Model Checking WS 2012: Assignment 5

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Due 10.01.2013

Exercise 25

Let $A_1, A_2$ and $A_3$ be LTS defined as follows:

- $A_1 := (\{1, 2\}, \{1\}, \{a_1,t,s\}, (1,a_1,2), (2,t,1), (1,s,2))$.
- $A_2 := (\{1, 2, 3\}, \{1\}, \{a,b,t\}, (1,b,2), (2,a,3), (3,t,1))$.
- $A_3 := (\{1, 2\}, \{1\}, \{t,s\}, (1,s,2), (2,t,1))$.

Determine the set of local and global symbols for $A_1, A_2, A_3$.

Exercise 26

a) Given LTS $A_2$ from Exercise 25 and LTS $A_1 := (\{1, 2\}, \{1\}, \{a_1,t\}, (1,a_1,2), (2,t,1))$, draw the LTS for $A_1 || A_2$.

b) Why is the requirement $\Psi(a) \neq \emptyset$ in the definition of transitions in the asynchronous composition of multiple LTS necessary? Give a concrete example where the semantics will differ if this requirement is dropped.

Exercise 27

Let $A, B$ and $C$ be LTS defined as follows:

- $A := (\{1, 2, 3, 4\}, \{1\}, \{a,t,s\}, ((1,a,2), (2,t,3), (3,a,4), (4,s,4)))$.
- $B := (\{1, 2, 3\}, \{1\}, \{b,t,s\}, ((1,b,2), (2,t,2), (2,b,3), (3,s,1)))$.
- $C := (\{1, 2, 3\}, \{1\}, \{a,b,t,s\}, ((1,a,1), (1,b,1), (1,t,2), (2,a,2), (2,b,2), (2,s,3)))$. 
Given LTS $A, B$ and $C$ as defined above, $(A || B) \times C$ describes a model checking problem where $C$ is the “checker automaton”.

Draw the state graph $G$ for $(A || B) \times C$ without applying partial order reduction but – as usual – with on-the-fly generation of reachable states.

**Exercise 28**

Given the state graph $G$ for $(A || B) \times C$ from Exercise 27.

a) Find all traces of maximum length in $G$.

b) Which of the traces of a) are locally-equivalent? How many equivalence classes are there (see definition on slide 96)?

c) Find all states and transitions in $G$ which would be generated on-the-fly if partial order reduction was applied during the construction of the state graph for $(A || B) \times C$. Choose $A$ whenever there is a choice between locally expanding a state with respect to $A$ or $B$. Annotate states in $G$ if they are local to $A$ or $B$ or not.

**Exercise 29**

For the model checking problem given above, perform reachability analysis with on-the-fly generation of states and partial order reduction and draw the resulting LTS. If there are multiple choices for local expansion, then choose the rightmost among all components in the asynchronous composition which are ready for local expansion.
Exercise 30

Recap the basics of propositional logic in order to solve the following exercise.

a) Given boolean variables $x$ and $y$, find two different formulations of the binary XOR-operation $x \oplus y$ using only negation and binary conjunction.

b) Find a DNF representation\(^1\) for the parity function $f$ over four boolean variables:

\[
f(x_1, x_2, x_3, x_4) := x_1 \oplus x_2 \oplus x_3 \oplus x_4.
\]

Bonus Exercise

For the model checking problem given above, perform reachability analysis with on-the-fly generation of states and partial order reduction and draw the resulting LTS. If there are multiple choices for local expansion, then choose the rightmost among all components in the asynchronous composition which are ready for local expansion.

\(^1\)Note that this exercise can be solved without constructing the truth table of $f$. 