Encodings of Reactive Synthesis

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Outline

Bounded Synthesis

Encodings

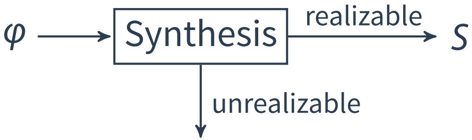
Experimental Results

Strategy Extraction

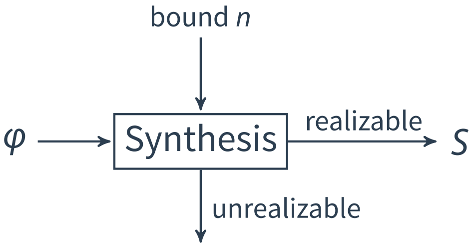
Synthesis of Reactive Systems

- Systems that react on external events
- Interest accelerated towards automatic construction (synthesis)
- Two iterations of SyntComp (restricted safety format)
- Full synthesis track in planning

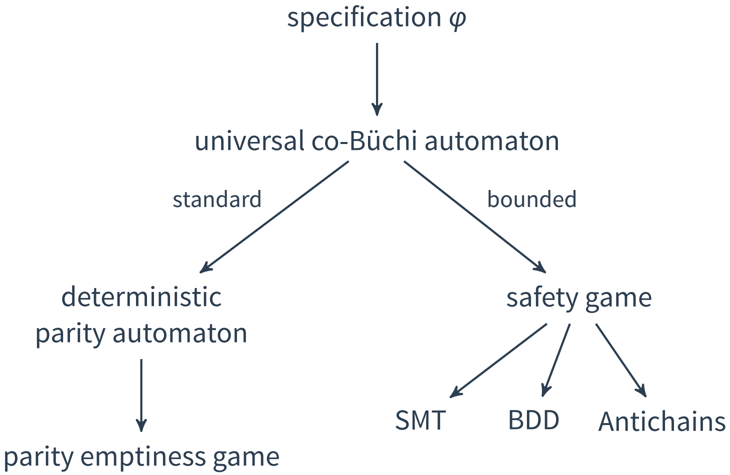
Reactive Synthesis



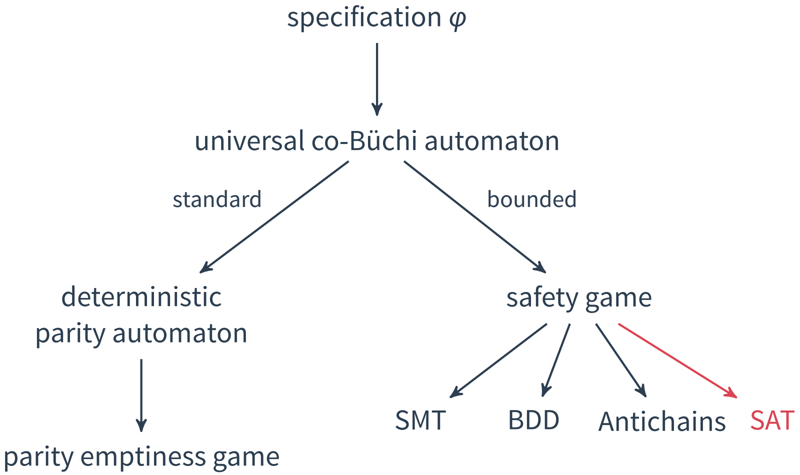
Bounded Synthesis



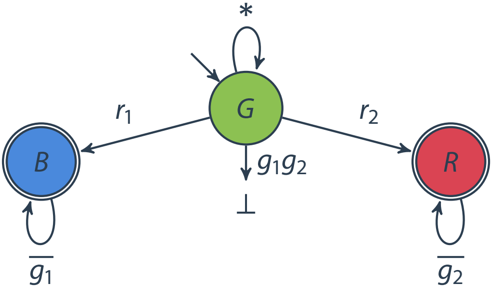
Single Process Synthesis



Single Process Synthesis



Universal Co-Büchi Automata



Example (Simplified Arbiter)

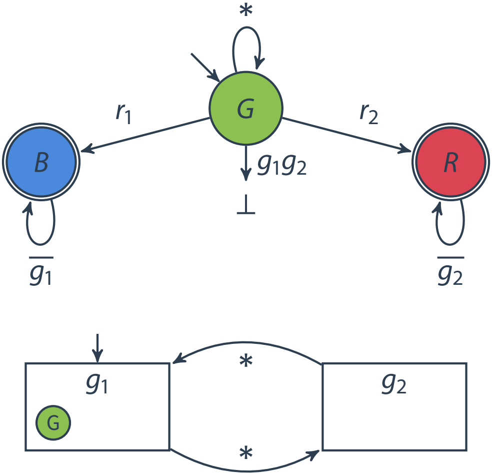
$$\varphi = \Box(r_1 \rightarrow \bigcirc\Diamond g_1) \land \Box(r_2 \rightarrow \bigcirc\Diamond g_2) \land \neg\Diamond(g_1 \land g_2)$$

Acceptance

A **transition system** is accepted by an universal co-Büchi automaton if **all paths** in the (unique) run graph contain only **finitely** many visits to rejecting states.

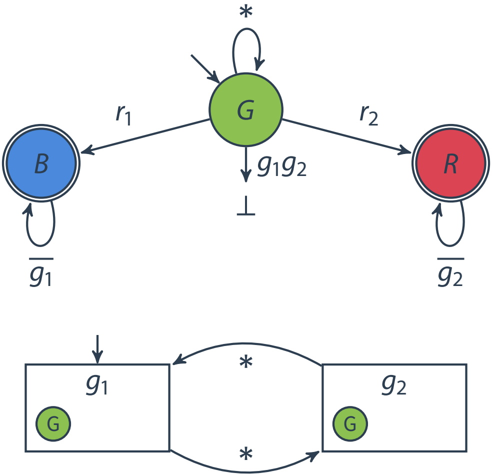
Acceptance of a Transition System

example



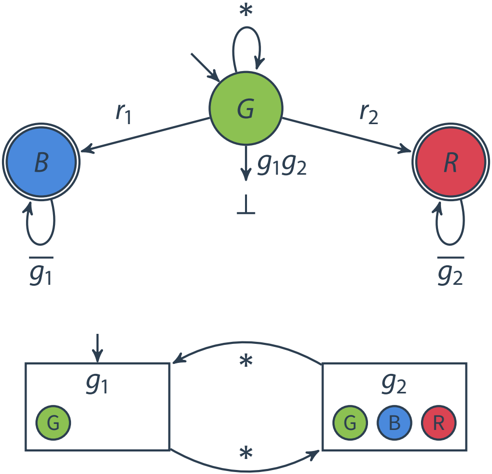
Acceptance of a Transition System

example



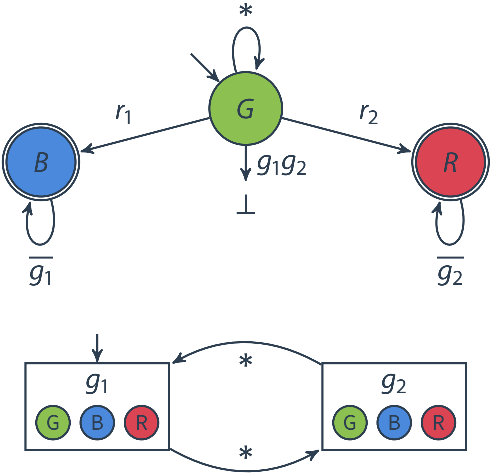
Acceptance of a Transition System

example



Acceptance of a Transition System

example



Acceptance of a Transition System

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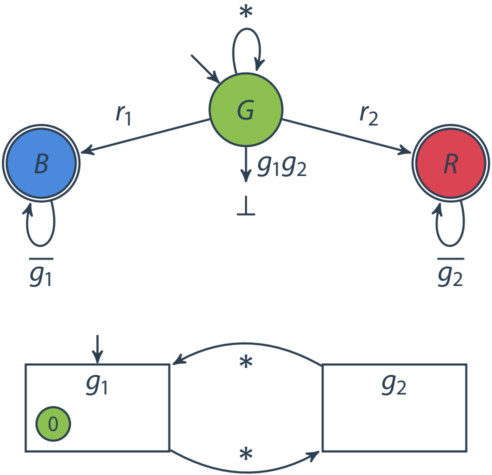

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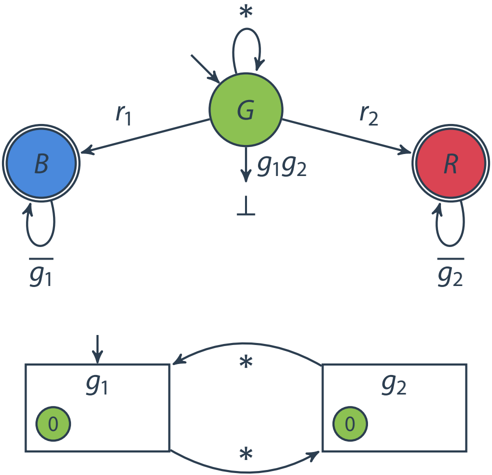

Acceptance of a Transition System
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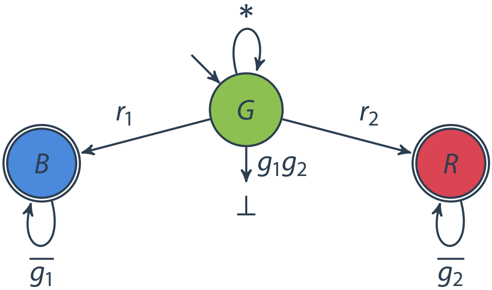

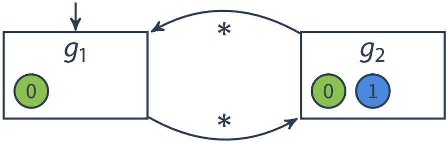

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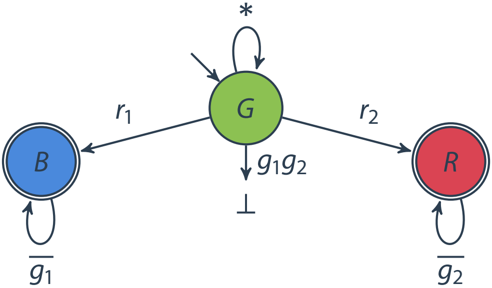

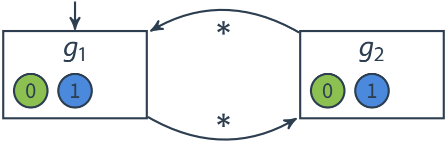

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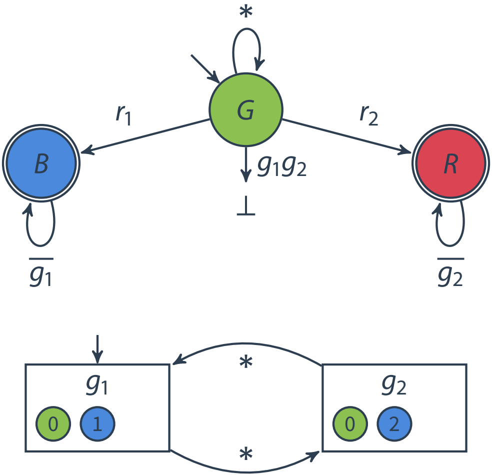

Acceptance of a Transition System

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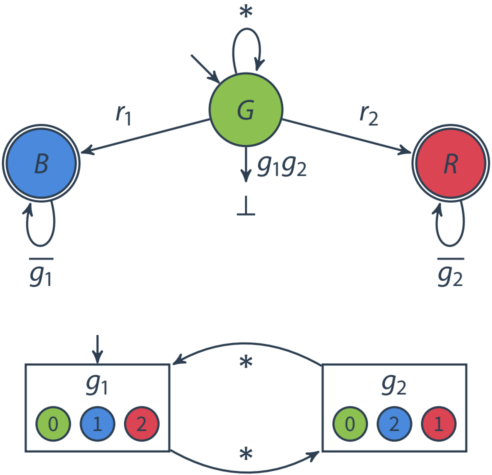
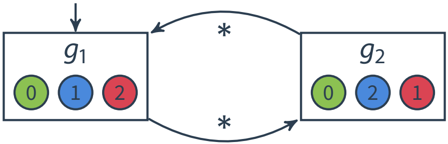


Annotated Transition System

- collects the paths of the run graph that lead to a state in the transition system
- for each automaton state, indicates whether state visited on some path, and if so, max number of visits to rejecting states

Theorem (Finkbeiner & Schewe'07)

A transition system is accepted by a universal co-Büchi automaton ⇔ it has a valid annotation



Encodings

Build a constraint system that specifies the existence of an annotated transition system

- Representation of transition system
  - states
  - labeling
  - transitions
- Representation of annotation
  - state occurrence
  - rejecting bound

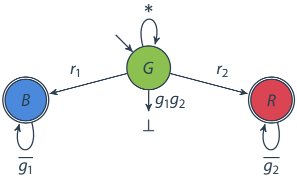
SMT Encoding

Inputs I, Outputs O, universal co-Büchi automaton $\langle Q, q_0, \delta, R \rangle$

- Representation of transition system
  - **states**: $\mathbb{N}_n = \{0, \ldots, n-1\}$
  - **labeling**: functions $o : \mathbb{N}_n \to \mathbb{B}$ for every $o \in O$
  - **transitions**: functions $\tau_I : \mathbb{N}_n \to \mathbb{N}_n$
- Representation of annotation
  - **state occurrence**: functions $\lambda_q^{\mathbb{B}} : \mathbb{N}_n \to \mathbb{B}$
  - **rejecting bound**: functions $\lambda_q^{\#} : \mathbb{N}_n \to \mathbb{N}$

SMT Encoding

example



- $\lambda^{\mathbb{B}}_G(0)$
- $$\begin{aligned}\forall s.\, \lambda^{\mathbb{B}}_G(s) \to\ & \lambda^{\mathbb{B}}_G(\tau_{\overline{r_1 r_2}}(s)) \wedge \lambda^{\#}_G(\tau_{\overline{r_1 r_2}}(s)) \geq \lambda^{\#}_G(s)\\ \wedge\ & \lambda^{\mathbb{B}}_G(\tau_{\overline{r_1} r_2}(s)) \wedge \lambda^{\#}_G(\tau_{\overline{r_1} r_2}(s)) \geq \lambda^{\#}_G(s)\\ \wedge\ & \lambda^{\mathbb{B}}_G(\tau_{r_1 \overline{r_2}}(s)) \wedge \lambda^{\#}_G(\tau_{r_1 \overline{r_2}}(s)) \geq \lambda^{\#}_G(s)\\ \wedge\ & \lambda^{\mathbb{B}}_G(\tau_{r_1 r_2}(s)) \wedge \lambda^{\#}_G(\tau_{r_1 r_2}(s)) \geq \lambda^{\#}_G(s)\end{aligned}$$
- $\forall s.\, \lambda^{\mathbb{B}}_G(s) \to \neg g_1(s) \vee \neg g_2(s)$
- $$\begin{aligned}\forall s.\, \lambda^{\mathbb{B}}_G(s) \wedge r_1(s) \to\ & \lambda^{\mathbb{B}}_B(\tau_{\overline{r_1 r_2}}(s)) \wedge \lambda^{\#}_B(\tau_{\overline{r_1 r_2}}(s)) > \lambda^{\#}_G(s)\\ \wedge\ & \lambda^{\mathbb{B}}_B(\tau_{\overline{r_1} r_2}(s)) \wedge \lambda^{\#}_B(\tau_{\overline{r_1} r_2}(s)) > \lambda^{\#}_G(s)\\ \wedge\ & \lambda^{\mathbb{B}}_B(\tau_{r_1 \overline{r_2}}(s)) \wedge \lambda^{\#}_B(\tau_{r_1 \overline{r_2}}(s)) > \lambda^{\#}_G(s)\\ \wedge\ & \lambda^{\mathbb{B}}_B(\tau_{r_1 r_2}(s)) \wedge \lambda^{\#}_B(\tau_{r_1 r_2}(s)) > \lambda^{\#}_G(s)\end{aligned}$$
- $$\begin{aligned}\forall s.\, \lambda^{\mathbb{B}}_B(s) \wedge \neg g_1(s) \to\ & \lambda^{\mathbb{B}}_B(\tau_{\overline{r_1 r_2}}(s)) \wedge \lambda^{\#}_B(\tau_{\overline{r_1 r_2}}(s)) > \lambda^{\#}_B(s)\\ \wedge\ & \lambda^{\mathbb{B}}_B(\tau_{\overline{r_1} r_2}(s)) \wedge \lambda^{\#}_B(\tau_{\overline{r_1} r_2}(s)) > \lambda^{\#}_B(s)\\ \wedge\ & \lambda^{\mathbb{B}}_B(\tau_{r_1 \overline{r_2}}(s)) \wedge \lambda^{\#}_B(\tau_{r_1 \overline{r_2}}(s)) > \lambda^{\#}_B(s)\\ \wedge\ & \lambda^{\mathbb{B}}_B(\tau_{r_1 r_2}(s)) \wedge \lambda^{\#}_B(\tau_{r_1 r_2}(s)) > \lambda^{\#}_B(s)\end{aligned}$$

Generic Propositional Encoding

Inputs I, Outputs O, universal co-Büchi automaton $\langle Q, q_0, \delta, R \rangle$

- Representation of transition system
  - **states**: $S = \{s_0, \ldots, s_{n-1}\}$
  - **labeling**: $o : S \to \mathbb{B}$ for every $o \in O$
  - **transitions**: $\tau : S \times \mathbb{B}^{|I|} \to S$
- Representation of annotation
  - **state occurrence**: $\lambda : S \times Q \to \mathbb{B}$
  - **rejecting bound**: $\lambda^{\#} : S \times Q \to \mathbb{B}^{b}$ (b-bit counter)

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$\exists \lambda(s, q), \lambda^{\#}(s, q), o(s), \tau(s, i).$

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$$\exists \lambda(s, q), \lambda^\#(s, q), o(s), \tau(s, i). \quad \forall s, s', q, q', i.$$

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$$\exists \lambda(s,q), \lambda^{\#}(s,q), o(s), \tau(s,i). \quad \forall s, s', q, q', i. \quad \lambda(s_0, q_0) \wedge$$
$$\left(\lambda(s,q) \wedge \delta(q, o(s), i, q') \wedge (\tau(s,i) = s')\right)$$

Generic Propositional Encoding

Inputs I, Outputs O, universal co-Büchi automaton $\langle Q, q_0, \delta, R\rangle$

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$$\exists \lambda(s,q), \lambda^\#(s,q), o(s), \tau(s,i). \quad \forall s, s', q, q', i. \quad \lambda(s_0, q_0) \wedge$$
$$\big(\lambda(s,q) \wedge \delta(q, o(s), i, q') \wedge (\tau(s,i) = s')\big) \to \big(\lambda(s',q') \wedge \lambda^\#(s',q') \rhd \lambda^\#(s,q)\big)$$

$$\rhd := \begin{cases} > & \text{if } q \text{ rejecting} \\ \geq & \text{otherwise} \end{cases}$$

Derived Encodings

- SAT: complete unrolling

$$\exists \lambda_{s,q}, \lambda^{\#}_{s,q}, o_s, \tau_{s,i,s'}$$

- QBF: input symbolic encoding

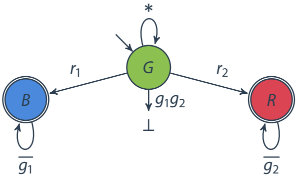
$$\exists \lambda_{s,q}, \lambda^{\#}_{s,q}, o_s . \forall i . \tau_{s,s'}$$

- DQBF: state and input symbolic encoding

$$\begin{array}{l} \forall s . \exists \lambda_q, \lambda^{\#}_q, o . \forall i . \tau \\ \forall s' . \exists \lambda'_q, \lambda'^{\#}_q . \end{array} \quad \bigwedge_q (s = s') \rightarrow (\lambda_q = \lambda'_q) \wedge (\lambda^{\#}_q = \lambda'^{\#}_q)$$

QBF Encoding

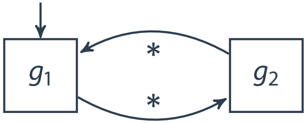
example



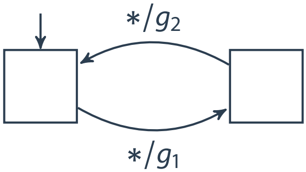
- $\lambda_{s_0,G}$
- $\bigwedge_{s\in S}\left(\lambda_{s,G} \rightarrow \bigwedge_{s'\in S}(\tau_{s,s'} \rightarrow \lambda_{s',G} \wedge \lambda^{\#}_{s',G} \geq \lambda^{\#}_{s,G})\right)$
- $\bigwedge_{s\in S}\left(\lambda_{s,G} \rightarrow \neg g_1^s \vee \neg g_2^s\right)$
- $\bigwedge_{s\in S}\left(\lambda_{s,G} \wedge r_1 \rightarrow \bigwedge_{s'\in S}(\tau_{s,s'} \rightarrow \lambda_{s',B} \wedge \lambda^{\#}_{s',B} > \lambda^{\#}_{s,G})\right)$
- $\bigwedge_{s\in S}\left(\lambda_{s,B} \wedge \neg g_1^s \rightarrow \bigwedge_{s'\in S}(\tau_{s,s'} \rightarrow \lambda_{s',B} \wedge \lambda^{\#}_{s',B} > \lambda^{\#}_{s,B})\right)$

Mealy and Moore Transition Systems

Moore: State-labeled transition systems



Mealy: Edge-labeled transition systems



Mealy and Moore Transition Systems

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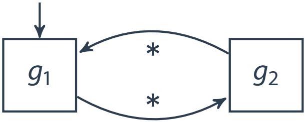
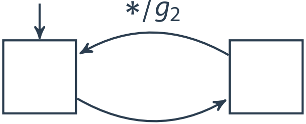


$$\exists \lambda_{s,q}, \lambda^{\#}_{s,q}, o_s.\ \forall i.\ \exists \tau_{s,s'}$$

Mealy: Edge-labeled transition systems



$$\exists \lambda_{s,q}, \lambda^{\#}_{s,q}.\ \forall i.\ \exists o_s, \tau_{s,s'}$$

Implementation

Encoding	Solver	Strategy Extraction
SMT	Z3, CVC4	✓
SAT	MiniSat, PicoSAT	✓
QBF	RAReQS, DepQBF, Bloqqer	✓
DQBF	iDQ, eprover (EPR)	

Experiments

Arbiter

$$\bigwedge_i \Box(r_i \rightarrow \Diamond g_i) \qquad \text{(response)}$$

$$\bigwedge_{i \neq j} \Box\neg(g_i \wedge g_j) \qquad \text{(mutex)}$$

Arbiter without spurious grants

$$\bigwedge_i \neg((\neg r_i \wedge \neg g_i)\,\mathcal{U}\,(\neg r_i \wedge g_i)) \qquad \text{(no-spurious-start)}$$

$$\bigwedge_i \neg\Diamond\Big(g_i \wedge \bigcirc\Big((\neg r_i \wedge \neg g_i) \wedge \big((\neg r_i \wedge \neg g_i)\mathcal{U}(\neg r_i \wedge g_i)\big)\Big)\Big) \qquad \text{(no-spurious)}$$

$$\bigwedge_i \Box((\neg r_i \wedge g_i) \rightarrow \Diamond((r_i \wedge g_i) \vee \neg g_i)) \qquad \text{(lowered)}$$

Results

Quad-Core Intel Xeon @ 3.6 GHz, 32 GB RAM, 1*h* timeout

Instance	SMT Z3		SAT MiniSat		QBF RAReQS+B		DQBF iDQ	
	mealy	moore	mealy	moore	mealy	moore	mealy	moore
arbiter-2	0.22	0.21	0.21	0.21	0.19	0.19	0.23	0.23
arbiter-3	0.45	0.39	0.30	0.31	0.21	0.21	0.62	0.63
arbiter-4	1428	2234	0.73	0.76	0.25	0.25	1.71	1.83
arbiter-5	TO	TO	4.15	3.72	0.40	0.40	24.6	24.6
arbiter-6	TO	TO	21.0	21.0	0.71	0.71	76.8	79.9
arbiter-7	TO	TO	155.6	102.5	7.02	5.98	294.6	294.3
arbiter-8	TO	TO	2384	TO	397.6	406.6	TO	TO
full-arbiter-2	0.49	2.75	0.39	0.63	0.28	0.38	20.6	19.9
full-arbiter-3	TO	TO	16.6	34.9	9.28	17.6	TO	TO
full-arbiter-4	TO	TO	TO	TO	TO	TO	TO	TO

QBF Encoding

Quad-Core Intel Xeon @ 3.6 GHz, 32 GB RAM, 1h timeout

Instance	QBF Mealy				
	RAReQS+B	RAReQS	DepQBF+B	DepQBF	RAReQS-QCIR
arbiter-2	0.19	0.21	0.20	0.21	0.26
arbiter-3	0.21	0.21	0.22	0.21	0.33
arbiter-4	0.25	0.31	0.25	0.30	0.52
arbiter-5	0.40	0.34	0.41	0.45	1.2
arbiter-6	0.71	0.90	1.28	2.46	4.46
arbiter-7	7.02	30.3	128.9	TO	94.8
arbiter-8	397.6	TO	TO	TO	TO
full-arbiter-2	0.28	0.28	0.27	0.35	1.05
full-arbiter-3	9.28	867	TO	TO	968
full-arbiter-4	TO	TO	TO	TO	TO

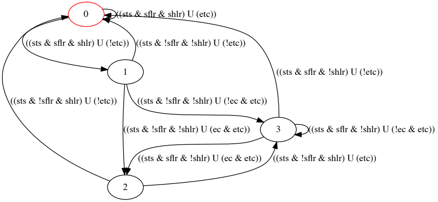
Strategy Extraction

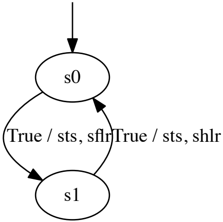
- Use certification feature of solvers to get witness for o and τ
  - Model from SMT solver
  - Assignments from SAT solver
  - Skolem functions from QBF solver
- Build transition system and encode it in SMV
- Model-check solution (NuSMV)

Strategy Extraction

example

Traffic Light Controller (Lily benchmark)





Acacia+ (optimal strategy option)

QBF Encoding

Conclusions
and future work

- Generic propositional encoding
- Encodings to SAT, QBF, DQBF
- All propositional encodings outperform SMT

- Optimizations similar to previous work (decompose specification into safety/liveness, conjunctions, etc.)
- Incremental solving