

# Model Checking WS 2015: Assignment 3

Institute for Formal Models and Verification, JKU Linz

Due 19.11.2015

## Exercise 13

Let  $L := (S, I, \Sigma, T)$  be an LTS with states  $S$ . Let  $\Psi : \mathbb{P}(S \times S) \rightarrow \mathbb{P}(S \times S)$  be the operator defined on slide 38, i.e.  $\Psi(\lesssim) := \{(r, t) \in (S \times S) \mid r \lesssim t \text{ or } \exists s \in S : [r \lesssim s \text{ and } s \lesssim t]\}$  for relation  $\lesssim \subseteq S \times S$ .

- Prove that if  $\lesssim$  is a simulation then  $\Psi(\lesssim)$  is also a simulation.
- Given a relation  $\lesssim \subseteq S \times S$ , is  $\Psi(\lesssim)$  always a transitive relation? Justify your answer.

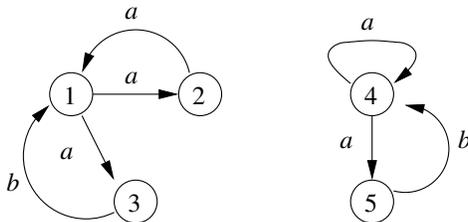
## Exercise 14

Let  $A_1$  and  $A_2$  be two LTS. Prove the theorem from slide 40: If  $A_1 \lesssim A_2$  then  $L(A_1) \subseteq L(A_2)$ .

*Hint:* let  $L := (S, I, \Sigma, T)$  be an LTS. Let  $w := a_1 a_2 \dots a_{n-1} a_n$  be a trace of  $L$  for  $s_0 \xrightarrow{a_1} s_1 \xrightarrow{a_2} \dots \xrightarrow{a_{n-1}} s_{n-1} \xrightarrow{a_n} s_n$  where  $s_0 \in I$  and length  $|w| = n$  for  $n \geq 0$ . Note that  $w$  can not only be interpreted as a sequence  $a_1 \dots a_n$  of symbols  $a_i$  in  $\Sigma$  but also as a sequence  $s_0 \dots s_n$  of states  $s_i$  in  $S$ .

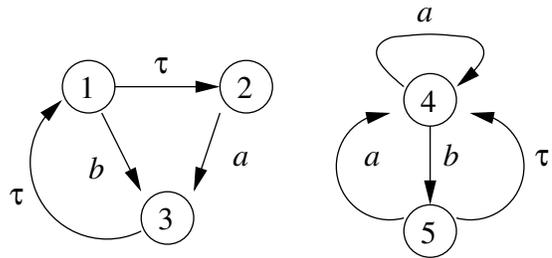
## Exercise 15

Compute the maximal simulation  $\lesssim$  over the following LTS using the fixpoint algorithm:



### Exercise 16

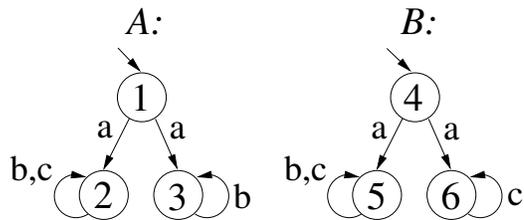
Compute the *maximal weak simulation*  $\lesssim$  over the LTS shown on the right.



### Exercise 17

Given LTS  $A$  and  $B$  as shown on the right,...

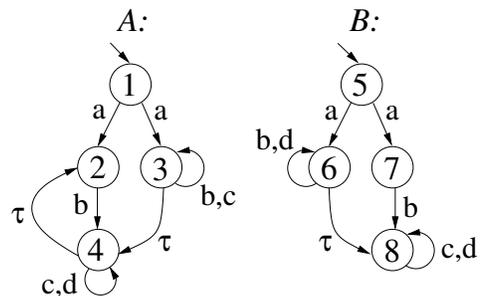
- ... compute the *maximal strong simulation*  $\lesssim$  over  $A \dot{\cup} B$ .
- ... compute the *maximal strong bisimulation*  $\approx$  over  $A \dot{\cup} B$ .
- Check whether  $1 \lesssim 4$ ,  $4 \lesssim 1$  and  $1 \approx 4$ .
- Is  $L(A) = L(B)$ ?



### Exercise 18

Given LTS  $A$  and  $B$  as shown on the right, and relation  $\approx := \{(1, 5), (2, 7), (3, 6), (4, 8), (3, 8), (4, 6)\}$ .<sup>a</sup> Assume that we want to find out whether relation  $\approx$  is a *weak bisimulation* over  $A \cup B$  by checking pairs in  $\approx$ .

<sup>a</sup>Assume this is symmetric by definition, i.e.  $(5, 1), (7, 2), \dots \in \approx$



- Does the check succeed for pair  $(3, 6)$ ? Justify your answer.
- Does the check succeed for pair  $(4, 6)$ ? Justify your answer.
- Does the check succeed for pair  $(4, 8)$ ? Justify your answer.