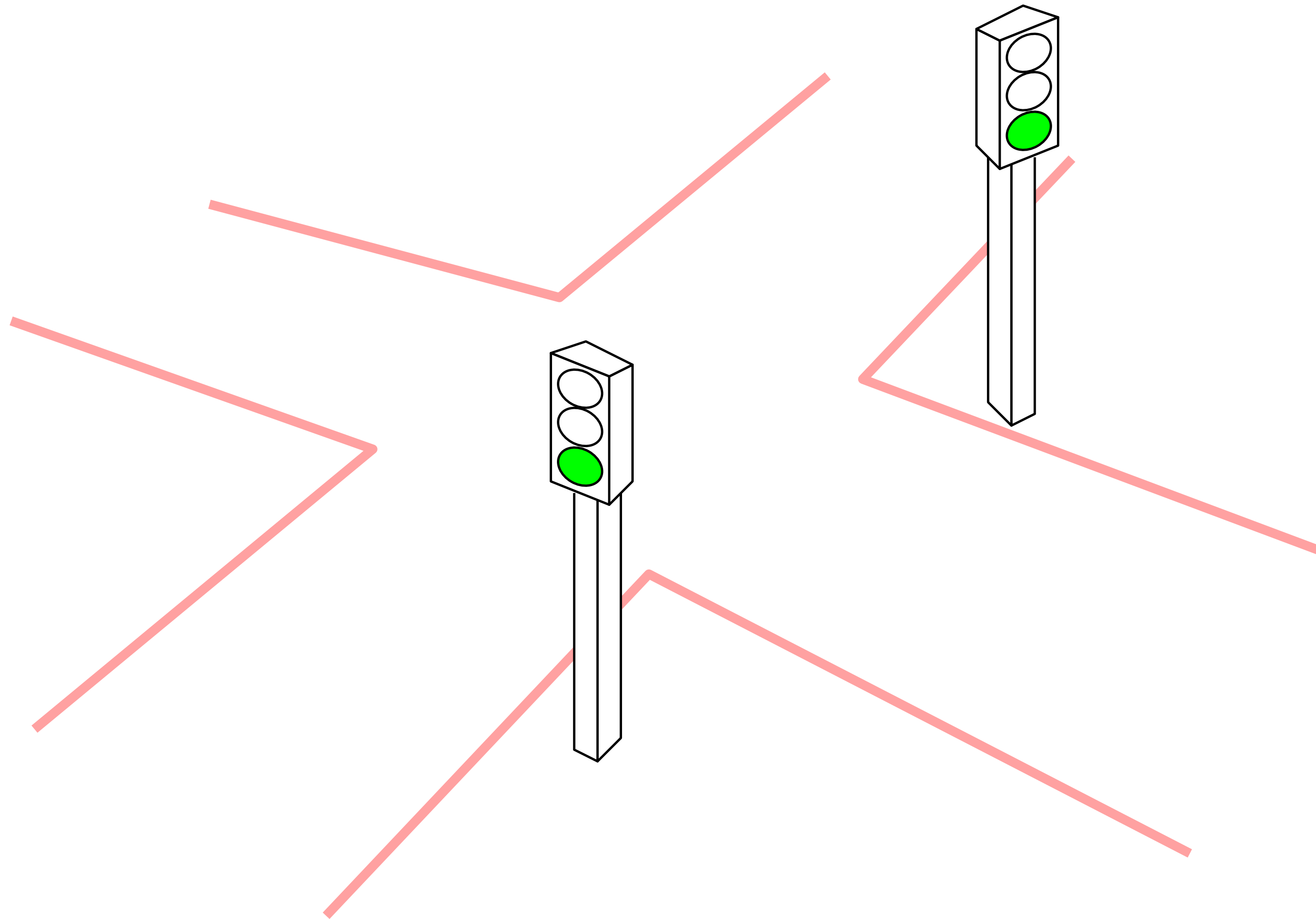




the two traffic lights should never show a green light at the same time

State Space

- state space is the set of assignments to variables of the system
  - state space is finite if the **range** of variables is finite
  - this notion works for inifinite state spaces as well
- TLC example:
  - single assignment $\sigma\colon \{southnorth, eastwest\} \to \{green, yellow, red\}$
  - set of assignments is isomorphic to $\{green, yellow, red\}^2$
  - eg state space is isomorphic to the crossproduct of variable ranges
- not all states are reachable: $(green, green)$

Safety

- safety properties specify **invariants** of the system

- simple generic algorithm for checking safety properties:

  1. iteratively generate all reachable states

  2. check for violation of invariant for newly reached states

  3. terminate if all newly reached states can be found

- compare with **assertions**

  – used in run time checking: `assert` in C and VHDL

  – contract checking: `require`, `ensure`, etc. in Eiffel

```
MODULE trafficlight (enable)
VAR
  light : { green, yellow, red };
  back : boolean;
ASSIGN
  init (light) := red;
  next (light) :=
    case
      light = red & !enable : red;
      light = red & enable : yellow;
      light = yellow & back : red;
      light = yellow & !back : green;
      1 : yellow;
    esac;
  next (back) :=
    case
      light = red & enable : 0;
      light = green : 1;
      1 : back;
    esac;
MODULE main
VAR
  southnorth : trafficlight (1);
  eastwest : trafficlight (1);
SPEC
  AG !(southnorth.light = green & eastwest.light = green)
```

- symbolic model checker implemented by Ken McMillan at CMU (early 90'ies)

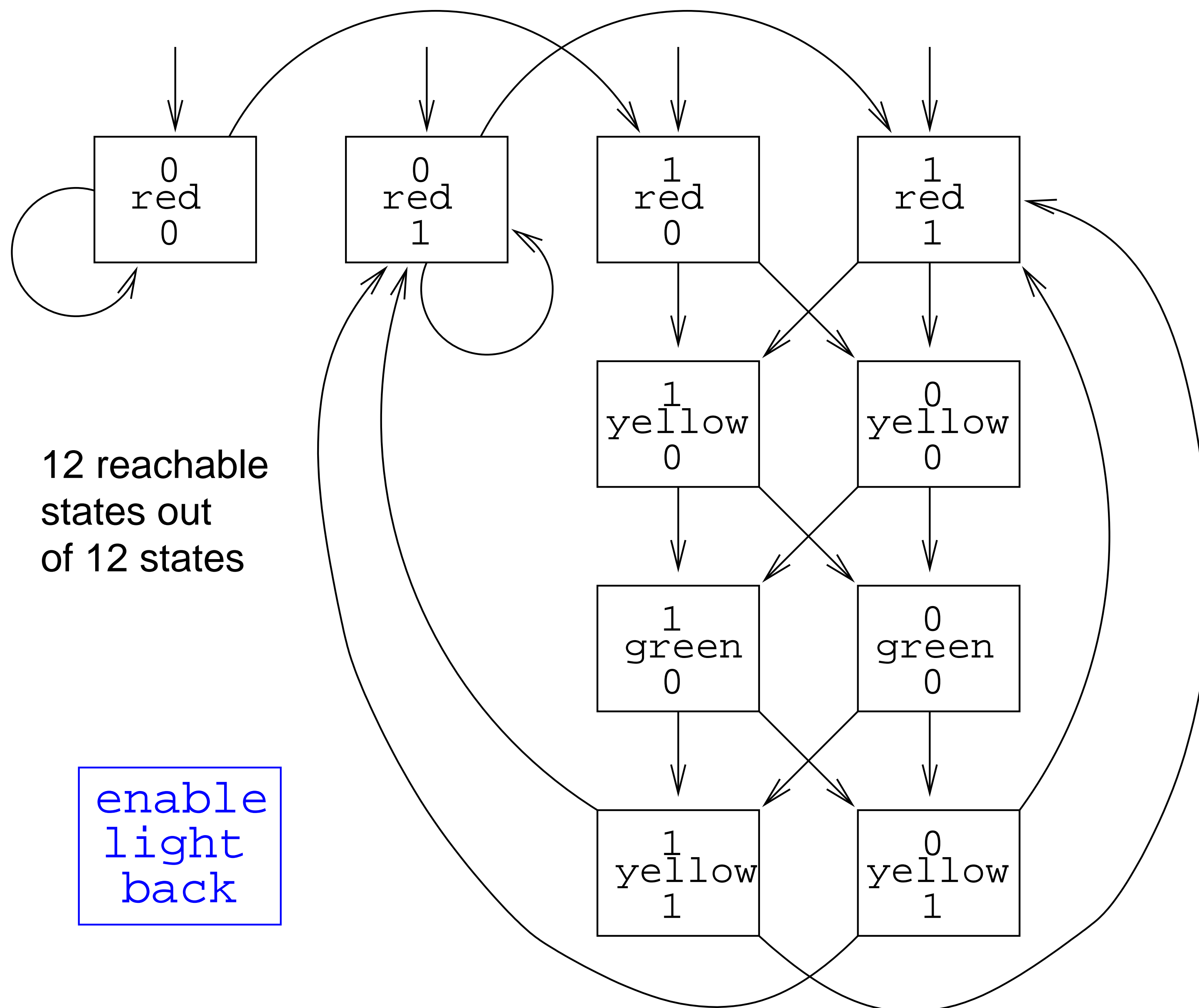
- input language: finite models + temporal specification

- hierarchical description, similar to hardware description language (HDL)

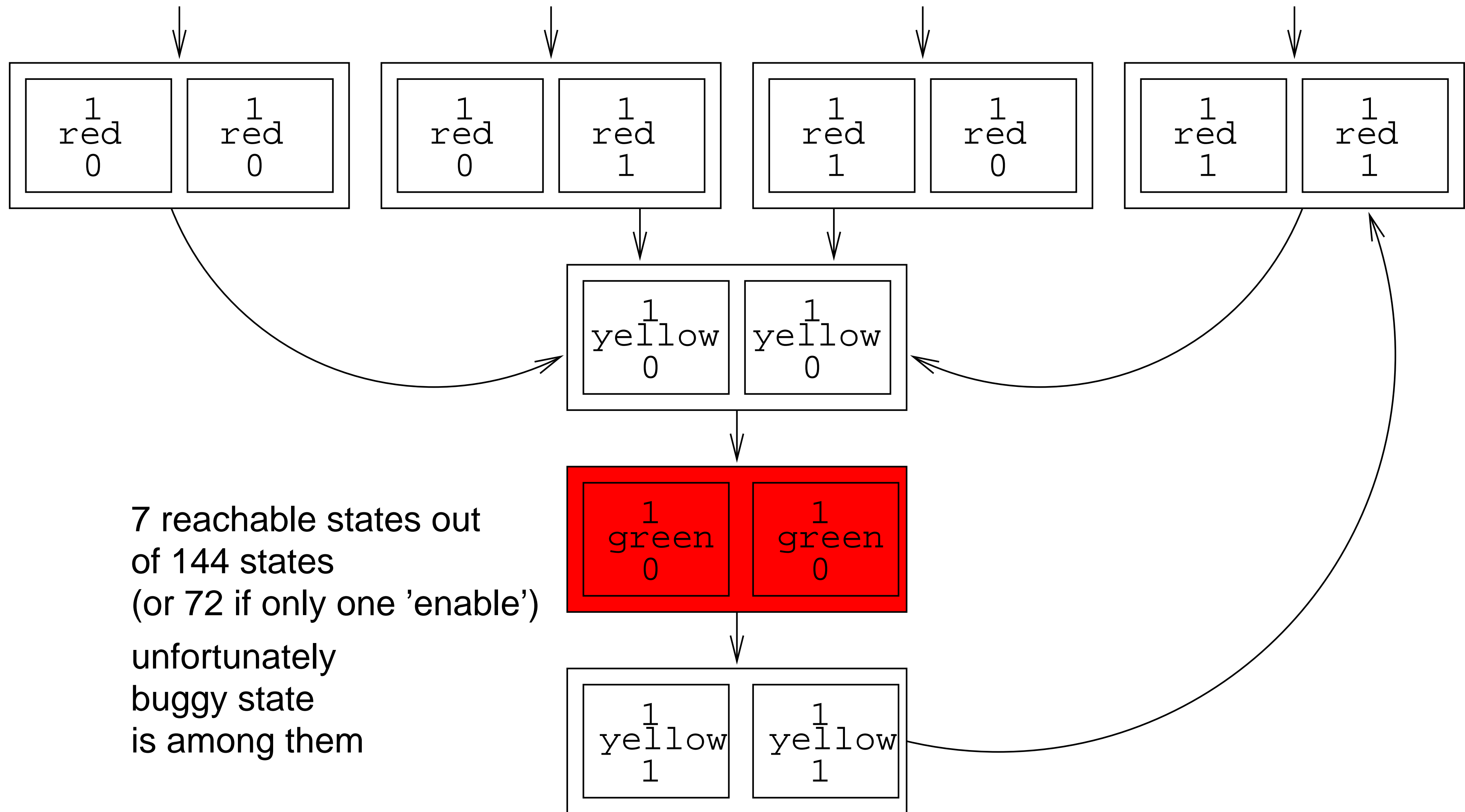
- integer and enumeration types, arithmetic operations

- heavily relies on the data structure Binary Decision Diagrams (BDDs)

Reachable States of One Traffic Light



Reachable States of Unsafe TLC

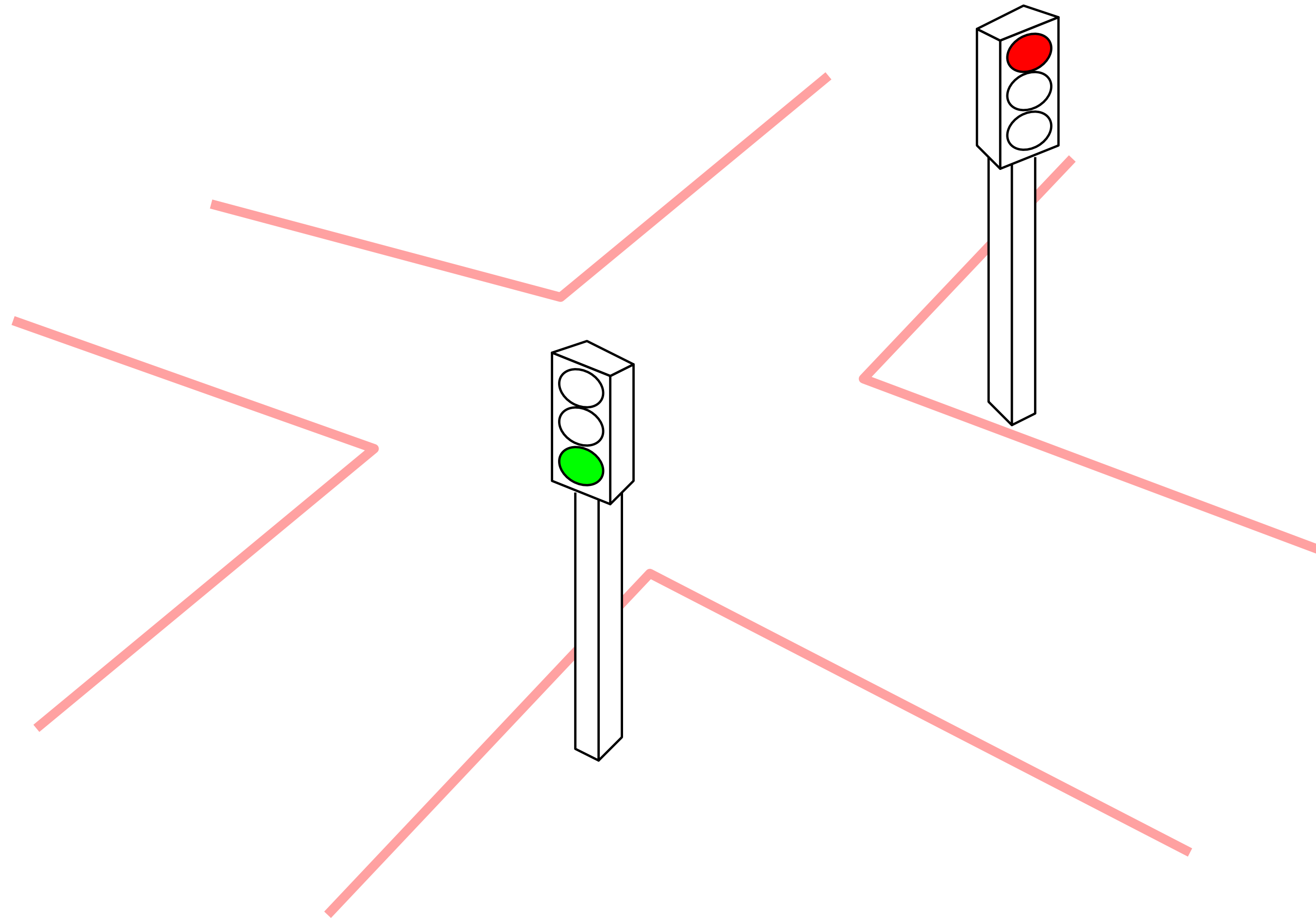


7 reachable states out
of 144 states
(or 72 if only one ’enable’)

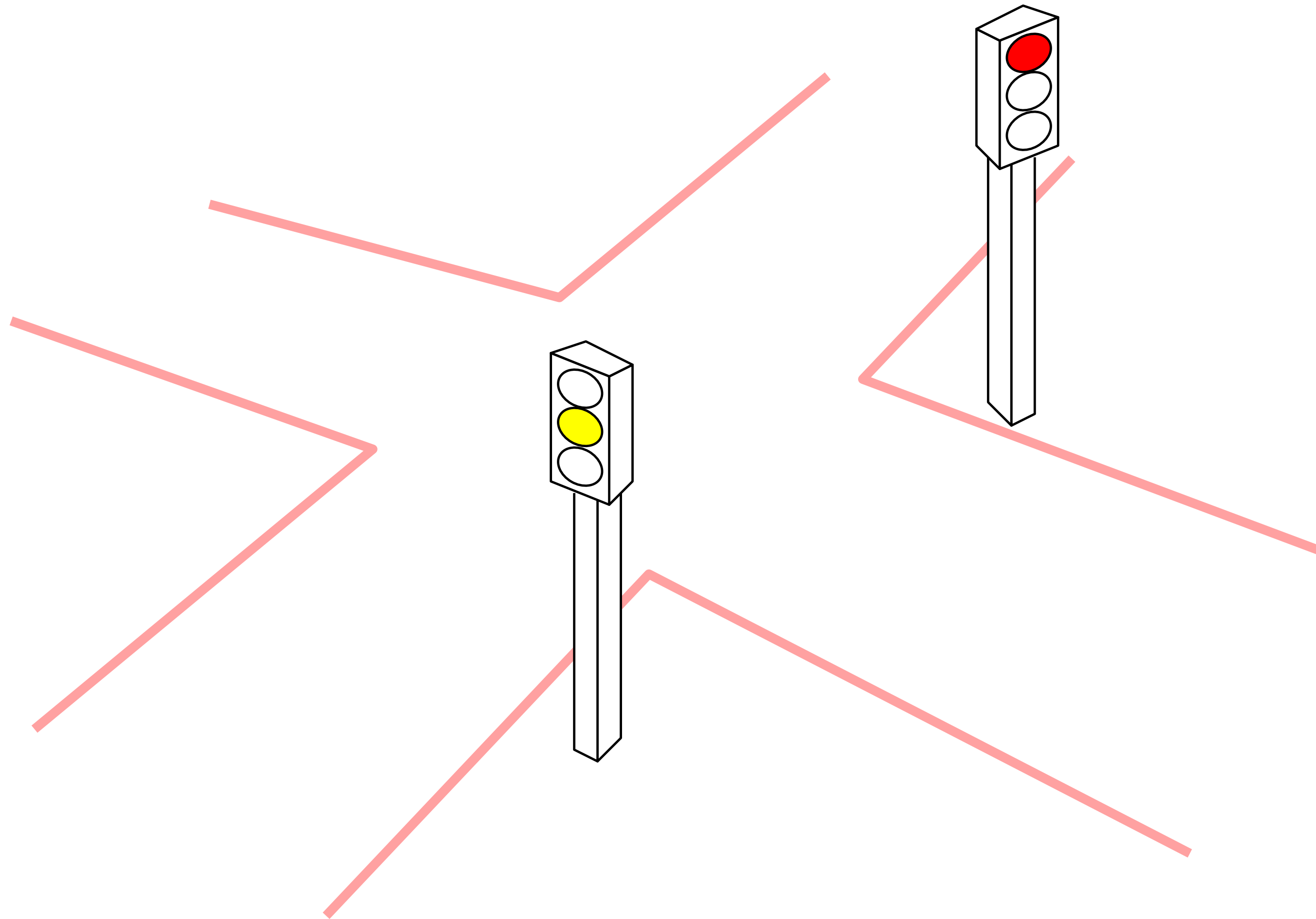
unfortunately
buggy state
is among them

```
MODULE main
VAR
  turn : { ew, sn };
  southnorth : trafficlight (enablesouthnorth);
  eastwest : trafficlight (enableeastwest);
DEFINE
  enableeastwest := southnorth.light = red & turn = ew;
  enablesouthnorth := eastwest.light = red & turn = sn;
SPEC
  AG !(southnorth.light = green & eastwest.light = green)
```

idea: disable traffic light as long the other is not red and its not the others turn



traffic lights showing red should eventually show green



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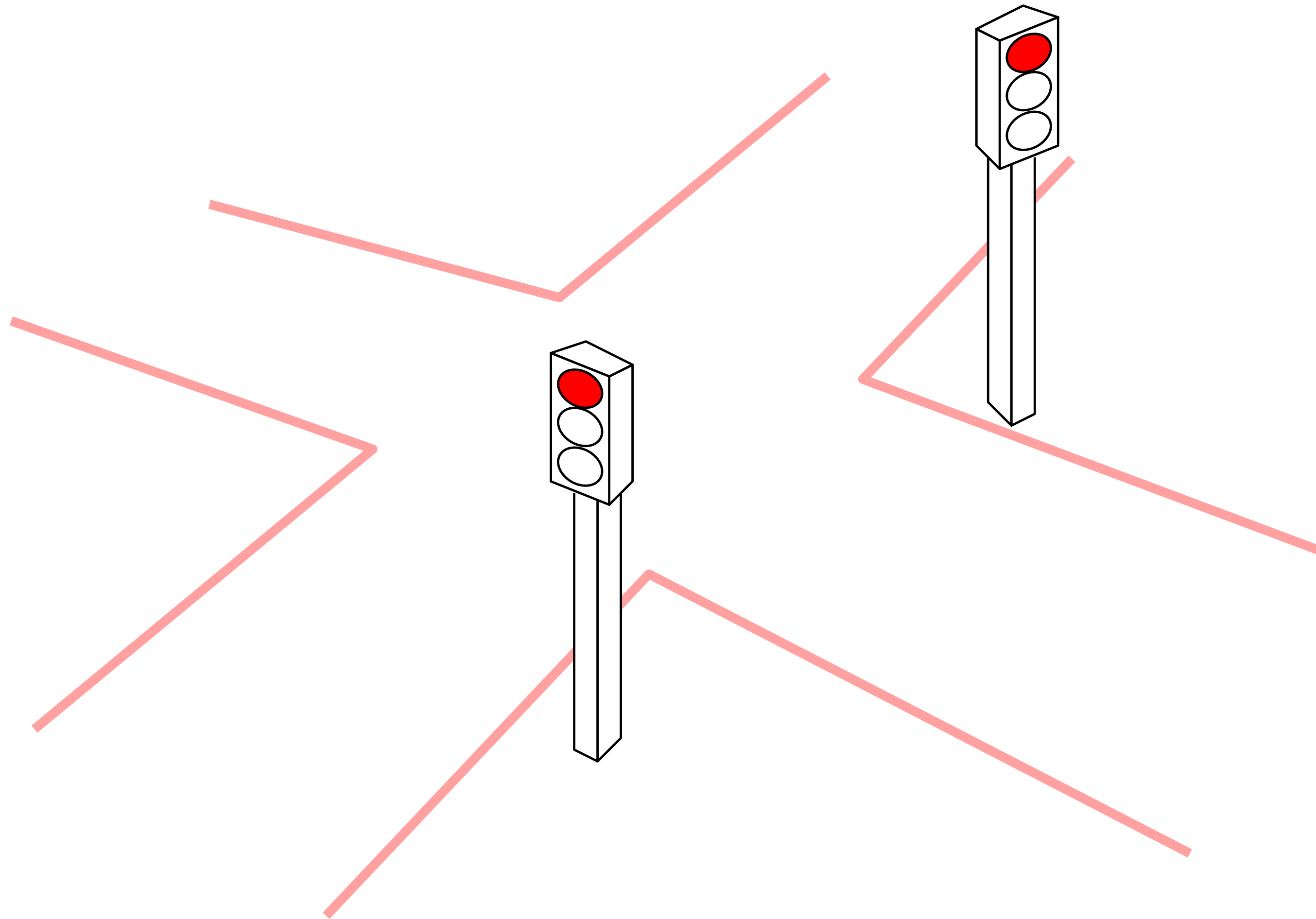

traffic lights showing red should eventually show green

Boolean Encoding

- compilation of finite model into pure propositional domain
- first step is to flatten the hierarchy
  - recursive instantiation of all submodules
  - name and parameter substitution
  - may increase program size exponentially
- second step is to encode variables with boolean variables

light		light@1	light@0
green	$\mapsto$	0	0
yellow	$\mapsto$	0	1
red	$\mapsto$	1	0

Result of Boolean Encoding

- initial state predicate I represented as boolean formula

  `!eastwest.light@0 & eastwest.light@1`

  (equivalent to `init(eastwest.light) := red`)

- transition relation T represented as boolean formula

- encoding of atomic predicates p as boolean formulae

  `!eastwest.light@1 & !eastwest.light@0`

  (equivalent to `eastwest.light != green`)