Better Lemmas with Lambda Extraction

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Introduction

Better Lemmas?

. . . in the context of lemmas on demand for the theory of arrays

- more succinct
- stronger
- reduce number of lemmas $\longrightarrow$ speeds up solving

How?

1. identify array patterns in sequences of array operations
2. generalize them as lambda terms
3. to create better lemmas on demand

$\longrightarrow$ considerably improves solver performance
$\longrightarrow$ particularly on instances from symbolic execution

Introduction

Theory of Arrays [McCarthy'62]

- introduces two function symbols to access/modify arrays
  - $read(a, i)$ read value from array a on index i
  - $write(a, i, e)$ write value e to array a at index i
- reason about memory in SW and HW verification

Limitations

- operate on single indices only
- no succinct operations over multiple indices
  e.g. *memset* or *memcpy* operations
- not possible to reason about variable number of indices
  (without quantifiers)

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UCLID [CAV'02]

- restricted lambda terms to tackle limitations
- eager elimination of lambda terms
- might result in exponential blow-up in formula size

Boolector [DIFTS'13]

- decision procedure for lambda terms
- lazy handling of lambda terms
- avoid worst-case exponential blow-up
- array engine in Boolector
  $\longrightarrow$ treats arrays and array operations as functions

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Arrays as Lambdas

Representation

Array Variable	Uninterpreted Function
a	f_a
Read Operation	Function Application
$read(a, i)$	$f_a(i)$
Write Operation	Lambda Term
$write(a, i, e)$	$\lambda x \,.\, ite(x = i, e, f_a(x))$
Memset Operation	Lambda Term
$memset(a, i, n, e)$	$\lambda x \,.\, ite(i \leq x < i + n, e, f_a(x))$

Motivation

Example

Set 4 consecutive indices of array a to value e starting from index i.

Array representation

$a_1 := write(a, i, e)$

$a_2 := write(a_1, i + 1, e)$

$a_3 := write(a_2, i + 2, e)$

$a_4 := write(a_3, i + 3, e)$

$\longrightarrow$ requires $n = 4$ writes

$\longrightarrow$ n arbitrarily big

Lambda term representation

$\lambda_4 := \lambda x \,.\, ite(i \leq x \wedge x < i + 4, e, f_a(x))$

$\longrightarrow$ more compact representation

$\longrightarrow$ symbolic size n

$\longrightarrow$ better lemmas

Our goal: Identify array patterns and represent them as lambda terms

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Lambda Extraction

memset Pattern

```
(set-logic QF_ABV)
(declare-fun a () (Array (_ BitVec 8) (_ BitVec 32)))
(declare-fun e () (_ BitVec 32))
...
(assert
 (= a_init
    (store
     (store
      (store
       (store a (_ bv0 8) e)
                (_ bv1 8) e)
                 (_ bv2 8) e)
                  (_ bv3 8) e)))
...
(exit)
```

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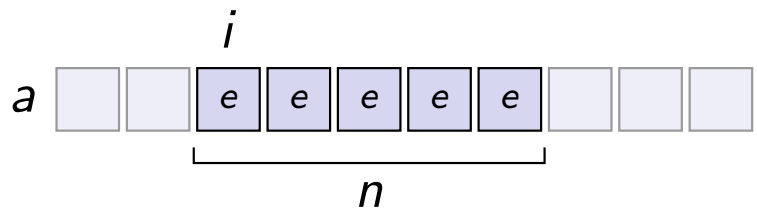
...
(exit)
```

Lambda Extraction

memset Pattern

$memset(a, i, n, e)$

- a ... base array
- i ... start address
- n ... size (constant)
- e ... value



Lambda Term

$$\lambda_{mset} := \lambda x \,.\, ite(i \leq x < i + n, e, f_a(x))$$

Lambda Extraction

Loop Initialization Pattern: $i \rightarrow e$

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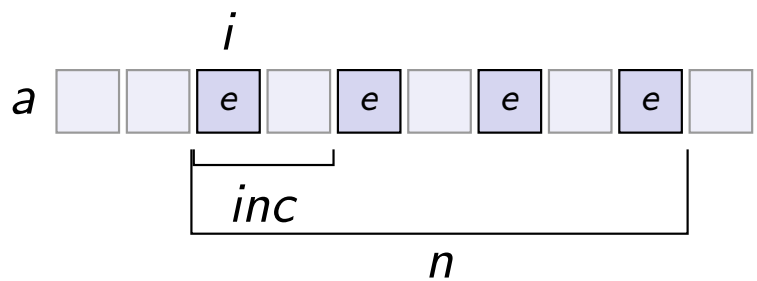
Lambda Extraction

Loop Initialization Pattern: $\mathbf{i \rightarrow e}$

$$for\ (j = i;\ j < i + n;\ j = j + inc)\ \{a[j] = e;\}$$

a	...	base array
i	...	start address
n	...	size (constant)
inc	...	increment (constant)
e	...	value



Lambda Term

$$\lambda_{i \rightarrow e} := \lambda x\,.\,ite(i \leq x \wedge x < i + n\ \wedge (inc \mid (x - i)), e, f_a(x))$$

Lambda Extraction

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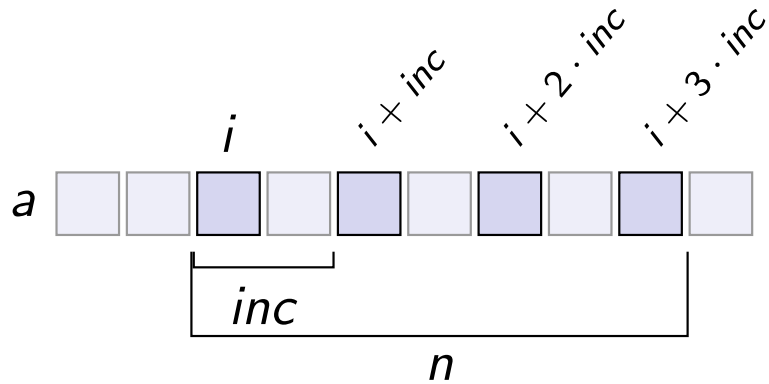

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$$\lambda_{i \rightarrow i} := \lambda x\,.\,ite(i \leq x \wedge x < i + n\ \wedge (inc \mid (x - i)), x,\ f_a(x))$$

Variation: $i \rightarrow i + 1$

$$for\ (j = i;\ j < i + n;\ j = j + inc)\ \{a[j] = j + 1;\}$$

Lambda Extraction

Loop Initialization Pattern: $\mathbf{i \to i}$

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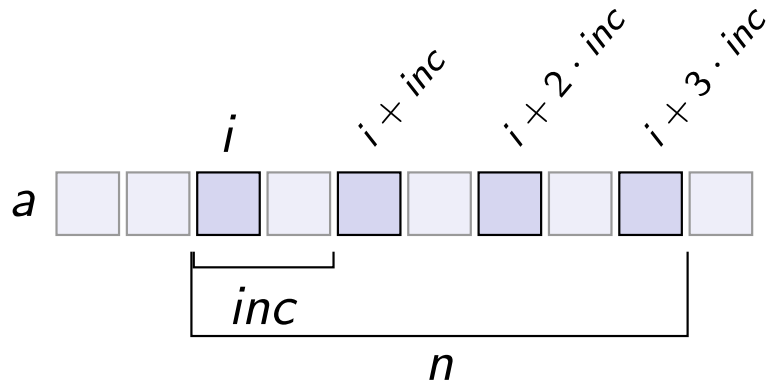

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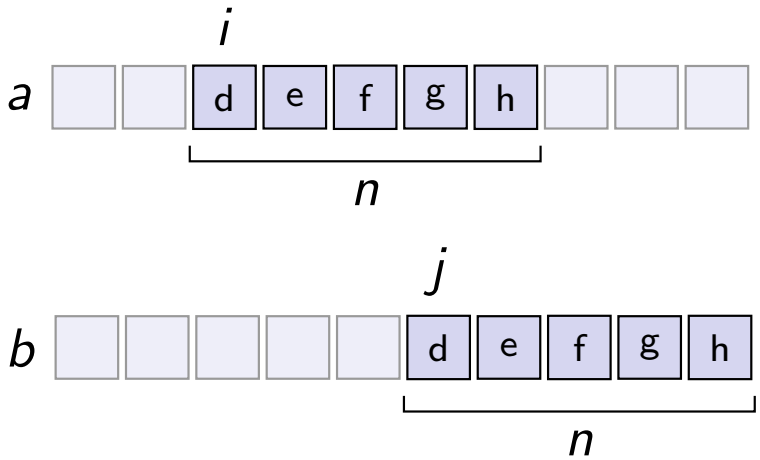
Lambda Extraction

memcpy Pattern

$memcpy(a, b, i, j, n)$

- a ... source array
- b ... destination array
- i ... source address
- j ... destination address
- n ... size (constant)



Lambda Term

$$\lambda_{mcpy} := \lambda x \,.\, ite(j \leq x < j + n, f_a(i + x - j), f_b(x))$$

Lambda Extraction

Better Lemma Generation

Write sequence

$a_1 := write(a, 5, e)$
$a_2 := write(a_1, 6, e)$
$a_3 := write(a_2, 7, e)$

Conflict

$j = 7 \wedge read(a_3, j) \neq e$
$j = 6 \wedge read(a_3, j) \neq e$
$j = 5 \wedge read(a_3, j) \neq e$

Lemmas

$j = 7 \rightarrow read(a_3, j) = e$
$j = 6 \rightarrow read(a_3, j) = e$
$j = 5 \rightarrow read(a_3, j) = e$

$\longrightarrow$ n=3 lemmas in worst-case
$\longrightarrow$ covers single indices

Lambda term

$\lambda_3 := \lambda x \,.\, ite(5 \leq x \wedge x < 8, e, f_a(x))$

Conflict

$j = 7 \wedge \lambda_3(j) \neq e$

Lemma

$5 \leq j \wedge j < 8 \rightarrow \lambda_3(j) = e$

$\longrightarrow$ only one lemma generated
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Lambda Merging

Workflow

Lambda sequence

$\lambda_1 := \lambda z \,.\, ite(z = i_1, e, f_a(z))$

$\lambda_2 := \lambda y \,.\, ite(y = i_2, e, \lambda_1(y))$

$\lambda_3 := \lambda x \,.\, ite(x = i_3, e, \lambda_2(x))$

$\longrightarrow$ i_1, i_2, i_3 arbitrary

Merge Lambdas $\lambda_1, \lambda_2, \lambda_3$

Simplification

$\lambda_4 := \lambda x \,.\, ite(x = i_3 \vee x = i_2 \vee x = i_1, e, f_a(x))$

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Lambda Merging

Better Lemma Generation

Lambda term

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Conflict

$i_1 = j \wedge \lambda_4(j) \neq e$

Lemma

$j = i_3 \vee j = i_2 \vee j = i_1 \rightarrow \lambda_4(j) = e$

$\longrightarrow$ covers all indices in one disjunction (one lemma generated)

- orthogonal
- not as compact as lambda extraction
- still generates better lemmas

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Experiments

Setup

Configurations

- Boolector$_{\mathbf{Base}}$
- Boolector$_{\mathbf{E}}$
- Boolector$_{\mathbf{M}}$
- Boolector$_{\mathbf{X}}$
- Boolector$_{\mathbf{XM}}$
- Boolector$_{\mathbf{XME}}$

E ... lambda elimination enabled
M ... lambda merging enabled
X ... lambda extraction enabled

Benchmarks

- all non-extensional benchmarks from QF_ABV of SMT-LIB (13317 in total)

Limits

- 1200s time limit
- 7GB memory limit
- 1200s penalty if limit reached

Experiments performed on

- 2.83GHz Intel Core 2 Quad machines with 8GB RAM

Experiments

Overview

Solver	Solved	TO	MO	Time [s]
Boolector$_{\mathbf{Base}}$	13242	68	7	122645
Boolector$_{\mathbf{E}}$	13242	49	26	120659
Boolector$_{\mathbf{XME}}$	13246	47	24	111114
Boolector$_{\mathbf{X}}$	13256	54	7	99834
Boolector$_{\mathbf{M}}$	13259	50	8	105647
Boolector$_{\mathbf{XM}}$	**13263**	46	8	**84760**

TO ... time out MO ... memory out Time ... CPU time

Experiments

Benchmark Family Overview

	Boolector$_{Base}$		Boolector$_{XM}$		Extracted Patterns					Merged
Family	Slvd	[s]	Slvd	[s]	λ_{mset}	λ_{mcpy}	$\lambda_{i\to e}$	$\lambda_{i\to i}$	$\lambda_{i\to i+1}$	
bench (119)	119	2	119	0.3	208	0	34	0	0	1118
bmc (39)	38	1361	**39**	**182**	256	3	56	0	0	6010
brubiere (98)	75	29455	75	28854	0	10	0	0	0	75821
brubiere2 (22)	17	7299	**20**	**3241**	1392	0	8	0	0	4194
brubiere3 (8)	0	9600	1	8435	0	0	0	0	0	19966
btfnt (1)	1	134	1	134	0	0	0	0	0	0
calc2 (36)	36	862	36	863	0	0	0	0	0	0
dwp (4188)	4187	2668	4187	2089	42	0	0	0	0	26068
ecc (55)	54	1792	54	1845	125	0	0	0	0	0
egt (7719)	7719	222	7719	212	3893	0	0	0	0	7257
jager (2)	0	2400	0	2400	14028	0	239	0	0	153721
klee (622)	622	12942	**622**	**154**	9373	0	10049	0	0	33406
pipe (1)	1	10	1	10	0	0	0	0	0	0
platania (275)	247	42690	**258**	**31189**	0	0	0	58	120	9039
sharing (40)	40	2460	40	2458	0	0	0	0	0	0
stp (40)	34	8749	**39**	**2695**	60	0	297	0	0	498472
stp_sa (52)	52	0.7	52	0.7	0	0	0	0	0	0
totals (13317)	13242	122645	**13263**	**84760**	29377	13	10683	58	120	835072

Total extraction time: 41s

Total merge time: 24s

Experiments

Benchmark Family Overview

	Boolector$_{Base}$		Boolector$_{XM}$		Extracted Patterns					Merged
Family	Slvd	[s]	Slvd	[s]	λ_{mset}	λ_{mcpy}	$\lambda_{i\to e}$	$\lambda_{i\to i}$	$\lambda_{i\to i+1}$	
bench (119)	119	2	119	0.3	208	0	34	0	0	1118
bmc (39)	38	1361	**39**	**182**	256	3	56	0	0	6010
brubiere (98)	75	29455	75	28854	0	10	0	0	0	75821
brubiere2 (22)	17	7299	**20**	**3241**	1392	0	8	0	0	4194
brubiere3 (8)	0	9600	1	8435	0	0	0	0	0	19966
btfnt (1)	1	134	1	134	0	0	0	0	0	0
calc2 (36)	36	862	36	863	0	0	0	0	0	0
dwp (4188)	4187	2668	4187	2089	42	0	0	0	0	26068
ecc (55)	54	1792	54	1845	125	0	0	0	0	0
egt (7719)	7719	222	7719	212	3893	0	0	0	0	7257
jager (2)	0	2400	0	2400	14028	0	239	0	0	153721
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Experiments

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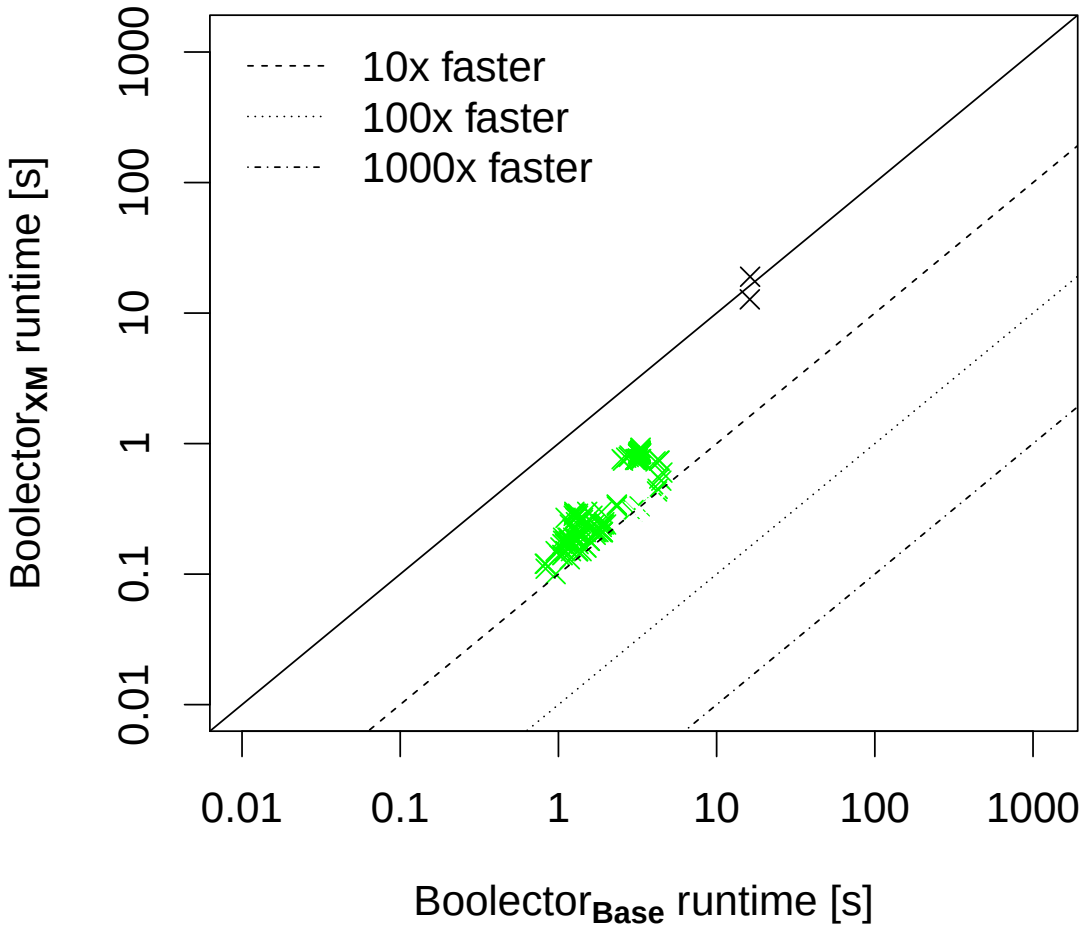
stp (40)	34	8749	**39**	**2695**	60	0	297	0	0	498472
stp_sa (52)	52	0.7	52	0.7	0	0	0	0	0	0
totals (13317)	13242	122645	**13263**	**84760**	29377	13	10683	58	120	835072

Total extraction time: 41s

Total merge time: 24s

Experiments

Scatter Plot klee Benchmarks (symbolic execution)

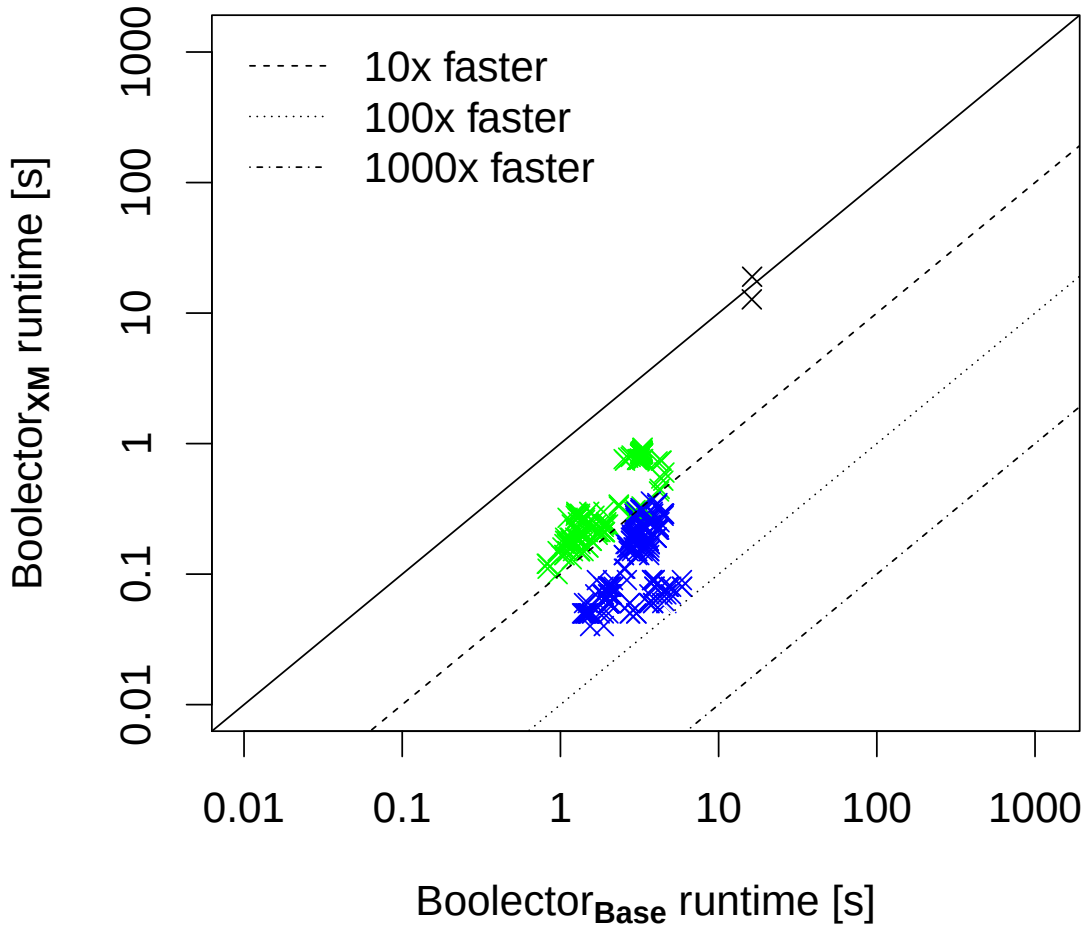


622 benchmarks

- 155 instances
  2-10x faster
- 201 instances
  10-100x faster
- 264 instances
  100-580x faster

Experiments

Scatter Plot klee Benchmarks (symbolic execution)



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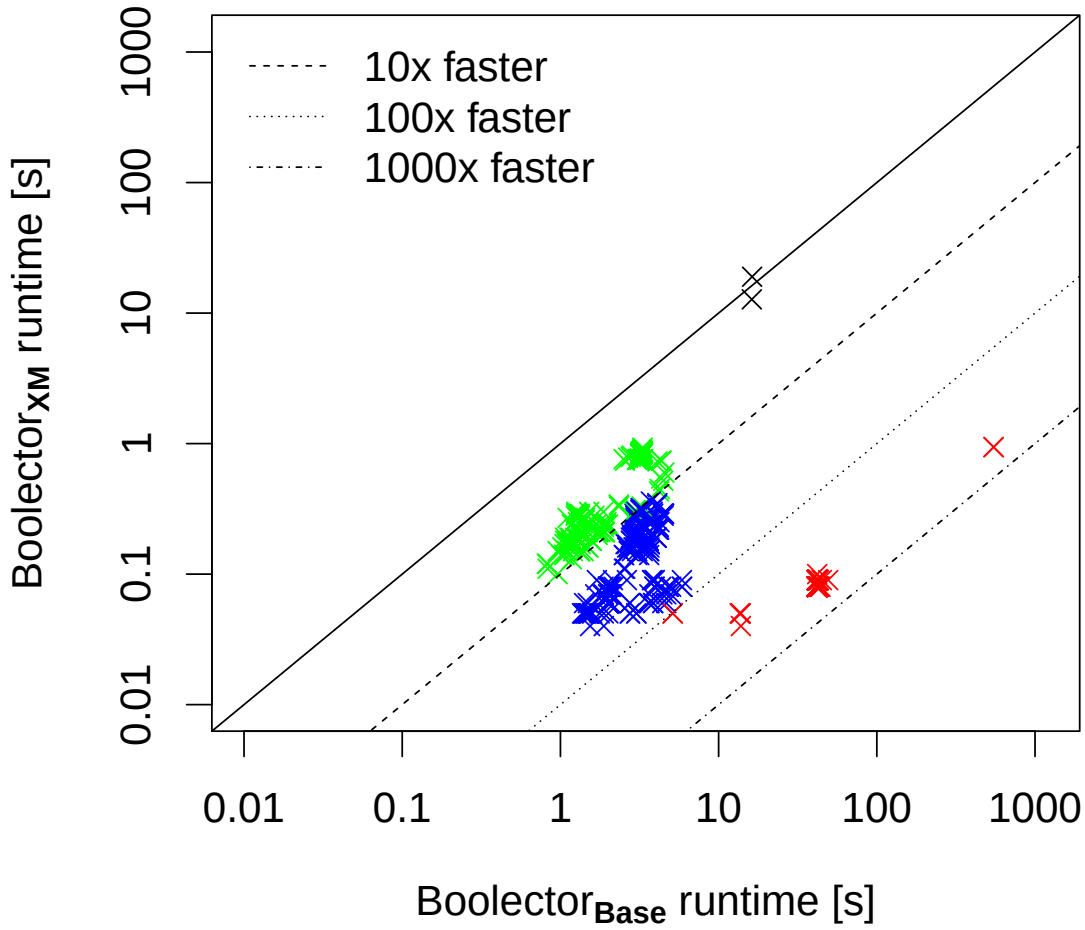

622 benchmarks

- 155 instances
  2-10x faster
- 201 instances
  10-100x faster
- 264 instances
  100-580x faster

Experiments

Lemma Generation Boolector$_{\textbf{Base}}$ vs. Boolector$_{\textbf{XM}}$

Commonly solved: 13242 instances

Impact on Lemma Generation

- Boolector$_{\textbf{Base}}$: 699027 lemmas
- Boolector$_{\textbf{XM}}$: 88762 lemmas

  $\longrightarrow$ Reduction by factor 7.9

Bit-blasted CNF: Reduction by 25% on average

SAT solver time

- Boolector$_{\textbf{Base}}$: 18175s
- Boolector$_{\textbf{XM}}$: 13653s

  $\longrightarrow$ Reduction by 25%

Conclusion

Summary

- lambda merging orthogonal to lambda extraction
- both techniques improve lemma generation
- negligible overhead
- reduces number of lemmas and consequently bit-blasted CNF
- considerable performance improvements, particularly on symbolic execution benchmarks

Future Work

- more array patterns
- more expressive array theory?

Boolector is available at `http://fmv.jku.at/boolector`

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