# Turbo-Charging Lemmas on Demand with Don't Care Reasoning

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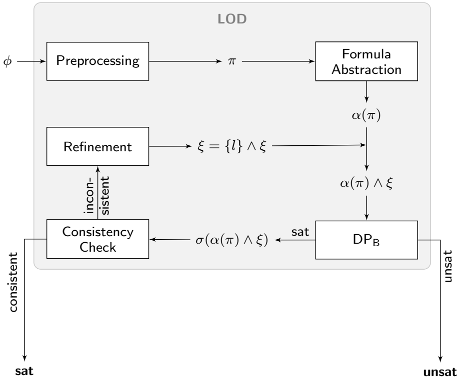
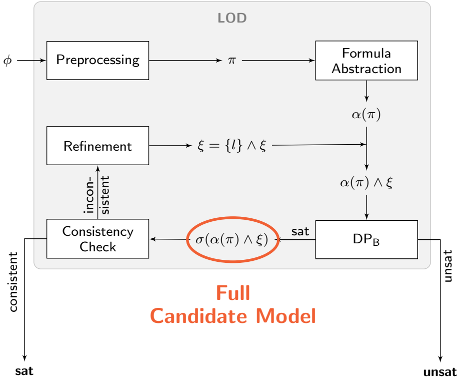
FMCAD 2014
October 21 - 24, 2014
Lausanne, Switzerland

## Lemmas on Demand

- so-called lazy SMT approach
- our SMT solver **Boolector**
  - implements Lemmas on Demand for
  - the quantifier-free theory of
    - fixed-size bit vectors
    - arrays
- **recently**: Lemmas on Demand for **Lambdas** [DIFTS'13]
  - generalization of Lemmas on Demand for Arrays [JSAT'09]
  - arrays represented as uninterpreted functions
  - array operations represented as lambda-terms
  - reads represented as function applications

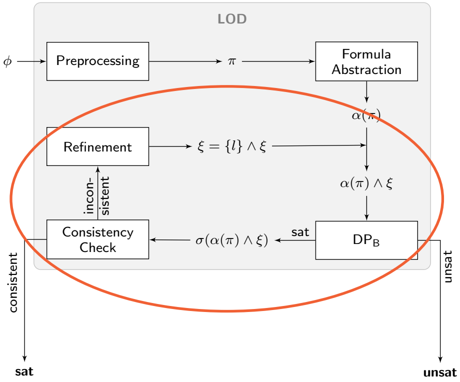



- bit vector formula abstraction (**bit vector skeleton**)
- enumeration of truth assignments (**candidate models**)
- iterative refinement with **lemmas** until convergence

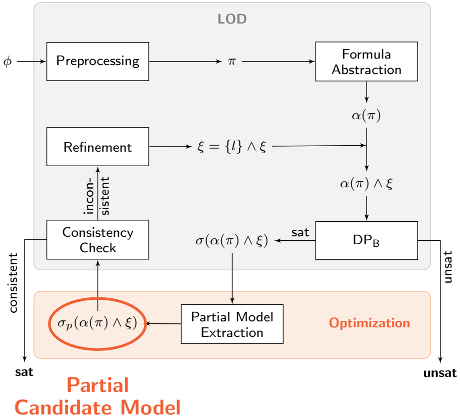



⟶ **each** candidate model is a **full** truth assignment of the formula abstraction

⟶ **full candidate model** needs to be checked for consistency w.r.t. theories



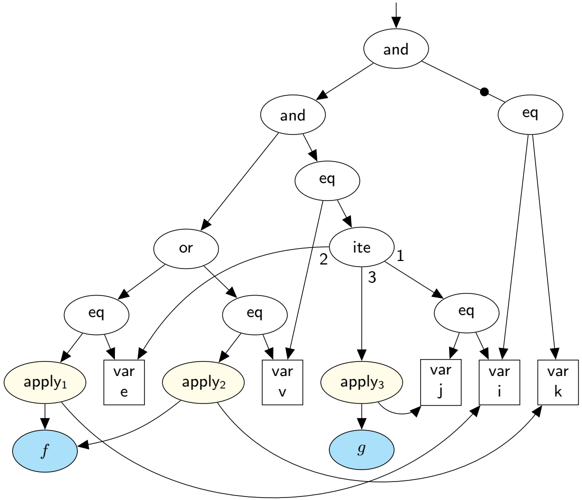

- ⟶ abstraction refinement usually the most **costly** part of LOD
- ⟶ cost generally correlates with **number of refinements**
- ⟶ checking the **full** candidate model often **not required**
- ⟶ **small subset** responsible for satisfying formula abstraction



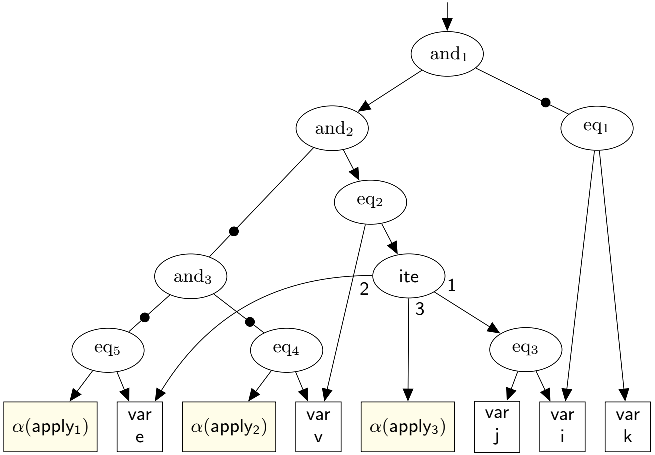

- focus LOD on the **relevant** parts of the input formula
- exploit **a posteriori observability don't cares**
- **partial model extraction** prior to consistency checking

$\longrightarrow$ subsequently **reduces the cost** for consistency checking

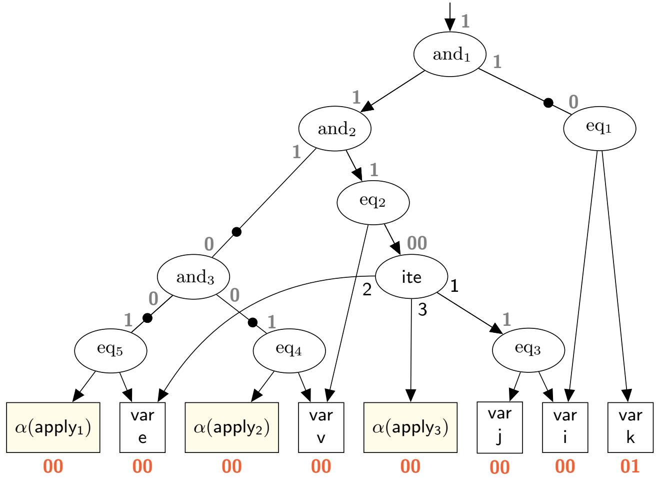
Example. $\psi_1 \equiv i \neq k \wedge (f(i) = e \vee f(k) = v) \wedge v = ite(i = j, e, g(j))$



Example. Bit Vector Skeleton

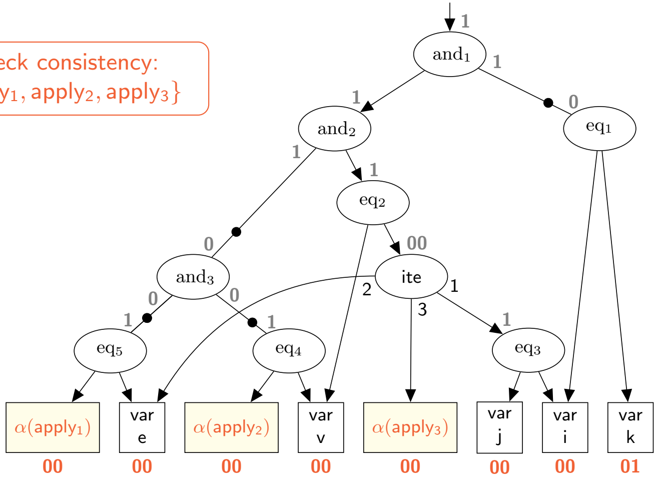


Example. **Full** Candidate Model



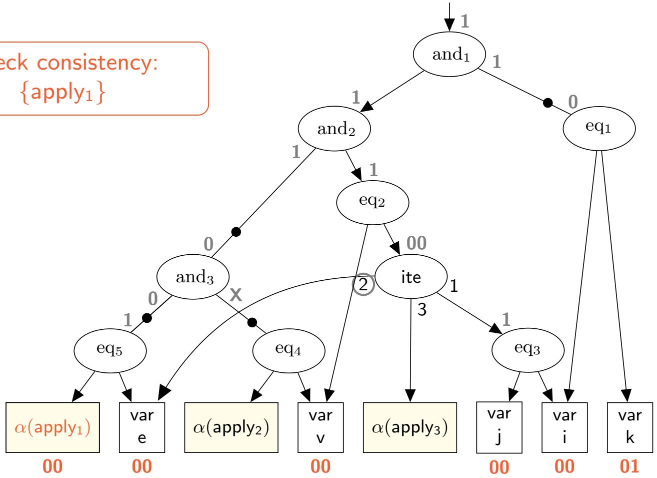
Example. **Full** Candidate Model

Check consistency:
$\{\text{apply}_1, \text{apply}_2, \text{apply}_3\}$



Example. **Partial** Candidate Model

Check consistency:
{$\text{apply}_1$}



# Partial Model Extraction

**Most intuitive:** use **justification-based** approach

⟶ Justification-based techniques in the context of

- SMT
  - prune the search space of DPLL(T) [ENTCS'05, MSRTR'07]
- Model checking
  - prune the search space of BMC [CAV'02]
  - generalize proof obligations in PDR [EénFMCAD'11, ChoFMCAD'11]
  - generalize candidate counter examples (CEGAR) [LPAR'08]

**Our approach:** **Dual propagation**-based partial model extraction

- exploiting the duality of a formula abstraction $\psi$
  $\longrightarrow$ assignments satisfying $\psi$ (the **primal** channel)
  falsify its negation $\neg\psi$ (the **dual** channel)
- motivated by dual propagation techniques in QBF [AAAI'10]
  - one solver with two channels (**online** approach)
  - symmetric propagation between primal and dual channel
- **here:** **offline** dual propagation
  - two solvers, one solver per channel
  - consecutive propagation between primal and dual channel
    $\longrightarrow$ primal generates full assignment before dual enables partial model extraction based on the primal assignment

Example. **Boolean** Level

**Primal** channel: $\psi_2 \equiv (a \wedge b) \vee (c \wedge d)$
**Dual** channel: $\neg\psi_2 \equiv (\neg a \vee \neg b) \wedge (\neg c \vee \neg d)$

Example. **Boolean** Level

**Primal** channel: $\psi_2 \equiv (a \wedge b) \vee (c \wedge d)$
**Dual** channel: $\neg\psi_2 \equiv (\neg a \vee \neg b) \wedge (\neg c \vee \neg d)$

Primal assignment: $\sigma(\psi_2) \equiv \{\sigma(a) = \top, \sigma(b) = \top, \sigma(c) = \top, \sigma(d) = \top\}$

Example. **Boolean** Level

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Fix values of inputs via **assumptions** to the dual solver:
Dual assumptions: $\{a{=}\top, b{=}\top, c{=}\top, d{=}\top\}$

Example. **Boolean** Level

**Primal** channel: $\psi_2 \equiv (a \land b) \lor (c \land d)$
**Dual** channel: $\neg\psi_2 \equiv (\neg a \lor \neg b) \land (\neg c \lor \neg d)$

Primal assignment: $\sigma(\psi_2) \equiv \{\sigma(a) = \top,\ \sigma(b) = \top,\ \sigma(c) = \top,\ \sigma(d) = \top\}$

Fix values of inputs via **assumptions** to the dual solver:
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**Failed** assumptions: $\{a{=}\top,\ b{=}\top\}$

$\longrightarrow$ sufficient to falsify $\neg\psi_2$
$\longrightarrow$ sufficient to satisfy $\psi_2$

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Fix values of inputs via **assumptions** to the dual solver:
Dual assumptions: $\{a{=}\top, b{=}\top, c{=}\top, d{=}\top\}$

**Failed** assumptions: $\{a{=}\top,\ b{=}\top\}$ **Partial Model**

$\longrightarrow$ sufficient to falsify $\neg\psi_2$
$\longrightarrow$ sufficient to satisfy $\psi_2$

$\longrightarrow$ **structural don't care reasoning** simulated via the dual solver

$\longrightarrow$ no structural SAT solver necessary

Example. (ctd)

| | | | | |
|---|---|---|---|---|
| Input formula: | $\psi_2$ | $\equiv$ | $(a \wedge b) \vee (c \wedge d)$ | $\equiv \top$ |
| Primal SAT solver: | CNF($\psi_2$) | $\equiv$ | $(\neg o \vee x \vee y) \wedge (\neg x \vee o) \wedge$ $(\neg y \vee o) \wedge (\neg x \vee a) \wedge$ $(\neg x \vee b) \wedge (\neg a \vee \neg b \vee x) \wedge$ $(\neg y \vee c) \wedge (\neg y \vee d) \wedge$ $(\neg c \vee \neg d \vee y)$ | $\equiv$ ? |
| Dual SAT solver: | CNF($\neg\psi_2$) | $\equiv$ | $(\neg a \vee \neg b) \wedge (\neg c \vee \neg d)$ | $\equiv \bot$ |

Dual assumptions: $\{a{=}\top, b{=}\top, c{=}\top, d{=}\top\}$

Partial Model: $\{a = \top, b = \top\}$

$\longrightarrow$ in contrast to partial model extraction techniques based on iterative removal of unnecessary assignments on the CNF level [FMCAD'13]

$\longrightarrow$ we lift this approach to the **word** level

**Primal** channel: $\Gamma \equiv \alpha(\pi) \wedge \xi \equiv \alpha(\pi) \wedge l_1 \wedge ... \wedge l_{i-1}$

**Dual** channel: $\neg\Gamma$

$\longrightarrow$ one SMT solver per channel

$\longrightarrow$ one single dual solver instance to maintain $\neg\Gamma$ over all iterations

Example. **Word** Level

$$\psi_1 \equiv i \neq k \wedge (f(i) = e \vee f(k) = v) \wedge v = ite(i = j,\, e,\, g(j))$$

$$\alpha(\psi_1) \equiv i \neq k \wedge (\alpha(\mathsf{apply}_1) = e \vee \alpha(\mathsf{apply}_2) = v) \wedge v = ite(i = j,\, e,\, \alpha(\mathsf{apply}_3))$$

**Primal** solver: $\alpha(\psi_1)$
**Dual** solver: $\neg\alpha(\psi_1)$
} Formula abstraction and its negation

Primal assignment:

$$\sigma(\psi_2) \equiv \{\sigma(i) = 00,\, \sigma(j) = 00,\, \sigma(e) = 00,\, \sigma(v) = 00,\, \sigma(k) = 01,\, \alpha(\mathsf{apply}_1) = 00,\, \alpha(\mathsf{apply}_2) = 00,\, \alpha(\mathsf{apply}_3) = 00\}$$

Fix values of inputs via assumptions to the dual solver:

Dual assumptions:

$$\sigma(\psi_2) \equiv \{i = 00,\, j = 00,\, e = 00,\, v = 00,\, k = 01,\, \alpha(\mathsf{apply}_1) = 00,\, \alpha(\mathsf{apply}_2) = 00,\, \alpha(\mathsf{apply}_3) = 00\}$$

**Failed** assumptions:

$$\{i = 00,\, j = 00,\, e = 00,\, v = 00,\, k = 01,\, \alpha(\mathsf{apply}_1) = 00\}$$

Example. **Word** Level

$$\psi_1 \equiv i \neq k \wedge (f(i) = e \vee f(k) = v) \wedge v = ite(i = j,\, e,\, g(j))$$
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**Primal** solver: $\alpha(\psi_1)$
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Fix values of inputs via assumptions to the dual solver:

Dual assumptions:

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**Failed** assumptions:

$$\{i = 00,\, j = 00,\, e = 00,\, v = 00,\, k = 01,\, \alpha(\mathsf{apply}_1) = 00\}$$

**Partial Model**

Example. **Word** Level

$$\psi_1 \equiv i \neq k \wedge (f(i) = e \vee f(k) = v) \wedge v = ite(i = j,\, e,\, g(j))$$
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Fix values of inputs via assumptions to the dual solver:

Dual assumptions:

$$\sigma(\psi_2) \equiv \{i = 00,\, j = 00,\, e = 00,\, v = 00,\, k = 01,\, \alpha(\mathsf{apply}_1) = 00,\, \alpha(\mathsf{apply}_2) = 00,\, \alpha(\mathsf{apply}_3) = 00\}$$

**Failed** assumptions:

**Consistency Check**

$$\{i = 00,\, j = 00,\, e = 00,\, v = 00,\, k = 01,\, \alpha(\mathsf{apply}_1) = 00\}$$

**Four Configurations:**

- **Boolector$_{\textbf{sc}}$**

  ⟶ version entering SMTCOMP'12, winner of the QF_AUFBV track

- **Boolector$_{\textbf{ba}}$**

  ⟶ current Boolector base version (new LOD for Lambdas engine)

- **Boolector$_{\textbf{dp}}$**

  ⟶ with dual propagation-based partial model extraction enabled

- **Boolector$_{\textbf{ju}}$**

  ⟶ **justification**-based partial model extraction approach for comparison

  - determine **a posteriori** observability don't cares

    ⟶ skip lines that do not influence the output of an **and**-gate under its current assignment

  - if both inputs of an **and**-gate are **controlling** ($\perp$)

    ⟶ skip **either** one based on a minimum cost **heuristic**

**Two Benchmark Sets:**

- SMT'12: **149** benchmarks
  all **non-extensional** QF_AUFBV benchmarks in SMTCOMP'12
- Selected: **173** benchmarks
  all **non-extensional** QF_AUFBV benchmarks (13696) in the SMT-LIB (pre-SMTCOMP'14) for which Boolector$_{sc}$ required at least **10 seconds**

⟶ 58 benchmarks shared between both sets

⟶ all experiments on 2.83 GHz Intel Core 2 Quad machines with 8GB RAM running Ubuntu 12.04

⟶ **time limit**: 1200 seconds, **memory limit**: 7GB

**Overall** results on sets SMT'12 and Selected.

| | Solver | Solved (sat/unsat) | TO | MO | Time [s] | DS [s] |
|---|---|---|---|---|---|---|
| *SMT'12* | $\text{Boolector}_{sc}$ | 140 (83/57) | 9 | 0 | 15882 | - |
| | $\text{Boolector}_{ba}$ | 141 (83/58) | 8 | 0 | 19312 | - |
| | $\text{Boolector}_{ju}$ | 142 (84/58) | 7 | 0 | 15709 | - |
| | $\text{Boolector}_{dp}$ | 142 (84/58) | 7 | 0 | 20992 | 5045 |
| *Selected* | $\text{Boolector}_{sc}$ | 116 (72/44) | 50 | 7 | 85863 | - |
| | $\text{Boolector}_{ba}$ | 121 (76/45) | 45 | 7 | 76104 | - |
| | $\text{Boolector}_{ju}$ | 130 (85/45) | 36 | 7 | 63202 | - |
| | $\text{Boolector}_{dp}$ | 130 (85/45) | 36 | 7 | 66991 | 4705 |

TO ... time out
MO ... memory out
Time ... total CPU time
DS ... dual solver overhead

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TO ... time out
MO ... memory out
Time ... total CPU time
DS ... dual solver overhead

- SMT'12: 1 additional instance (sat)
- Selected: 9 additional instances (all sat)

Results for **commonly solved** instances on sets SMT'12 and Selected.

| | Solver | Time [s] | | | SAT [s] | | | DS overhead [s] | | | LOD | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Total | Avg. | Med. | Total | Avg. | Med. | Total | Avg. | Med. | Total | Avg. | Med. |
| *SMT'12* | Boolector$_{sc}$ | 4129 | 29 | 2 | 3662 | 26 | 0 | - | - | - | 30741 | 221 | 0 |
| | Boolector$_{ba}$ | 8564 | 61 | 6 | 7262 | 52 | 1 | - | - | - | 33013 | 237 | 0 |
| | Boolector$_{ju}$ | 6362 | 45 | 4 | 5226 | 37 | 0 | - | - | - | 23660 | 170 | 0 |
| | Boolector$_{dp}$ | 10145 | 72 | 5 | 4700 | 33 | 0 | 4109 | 29 | 0 | 33492 | 240 | 0 |
| *Selected* | Boolector$_{sc}$ | 15037 | 133 | 35 | 12836 | 113 | 34 | - | - | - | 104646 | 926 | 175 |
| | Boolector$_{ba}$ | 10001 | 88 | 35 | 8330 | 73 | 22 | - | - | - | 31752 | 280 | 88 |
| | Boolector$_{ju}$ | 8182 | 72 | 29 | 6639 | 58 | 19 | - | - | - | 28215 | 249 | 28 |
| | Boolector$_{dp}$ | 10838 | 95 | 30 | 6164 | 54 | 15 | 3036 | 26 | 0 | 24866 | 220 | 29 |

Time ... total CPU time
DS overhead ... dual solver overhead
SAT ... SAT solver runtime (primal solver)
LOD ... number of lemmas generated

- SMT'12: 139 (out of 149) benchmarks, 82 sat, 57 unsat

  $\longrightarrow$ not representative:
  $\sim$50% solved without a single refinement iteration

- Selected: 113 (out of 173) benchmarks, 70 sat, 43 unsat

Results for **commonly solved** instances on sets SMT'12 and Selected.

| | Solver | Time [s] | | | SAT [s] | | | DS overhead [s] | | | LOD | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Total | Avg. | Med. | Total | Avg. | Med. | Total | Avg. | Med. | Total | Avg. | Med. |
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- $\text{Boolector}_{sc}$ implements old LOD engine

  ⟶ new engine ($\text{Boolector}_{ba}$) struggles on a small set of benchmarks

  ⟶ needs further investigation

Results for **commonly solved** instances on sets SMT'12 and Selected.

| | Solver | Time [s] | | | SAT [s] | | | DS overhead [s] | | | LOD | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
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- sat solver runtime (**SAT**)
  $\longrightarrow$ **$\text{Boolector}_{dp}$** most notable improvement on both sets

Results for **commonly solved** instances on sets SMT'12 and Selected.

| | Solver | Time [s] | | | SAT [s] | | | DS overhead [s] | | | LOD | | |
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Time ... total CPU time
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- number of lemmas generated (**LOD**)
  - SMT'12:
    - **$\text{Boolector}_{ju}$** least number of lemmas
    - **$\text{Boolector}_{dp}$** and **$\text{Boolector}_{ba}$** approx. the same
      $\longrightarrow$ on 14 instances 1.5-2.6 x more lemmas than $\text{Boolector}_{ba}$
  - Selected: **$\text{Boolector}_{dp}$** most notable improvement

Results for **commonly solved** instances on sets SMT'12 and Selected.

| | Solver | Time [s] | | | SAT [s] | | | DS overhead [s] | | | LOD | | |
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Time … total CPU time
DS overhead … dual solver overhead
SAT … SAT solver runtime (primal solver)
LOD … number of lemmas generated

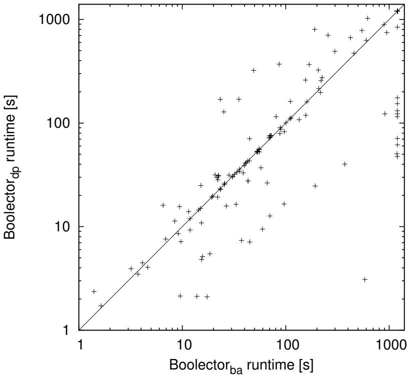
- dual solver overhead ∼30-40% in total
  - on ≤10% of the benchmarks 50-70% of the total runtime
  - on >50% of the benchmarks <10% of the total runtime

⟶ Boolector$_{dp}$ outperforms others disregarding DS overhead

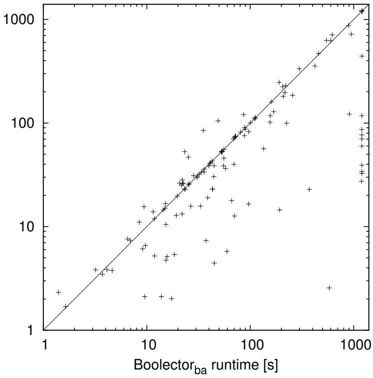
⟶ **online** dual propagation approach: DS overhead negligible

# Experimental Evaluation

## Boolector$_{dp}$ vs Boolector$_{ba}$




DS overhead included




DS overhead **not** included

# Conclusion

⟶ **dual propagation-based** optimization for Lemmas on Demand

- don't care reasoning on full candidate models improves performance
- our offline dual propagation-based approach competitive (in spite of introducing considerable overhead)

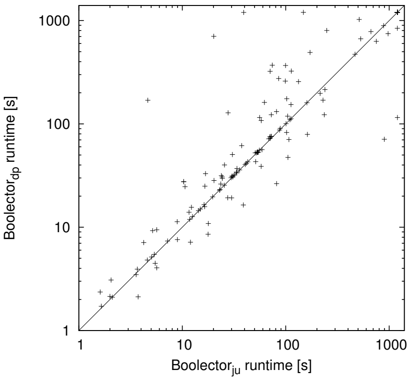
  ⟶ **Boolector$_{\textbf{ju}}$** won QF_ABV track of SMTCOMP'14
  ⟶ **Boolector$_{\textbf{dp}}$** came in close second

**Future work:** **online** dual propagation approach, promises

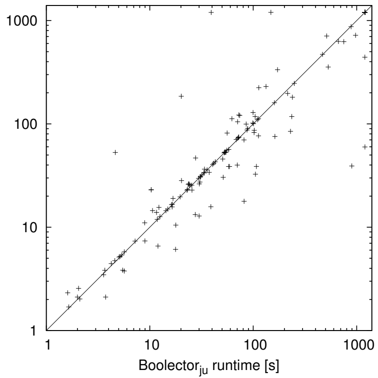
- negligible or no dual solver overhead
- further improvment of overall performance by enabling partial model extraction even before a full candidate model has been generated
- requires interleaved execution between primal and dual solver

# Appendix

Boolector$_{dp}$ vs Boolector$_{ju}$



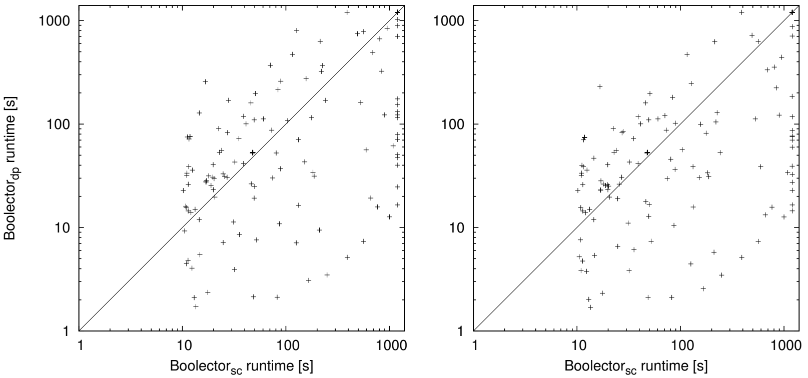

DS overhead included




DS overhead **not** included

# Appendix

## $Boolector_{dp}$ vs $Boolector_{sc}$




DS overhead included

DS overhead **not** included

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