

# Third International Workshop on Quantified Boolean Formulas (QBF 2015)

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Austin, Sept. 23rd, 2015

colocated with the  
18th International Conference on  
Theory and Applications of Satisfiability Testing  
(SAT 2015)

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# 1 Overview

Quantified Boolean formulas (QBF) are an extension of propositional logic which allows for explicit quantification over propositional variables. The decision problem of QBF is PSPACE-complete compared to NP-completeness of the decision problem of propositional logic (SAT). Many problems from application domains such as model checking, formal verification or synthesis are PSPACE-complete, and hence could be encoded in QBF. Considerable progress has been made in QBF solving throughout the past years.

The goal of the International Workshop on Quantified Boolean Formulas (QBF Workshop) is to bring together researchers working on theoretical and practical aspects of QBF solving. The aim of this workshop is to provide an interactive platform for discussing recent advancements and alternative approaches to QBF solving. In addition to that, it addresses (potential) users of QBF in order to reflect on the state-of-the-art and to consolidate on immediate and long-term research challenges. Amongst others, these topics include

- QBF applications, encodings and benchmarks
- Case studies and experimental evaluations
- Certificates and proofs for QBF
- Formats of proofs and certificates
- Implementations of proof checkers and verifiers
- Decision procedures for QBF
- Calculi for QBF
- QBF proof theory and complexity results
- Data structures, implementation details and heuristics
- Pre- and inprocessing techniques
- Structural QBF solving
- Quantifiers in other formalisms like SMT or CSP
- Tools related to any aspect of QBF/CSP/SMT reasoning

After two successful editions in 2013<sup>1</sup> (co-located with SAT in Helsinki, Finland) and 2014<sup>2</sup> (in the context of the Vienna Summer of Logic (VSL) in Vienna, Austria) this third edition<sup>3</sup> is again co-located with the International Conference on Theory and Applications of Satisfiability Testing (SAT).

## 2 Organization

We would like to thank the members of our program committee who carefully reviewed the submitted contributions and provided valuable feedback.

- **Armin Biere**, University of Linz, Austria
- **Uwe Egly**, Vienna University of Technology, Austria
- **Mikolas Janota**, INESC-ID Lisboa, Portugal
- **Will Klieber**, Software Engineering Institute, Carnegie Mellon University, USA
- **Allen Van Gelder**, University of California at Santa Cruz, USA

## 3 Contributions

### 3.1 *Friedrich Slivovsky*: Dependency Schemes for Quantified Boolean Formulas (Invited Talk)

The nesting of existential and universal quantifiers in Quantified Boolean Formulas causes dependencies among variables that have to be respected by solvers and preprocessing techniques.

Given formulas in prenex normal form, standard algorithms implicitly make the most conservative assumption about variable dependencies: variable  $y$  depends on variable  $x$  whenever  $x$  and  $y$  are associated with different quantifiers and  $x$  precedes  $y$  in the quantifier prefix. The resulting set of dependencies is often a coarse overapproximation containing many “spurious” dependencies which lead to unnecessary restrictions that, in turn, inhibit performance.

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<sup>1</sup><http://fmv.jku.at/qbf2013/>

<sup>2</sup><http://www.easychair.org/smart-program/VSL2014/QBF-index.html>

<sup>3</sup><http://fmv.jku.at/qbf15/>

We survey dependency schemes as a means to obtaining more fine-grained overapproximations of a formula’s variable dependencies and talk about challenges arising from the integration of dependency schemes into solvers.

This talk is based upon work supported by Austrian Science Fund FWF under grant number P27721.

### **3.2 *Olaf Beyersdorff, Leroy Chew, Meena Mahajan and Shukla Anil: Feasible Interpolation for QBF Resolution Calculi***

The main aim in proof complexity is to understand the complexity of theorem proving. Arguably, what is even more important is to establish techniques for lower bounds, and the recent history of computational complexity speaks volumes on how difficult it is to develop general lower bound techniques. Understanding the size of proofs is important for at least two reasons.

The first reason is its tight relation to the separation of complexity classes: NP vs. coNP for propositional proofs, and NP vs. PSPACE in the case of proof systems for quantified boolean formulas (QBF). New super-polynomial lower bounds for specific proof systems rule out specific classes of non-deterministic poly-time algorithms for problems in co-NP or PSPACE, thereby providing an orthogonal approach to the predominantly machine-oriented view of computational complexity.

The second reason to study lower bounds for proofs is the analysis of SAT and QBF solvers: powerful algorithms that efficiently solve the classically hard problems of SAT and QBF for large classes of practically relevant formulas.

The contributions presented in this work are as follows:

1. A general lower bound technique
2. New lower bounds for QBF systems
3. Comparison to strategy extraction

### **3.3 *Valeriy Balabanov, Jie-Hong Roland Jiang and Christoph Scholl: Skolem functions computation for CEGAR based QBF solvers***

Recent QBF solvers evaluation verified the robustness and efficiency of CEGAR-based QBF solvers. On contrary to search-based approaches (e.g.,

DepQBF), however, there exists no methodology to certify their answer with semantic winning strategies in a closed form (e.g., Skolem-functions for true QBFs, which are essential for many QBF applications).

The CEGAR-based QBF solver RareQS can produce partial winning moves for both existential and universal players at each turn of an abstraction-refinement game. One straightforward use of this ability is that RareQS returns the winning assignment to outermost existential variables for true QBFs upon completion.

In this work we describe how to construct full Skolem-functions models for QBFs, based on partial winning moves information emitted by RareQS. Currently our algorithm is limited to two and three level true QBFs (with an innermost quantification level to be existential), but preliminary analysis confirms the existence of an extension to arbitrary QBFs, based on the given approach and interpolation.

### **3.4 *Valeriy Balabanov and Jie-Hong Roland Jiang: Reducing Satisfiability and Reachability to DQBF***

Both directed reachability (in symbolic form) and QBF underly the same PSPACE-complete complexity class. There exists an “iterative-squaring” quadratic-size reduction of REACH to QBF. Dependency quantified Boolean formulas (DQBF) extend QBF by introducing explicit dependencies among variables. This extension results into an exponential complexity jump, lifting the satisfiability decision problem of DQBF into NEXPTIME the complete complexity class.

In this work we show a relation between propositional satisfiability, directed reachability, and dependency quantified Boolean formulas, and present two contributions.

First we provide a constructive transformation from 3SAT to equisatisfiable DQBF with exponentially smaller number of variables and linear formula size. This approach allows us to build many DQBF benchmarks from existing CNF ones and test it. Second, we propose and implement an alternative to the linear translation of REACH into DQBF, which works in a simpler and more intuitive manner. We hope that generated industrial DQBF benchmarks will give a lift for DQBF solving research.