

Model Checking WS 2012: Extra Exercises

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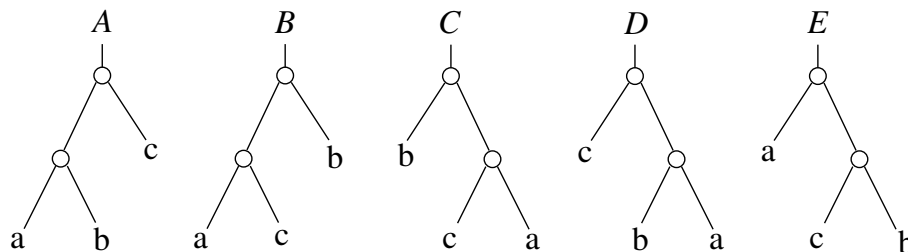
Due 24.01.2013

*These exercises are offered in order to get additional preparation for the lecture exam and they do **not** influence grading of the exercise class. Results will be discussed (partially) after the 2nd test.*

For exercises 37 and 38, assume that AIG construction with syntactic sharing, detection of commutativity and unique-tables is implemented as demonstrated in the lecture.

Exercise 37

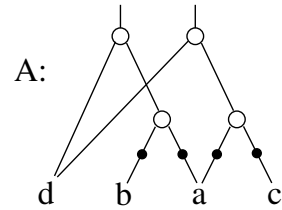
- a) Which of the following logically equivalent AIGs A , B , C , D and E , where a , b , and c are distinct AIG variable nodes, can be recognized as equivalent by syntactic sharing and detection of commutativity? Check *all* possible pairs and justify your answers.



- b) AIG t_5 is constructed bottom-up in the following four incremental steps. Draw the resulting AIG after step 4 including the effects of all previous steps:
- (a) $t_0 = \text{and_aig}(\text{var_aig}(0), \text{var_aig}(1))$, $t_1 = \text{and_aig}(\text{var_aig}(0), \text{var_aig}(2))$
 - (b) $t_2 = \text{or_aig}(t_0, t_1)$
 - (c) $t_3 = \text{and_aig}(\text{var_aig}(1), \text{var_aig}(0))$, $t_4 = \text{and_aig}(\text{var_aig}(2), \text{var_aig}(1))$
 - (d) $t_5 = \text{xor_aig}(t_3, t_4)$

Exercise 38

Given the AIG A shown on the right where a, b, c and d are distinct AIG variable nodes with variable IDs 0,1,2 and 3, respectively. That is, AIG A has been constructed and all of its nodes have been inserted into the unique-table.



a) After calling

$$\text{and_aig}(\text{or_aig}(\text{var_aig}(1), \text{var_aig}(0)), \text{or_aig}(\text{var_aig}(0), \text{var_aig}(2)))$$

for incrementally constructing an AIG for $(b \vee a) \wedge (a \vee c)$ on the given AIG A , draw the resulting AIG and determine all *reference counts*¹ and the total number of nodes.

b) The equivalence $(b \vee a) \wedge (a \vee c) \equiv a \vee (b \wedge c)$, when read from left to right, can be used as an AIG simplification rule: instead of constructing an AIG for $(b \vee a) \wedge (a \vee c)$, an AIG for $a \vee (b \wedge c)$ is constructed. Do you consider this rule as reasonable? Justify your answer.

c) Apply the simplification rule from b): after calling

$$\text{or_aig}(\text{var_aig}(0), \text{and_aig}(\text{var_aig}(1), \text{var_aig}(2)))$$

for incrementally constructing an AIG for $a \vee (b \wedge c)$ on the original AIG A shown above, draw the resulting AIG and determine all reference counts and the total number of nodes.

d) Compare and interpret the results from a) and c). Then answer b) again.

Exercise 39

Let $f(a, b, c) := (a \wedge \neg b) \vee (\neg a \wedge c)$. Draw a binary decision diagram² representing f (try to find a *minimal* one) where

a) a is the root node.

b) c is the root node.

Please consistently use the graphical notation introduced in the lecture (slide 150).

Exercise 40

a) Draw a binary decision tree with variable order³ $a > b > c$ for the boolean function $f(a, b, c)$ which returns *true* if, and only if, exactly one argument is *true*. Then draw the ROBDD F for f resulting from the decision tree by applying the reduction rules.

b) Eliminate two nodes in the ROBDD F from the first part by inserting negated edges without changing the function denoted by F .

¹The reference count of a node is how often that node is pointed to. For example, in the AIG A shown above, the two AND-nodes on top are roots of the AIG and have a reference count of one.

²See slide 150 for an example.

³Note that a is “above” b if $a > b$.