Quantifiers, Computation, and Cognition

Jakub Szymanik*

Institute of Logic Language and Computation, University of Amsterdam
J.K.Szymanik@uva.nl

Generalized quantifier theory studies the semantics of quantifier expressions, like, 'every', 'some', 'most', 'infinitely many', 'uncountably many', etc. The classical version was developed in the 1980s, at the interface of linguistics, mathematics, and philosophy. In logic, generalized quantifiers are often defined as classes of models closed on isomorphism (topic neutral). For instance, the quantifier 'infinitely many' may be defined as a class of all infinite models. Equivalently, in linguistics generalized quantifiers are formally treated as relations between subsets of the universe. For example, in the sentence 'Most of the students are smart', quantifier 'most' is a binary relation between the set of students and the set of smart people. The sentence is true if and only if the cardinality of the set of smart students is greater than the cardinality of the set of students who are not smart. Generalized quantifiers turned out to be one of the crucial notions in the development of formal semantics but also logic, theoretical computer science and philosophy [11]. In this talks we survey recent results combining classical generalized quantifier themes and a computational complexity perspective, with an outlook toward applications in cognitive science and linguistics [16].

We focus on the complexity of meaning of natural language quantifiers. The general question we aim to answer is why the meanings of some sentences are more difficult than the meanings of others. For instance, why we will probably all agree that it is easier to evaluate sentence (1) than to evaluate sentence (2) and why sentence (3) seems hard while sentence (4) sounds odd.

(1) Every book on the shelf is yellow.
(2) Most of the books on the shelf are yellow.
(3) Less than half of the members of parliament refer to each other.
(4) Some book by every author is referred to in some essay by every critic.

The tools of logic and computability theory are useful in making such differences precise. The complexity analysis of the quantifier sentences in natural language allows drawing and testing empirical predictions about cognitive difficulty of language processing, and about specific cognitive resources (working memory, executive functions, etc.) involved in it.

We will start by introducing the notion of monadic quantifiers—the most important class of generalized quantifiers that captures the meanings of natural language simple determiners. In particular, we will introduce so-called semantic automata theory that associates each quantifier with a simple computational device. We will also discuss some classical definability results connecting expressibility with semantic automata, e.g., all quantifiers definable in the first-order

*The research was supported by Veni Grant NWO-639-021-232.

logic are recognizable by acyclic finite-automata [1, 9, 4]. In doing that we will use fundamental notions of automata theory and draw some connections with psychology, for instance, we will show that the distinction between finite-automata and push-down automata quantifiers matters for psycholinguistics [8, 13, 14, 17, 20, 18].

Next we will survey current literature concerned with polyadic quantification. We will explain how polyadic quantifiers result from semantically natural operations applied to monadic quantifiers, like iteration, cumulation, or Ramseyification. In addition to discussing definability issues, we will also demonstrate how the semantic automata framework can be extended to iterations, showing that if Q_1 and Q_2 are recognizable by finite-automata (push-down automata) then also their iteration must be recognizable by finite-automata (push-down automata) [12]. Furthermore, we will discuss computational complexity results on more kinds of polyadic quantifiers—among others—proving a dichotomy result for Ramsey quantifiers [15, 2], namely, we show that the Ramseyification of polynomial-time and constant-log-bounded monadic quantifiers result in polynomial time computable Ramsey quantifiers while assuming the Exponential Time Hypothesis. Moreover, we will discuss how such complexity results correlate with linguistic distributions [19, 3].

In the final, most technical part, we will show how the standard generalized quantifier theory, originally designed to deal with distributive quantification, can be extended to cover collective quantifiers. We will discuss type-lifting strategies constructing collective readings from distributive readings. We will also introduce the notion of second-order generalized quantifier that is a natural mathematical extension of Lindström quantifiers to the collective setting. We will introduce the definability theory for second-order generalized quantifiers [5] and discuss related computational complexity results [6]. In particular, we will show that the question whether a second-order generalized quantifier $\mathcal{Q}_1$ is definable in terms of another quantifier $\mathcal{Q}_2$, the base logic being monadic second-order logic, reduces to the question if a quantifier $\mathcal{Q}^{\star}{}_1$ is definable in $FO(\mathcal{Q}^{\star}{}_2, <, +, \times)$ for certain first-order quantifiers $Q^{\star}_1$ and $Q^{\star}_2$. We use our characterization to show new definability and non-definability results for second-order generalized quantifiers [7]. We will conclude with a more general methodological discussions, using definability and complexity results we will ask about the expressivity bounds of everyday language [10].

References

1. Johan van Benthem. *Essays in logical semantics.* Reidel, 1986.
2. Ronald de Haan and Jakub Szymanik. A dichotomy result for Ramsey quantifiers. In *Proceedings of the 22nd Workshop on Logic, Language, Information and Computation*, 2015.
3. Nina Gierasimczuk and Jakub Szymanik. Branching quantification vs. two-way quantification. *The Journal of Semantics*, 26(4):329–366, 2009.
4. Makoto Kanazawa. Monadic quantifiers recognized by deterministic pushdown automata. In F. Roelofsen M. Aloni, M. Franke, editor, *Proceedings of the 19th Amsterdam Colloquium*, pages 139–146, 2013.

5. Juha Kontinen. *Definability of second order generalized quantifiers*. PhD thesis, Helsinki University, 2004.
6. Juha Kontinen and Jakub Szymanik. A remark on collective quantification. *Journal of Logic, Language and Information*, 17(2):131–140, 2008.
7. Juha Kontinen and Jakub Szymanik. A characterization of definability of second-order generalized quantifiers with applications to non-definability. *Journal of Computer and System Sciences*, 80(6):1152 – 1162, 2014.
8. Corey T. McMillan, Robin Clark, Peachie Moore, Christian Devita, and Murray Grossman. Neural basis for generalized quantifier comprehension. *Neuropsychologia*, 43:1729–1737, 2005.
9. Marcin Mostowski. Computational semantics for monadic quantifiers. *Journal of Applied Non-Classical Logics*, 8:107–121, 1998.
10. Marcin Mostowski and Jakub Szymanik. Semantic bounds for everyday language. *Semiotica*, 188(1-4):363–372, 2012.
11. Stanley Peters and Dag Westerståhl. *Quantifiers in Language and Logic*. Clarendon Press, Oxford, 2006.
12. Shane Steinert-Threlkeld and III Icard, ThomasF. Iterating semantic automata. *Linguistics and Philosophy*, 36(2):151–173, 2013.
13. Jakub Szymanik. A comment on a neuroimaging study of natural language quantifier comprehension. *Neuropsychologia*, 45:2158–2160, 2007.
14. Jakub Szymanik. *Quantifiers in TIME and SPACE. Computational Complexity of Generalized Quantifiers in Natural Language*. PhD thesis, University of Amsterdam, Amsterdam, 2009.
15. Jakub Szymanik. Computational complexity of polyadic lifts of generalized quantifiers in natural language. *Linguistics and Philosophy*, 33(3):215–250, 2010.
16. Jakub Szymanik. *Quantifiers and Cognition*. Studies in Linguistics and Philosophy. Springer, 2015.
17. Jakub Szymanik and Marcin Zajenkowski. Comprehension of simple quantifiers. Empirical evaluation of a computational model. *Cognitive Science: A Multidisciplinary Journal*, 34(3):521–532, 2010.
18. Jakub Szymanik and Marcin Zajenkowski. Quantifiers and working memory. In M. Aloni and K. Schulz, editors, *Amsterdam Colloquium 2009, Lecture Notes In Artificial Intelligence 6042*, pages 456–464. Springer, 2010.
19. Camilo Thorne and Jakub Szymanik. Semantic complexity of quantifiers and their distribution in corpora. In *Proceedings of the International Conference on Computational Semantics*, 2015.
20. Marcin Zajenkowski, Rafał Styła, and Jakub Szymanik. A computational approach to quantifiers as an explanation for some language impairments in schizophrenia. *Journal of Communication Disorders*, 44(6):595 – 600, 2011.