Dependency Schemes & Q-Resolution

Friedrich Slivovsky & Stefan Szeider

Vienna University of Technology

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	$Q_i \in \{\forall, \exists\}$	in CNF
	(quantifier) prefix	matrix

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the prefix induces a partial order R_F on the set of variables

$(x, y) \in R_F$ y is to the right of x in F‘s prefix

order R_F constrains decision procedures

QDPLL

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$(y \vee x_1 \vee \neg x_2 \vee \cdots \vee x_k)$

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$$\frac{C \vee (\neg)x}{C} \quad \forall y \in var_{\exists}(C): (x, y) \notin R_F$$

dependency scheme D
maps F to a set of dependencies $D_F \subseteq R_F$

$(x, y) \in D_F$ "y may depend on x"

Biere & Lonsing ‘10

QDPLL(D)

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implemented in DepQBF for the Standard Dependency Scheme (D^{std})

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in this talk

1. stating sufficient conditions
2. proving soundness for D^{std}

QDPLL traces correspond to Q-resolution proofs

Giunchiglia, Narizzano, Tacchella ‘06

Q-resolution

Kleine Büning, Karpinski, Flögel ‘95

$$\frac{C \vee y \qquad \neg y \vee C'}{C \vee C'}$$

$C \vee C'$ non-tautological

$$\frac{C \vee (\neg) x}{C}$$

$\forall y \in var(C): (x, y) \notin R_F$

F is false iff the empty clause can be derived from F

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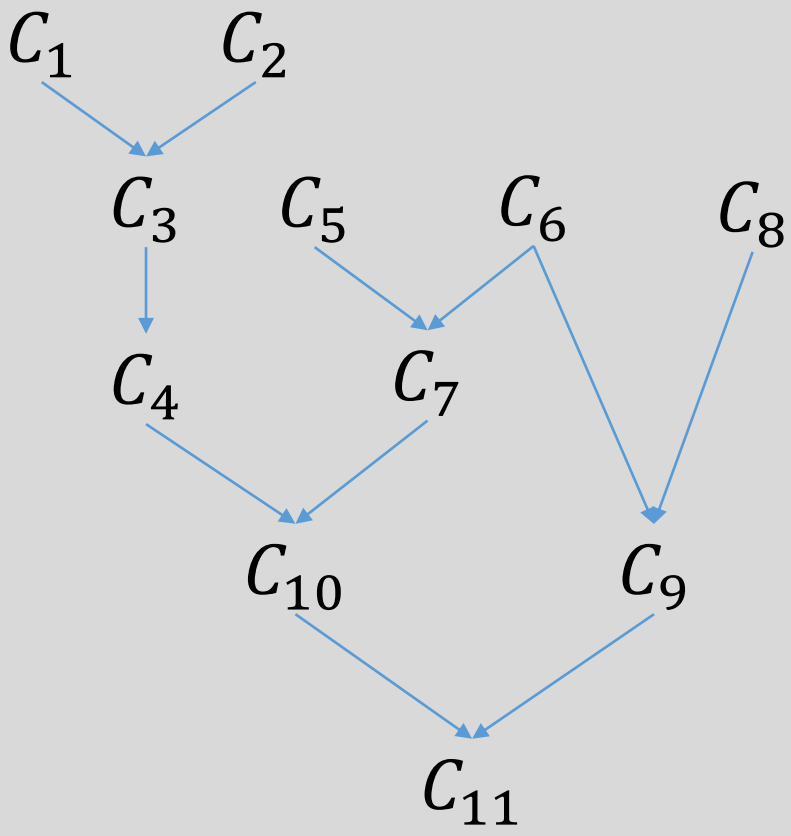
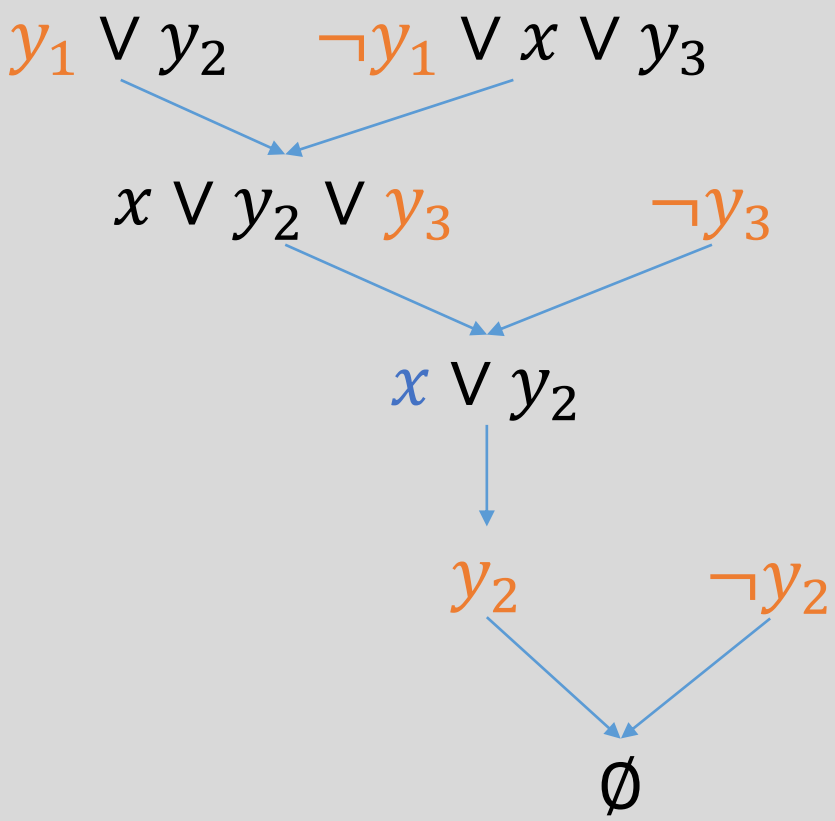
derivations as DAGs



DepQBF (almost) generates Q-resolution certificates

$$F = \exists y_1 \forall x \exists y_2 \exists y_3 \ (y_1 \vee y_2) \wedge (\neg y_1 \vee x \vee y_3) \wedge (\neg y_3) \wedge (\neg y_2)$$



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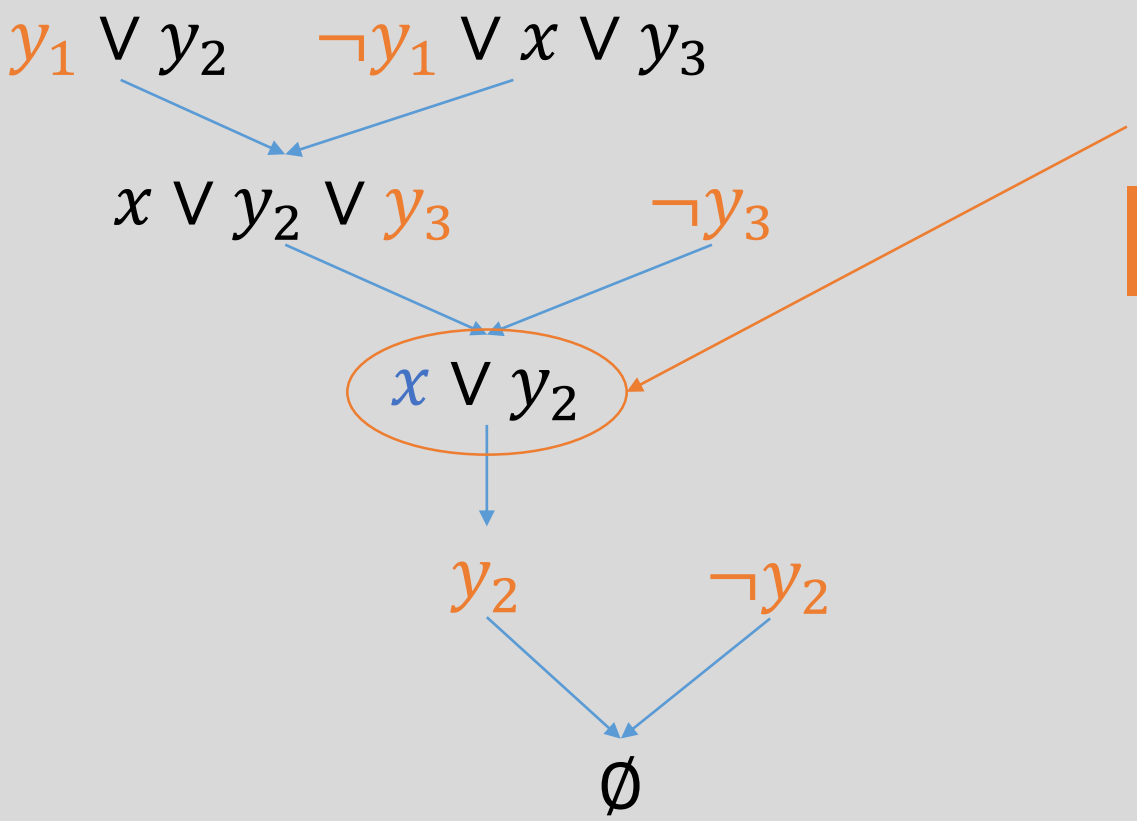

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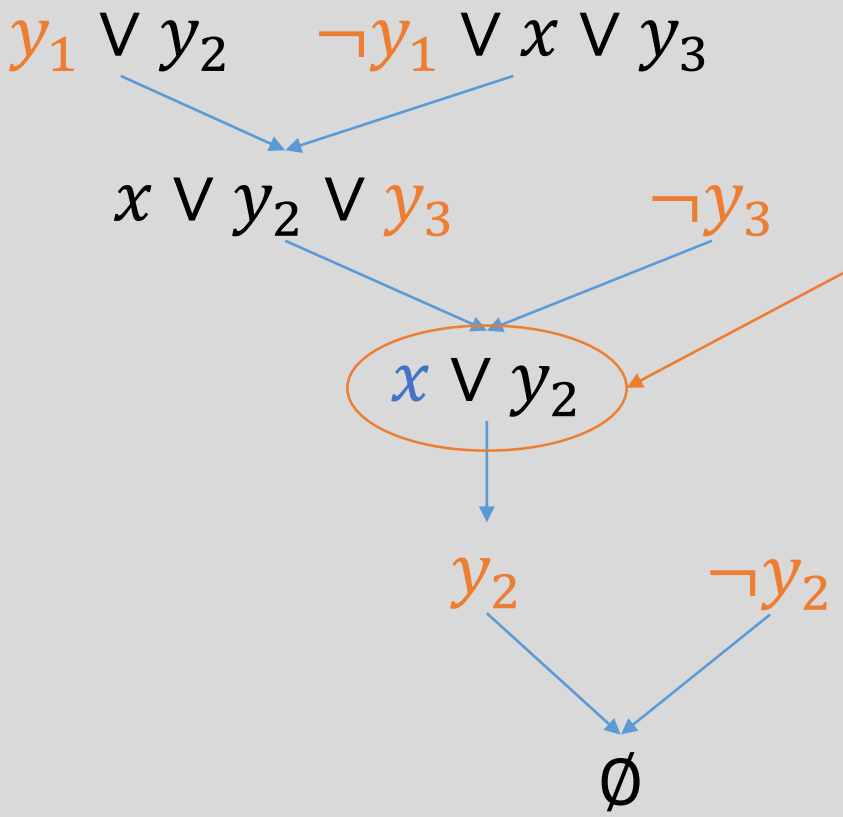

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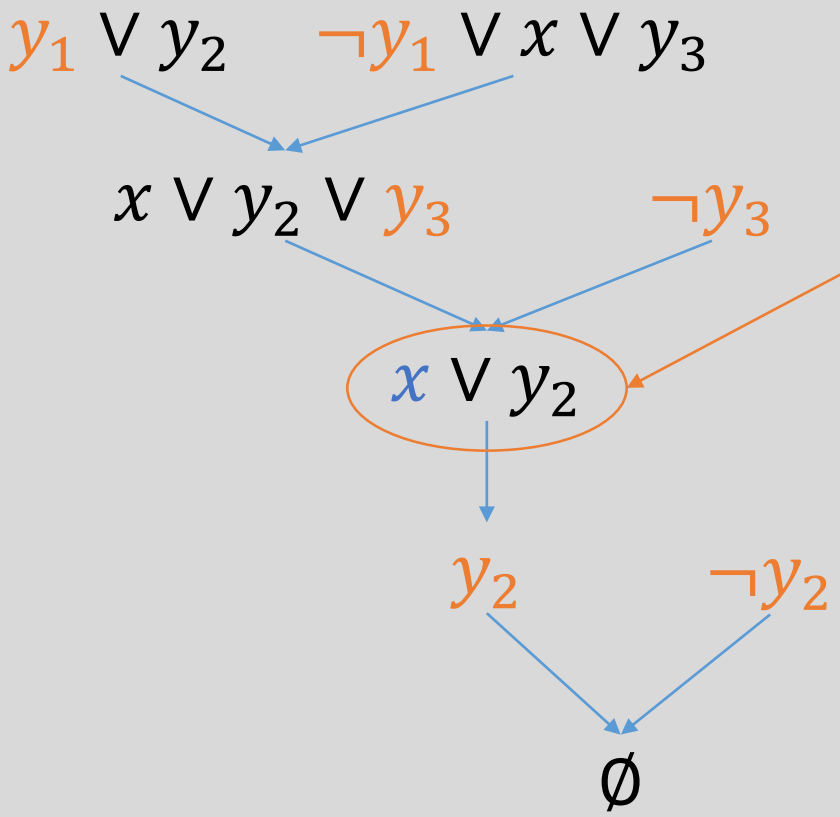

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Q(D)-resolution

D can be computed in polynomial time

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working hypothesis

QDPLL(D) generates Q(D)-resolution proofs

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working hypothesis

QDPLL(D) generates Q(D)-resolution proofs

If Q(D)-resolution is sound then QDPLL(D) is sound.

Q(D^{std})-resolution is sound

Theorem

Q-resolution simulates Q(D^{std})-resolution.

Q(D^{std})-resolution is sound

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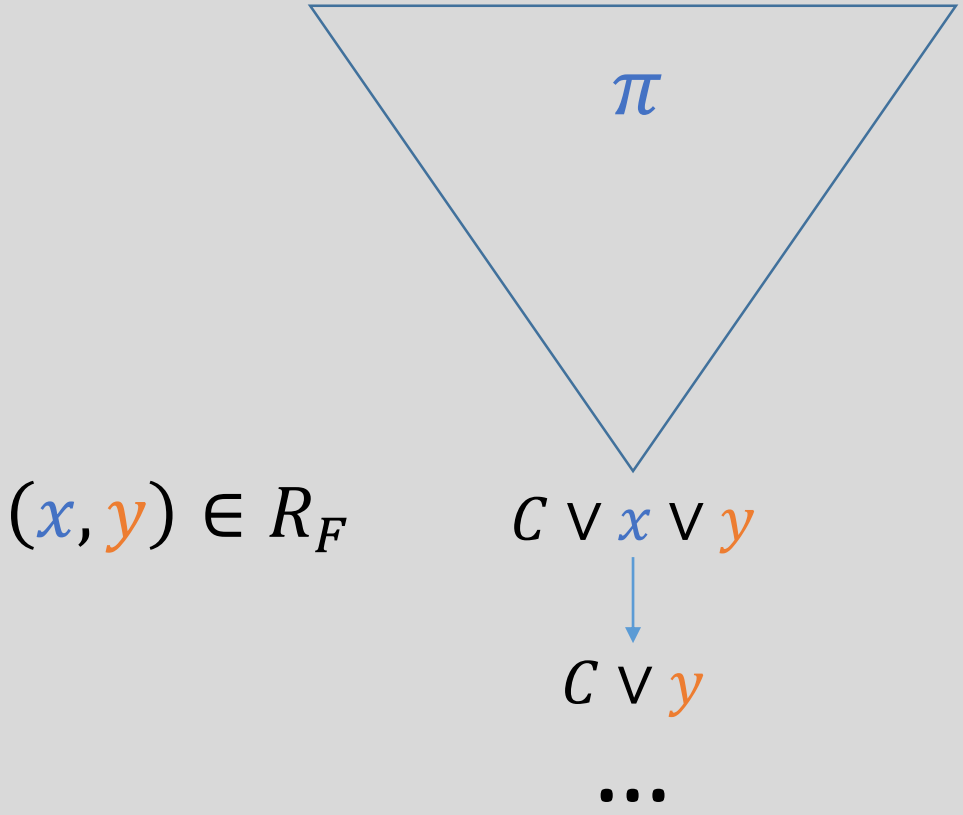
Q-resolution simulates Q(D^{std})-resolution.

There is an $O(3^n)$ time algorithm that, given an arbitrary Q(D)-resolution refutation π of a formula F, computes a Q-resolution refutation of F or concludes that π is not a Q(D^{std})-refutation.

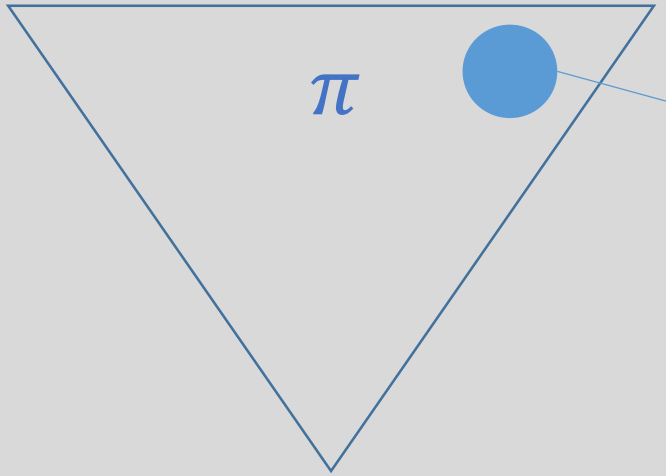
Q(D^{std})-resolution is sound



Lemma

π contains a resolution step on z with $(z, x) \in R_F$, or $(x, y) \in D_F^{std}$.

Q(D^{std})-resolution is sound



resolution on z with $(z, x) \in R_F$

idea lower this resolution step past the universal reduction step

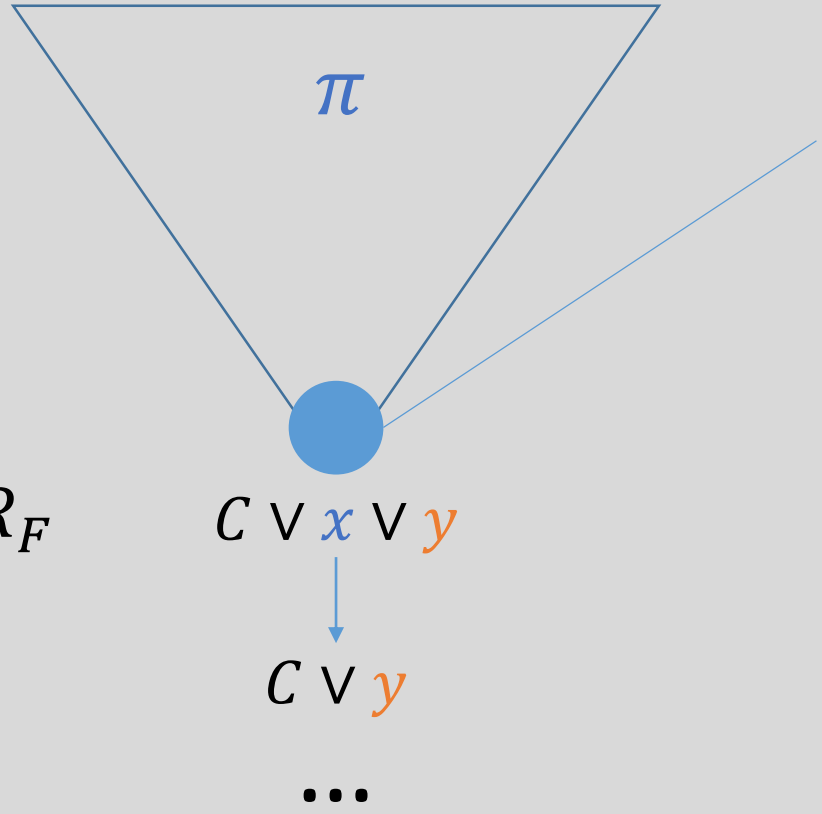
$(x, y) \in R_F$

$C \vee x \vee y$

$C \vee y$

...

Q(D^{std})-resolution is sound



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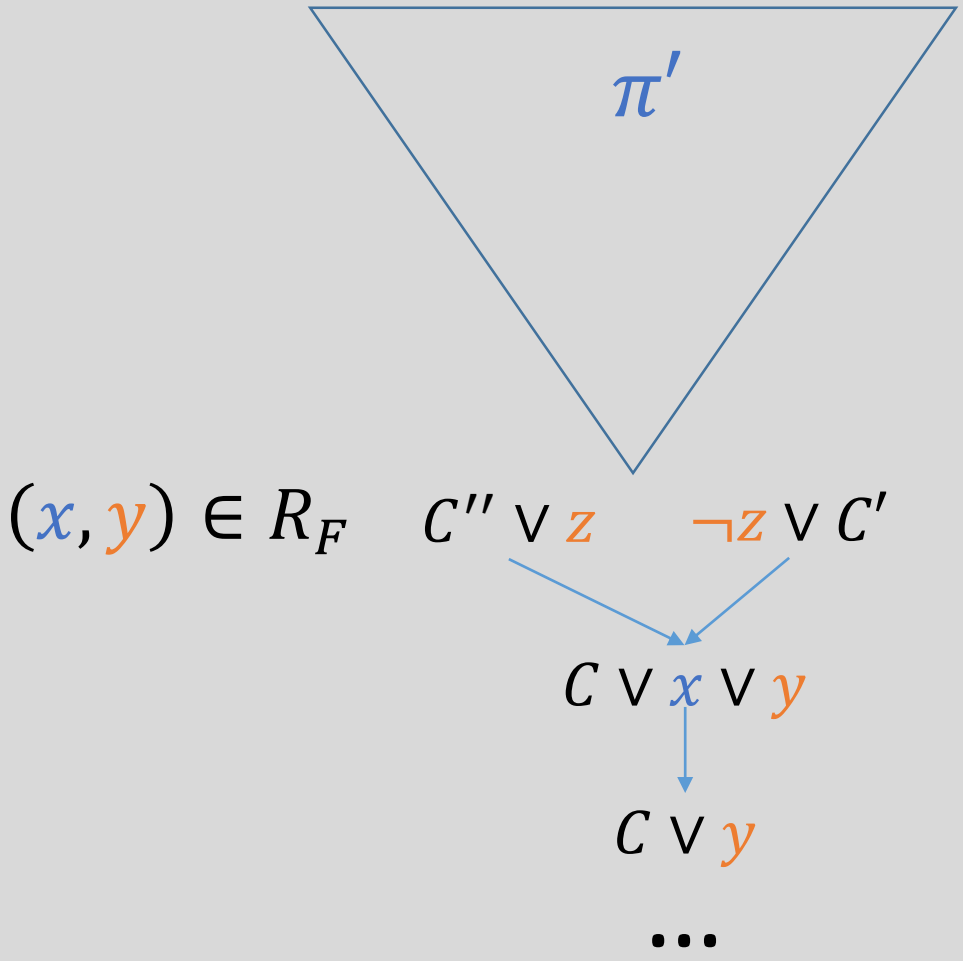
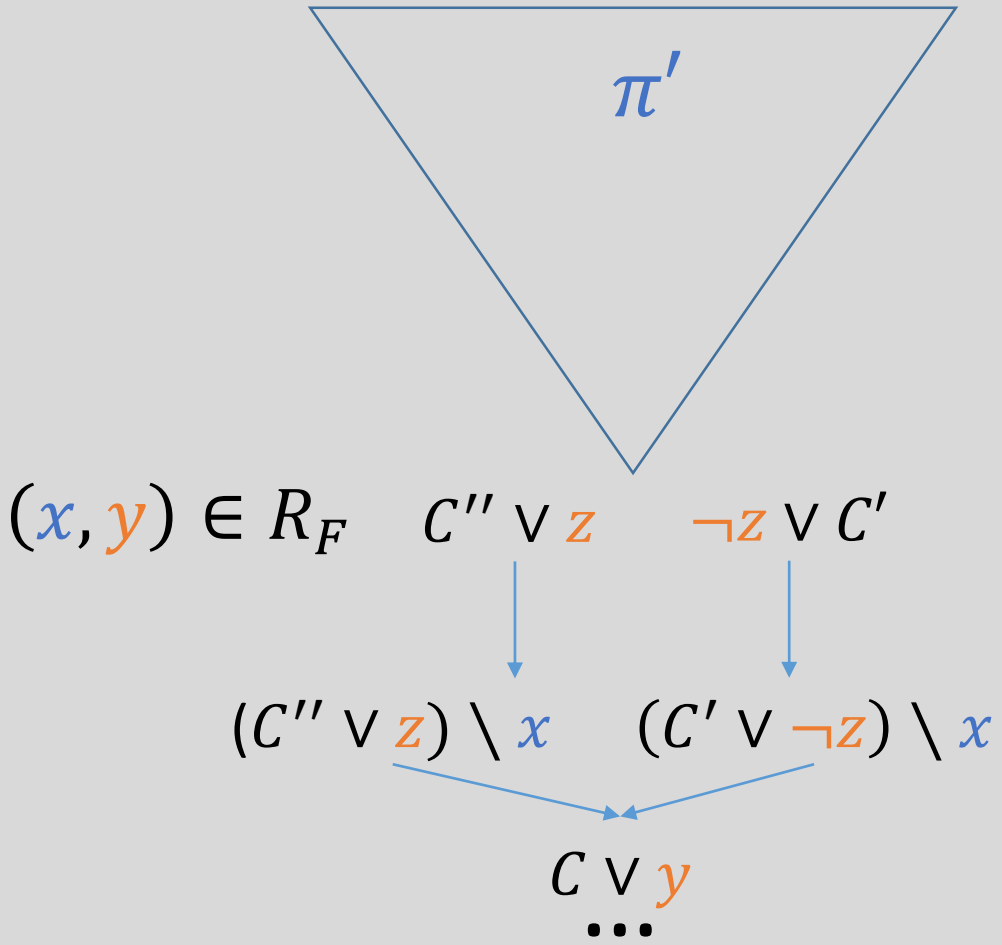


Because z is to the left of x, it does not interfere with forall reduction.

Q(D^{std})-resolution is sound



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So we can raise forall reduction.

repeat for subderivations

Future Work

- Proving soundness of Q(D^{std})-term-resolution
- Proving soundness of Q(D^{res*})-resolution
- Can we extract models without rewriting?
- Proof complexity