SMT and POR beat Counter Abstraction Parameterized Model Checking of Threshold-Based Distributed Algorithms

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Why fault-tolerant (FT) distributed algorithms

faults not in the control of system designer

- bit-flips in memory
- power outage
- disconnection from the network
- intruders take control over some computers







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distributed algorithms to make systems more reliable even in the presence of faults

- replicate processes
- exchange messages
- do coordinated computation
- goal: keep replicated processes in "good state"







Fault-tolerant distributed algorithms



n processes communicate by messages

Fault-tolerant distributed algorithms



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- all processes know that at most t of them might be faulty

Fault-tolerant distributed algorithms



- n processes communicate by messages
- all processes know that at most t of them might be faulty
- *f* are actually faulty, e.g., Byzantine
- resilience condition, e.g., $n > 3t \land t \ge f \ge 0$
- no masquerading: the processes know the origin of incoming messages

Distributed algorithms: computational model and faults

The classic model by [Fischer, Lynch, Paterson'85]

Environment:

- Asynchronous processes (no rounds, non-deterministic fair scheduler)
- Reliable asynchronous message passing (non-blocking send and receive)

Faults:

- crashes and clean crashes,
- omission faults,
- symmetric faults,
- Byzantine faults

Reliable Broadcast by Srikanth & Toueg 85

if initiator then send INIT to all;

while true do
 if received INIT from at least 1 distinct proc.
 then send ECHO to all;

if received ECHO from at least t + 1 distinct proc. and not sent ECHO before then send ECHO to all;

if received ECHO from at least n-t distinct proc.
then accept;
od









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Unforgeability: If no correct process sends <INIT> (broadcasts), then no correct process ever accepts.

Verification perspective: check, whether a bad state is reachable.

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Threshold-based fault-tolerant distributed algorithms

- The parameters (n, t, f) are fixed in each run
- Main loop with the body executed atomically
- Processes are anonymous (no identifiers)

- Receiving messages, counting them and comparing to thresholds, e.g., if received <ECHO> from t + 1 distinct processes then ...
- Sending messages to all processes, e.g., send <ECHO> to all

Outline

- **1** Threshold automata (TA): formalization of process code using shared variables
- 2 Counter systems with acceleration: computational model for parameterized systems of TA
- **3** Parameterized reachability: safety properties stated formally
- 4 **Counter abstraction and acceleration:** other approaches

5 Representatives and schemas: parameterized bounded model checking with SMT

Preliminaries

Threshold automata (TA)

Every correct process follows the control flow graph (L, E):



Processes move from one location to another along the edges labeled with:

- Threshold guards, e.g., x ≥ (t + 1) − f compare a shared variable to a linear combination of parameters.
- Updates, e.g., x++

increment shared variables (or do nothing).

(multiple guards and increments are allowed)

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Intuition: threshold automata and threshold-based DAs?



Crash faults:

run *n* processes,

$$\cdots \longrightarrow \ell_i \qquad \qquad \mathsf{nfaulty} < f, \ \mathsf{nfaulty^{++}} \land \ell_c \text{ crashed here}$$

Byzantine faults:

run n - f processes, count messages modulo Byzantine processes, e.g., $x + f \ge (t + 1)$

Warning:

This requires preliminary abstraction of message counters [FMCAD'13] 22

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Natural Restrictions of TA

Recall how processes count messages:

if received <ECHO> from t + 1 distinct processes

The case studies lead us to the natural restrictions on threshold automata:

Restriction 1: Every process changes a shared variable at most once

Restriction 2: The edges in cycles do not change the shared variables

Counter system with acceleration!

Counter system is a transition system simulating every system $P(\mathbf{p})^{N(\mathbf{p})}$.

Configuration $\sigma = (\kappa, \mathbf{g}, \mathbf{p})$:

- κ_i counts processes at location ℓ_i with $\kappa_1 + \cdots + \kappa_{|L|} = N(\mathbf{p})$,
- **g**_j is the value of the shared variable x_j ,
- **p** are the values of the parameters.

$$x \ge (n-t) - f \mapsto x \leftrightarrow t$$

$$x \ge (t+1) - f \mapsto x \leftrightarrow t$$

$$(\ell_2)$$

$$true \mapsto x \leftrightarrow t$$

$$x \ge (n-t) - f$$

one transition r^1 (interleaving): accelerated transition r^3 :

 $\begin{array}{c} x \ge (n-t) - f \\ \sigma_1 \quad \kappa_1 \ge 1 \quad \sigma_2 \\ \bigcirc \\ \kappa_1 \stackrel{--}{\longrightarrow} \quad \kappa_4 \stackrel{++}{\longrightarrow} \quad x \stackrel{++}{\longrightarrow} \end{array}$

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Reachability and parameterized reachability

Reachability (fixed parameters):

Fix the parameters, e.g., n = 4, t = 1, f = 1, N = n - f = 3. Fix configurations σ and σ' of P^N .

Question: is σ' reachable from σ in P^N ?

Parameterized reachability:

Fix properties S and S' on configurations, e.g., $S : \kappa_1 = N(\mathbf{p}) = n - f$ and $S' : \kappa_4 \neq 0$.

Question: are there parameter values **p** and configurations σ , σ' of $P^{N(\mathbf{p})}$:

■ parameters **p** satisfy the resilience condition *RC*(**p**),

•
$$\sigma \models S$$
 and $\sigma' \models S'$,

• σ' is reachable from σ in $P^{N(\mathbf{p})}$.

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Resilience condition 1: n > 3t and $t \ge f \ge 0$.

Can the faulty processes forge the broadcast by a correct process?

that is, can correct processes reach ℓ_4 , if they start at ℓ_1 ? NO

(t+1) - f > 0 = x $(n-t) - f \ge n - t - t > t \ge 0 = x$

$$(l_1) \xrightarrow{x \ge (n-t) - f \mapsto x + t} (l_2) \xrightarrow{x \ge (t+1) - f \mapsto x + t} (l_3) \xrightarrow{x \ge (n-t) - f} (n-t) - f$$

Resilience condition 1: n > 3t and $t \ge f \ge 0$.

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$$(t+1) - f > 0 = x$$

 $(n-t) - f \ge n - t - t > t \ge 0 = x$



Resilience condition 2: n > 3t and $t + 1 \ge f \ge 0$.

Can the faulty processes forge the broadcast by a correct process? that is, can correct processes reach ℓ_4 , if they start at ℓ_1 ? YES



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Resilience condition 2: n > 3t and $t + 1 \ge f \ge 0$.

Can the faulty processes forge the broadcast by a correct process? that is, can correct processes reach ℓ_4 , if they start at ℓ_1 ? YES



Parameterized reachability:

counter abstraction and acceleration

Way 1: Counter abstraction

Use counter abstraction to get a finite system \mathcal{A} .

Counters κ_i are mapped to a finite domain \widehat{D} , e.g.,

- $\{0, 1, \infty\}$ by [Pnueli, Xu, Zuck'02].
- Domain of parametric intervals extracted from thresholds, e.g., $\{[0,1), [1,t+1), [t+1,n-t), [n-t,\infty)\}$, see [FMCAD'13].



Use a finite-state model checker, e.g., NuSMV or Spin

Warning:

Sometimes, abstraction refinement is needed [FMCAD'13]

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Bounded diameter

Fix a threshold automaton TA and a size function N.

Theorem [CONCUR'14]

For each **p** with $RC(\mathbf{p})$, the diameter of an accelerated counter system is independent of parameters and is less than or equal to $|E| \cdot (|C| + 1) + |C|$:

- |E| is the number of edges in TA (self-loops excluded).
- |C| is the number of edge conditions in TA that can be unlocked (locked) by an edge appearing later (resp. earlier) in the control flow, or by a parallel edge.



Bounded diameter

Fix a threshold automaton TA and a size function N.

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Way 2: Complete parameterized bounded model checking

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Once we know the diameter d of the accelerated counter system,

we know the diameter of the abstract system:

$$\mathit{diam}(\mathcal{A}) \leq \mathit{d} \cdot (|\widehat{D}| - 1)$$

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Way 3: Acceleration Techniques of Counter Systems

Threshold automata are a special case of counter automata.

Apply symbolic acceleration techniques for counter automata, e.g., FAST [Bardin, Finkel, Leroux et al.'08].

The diameter bound implies that the threshold automata are flattable



Thus, FAST always terminates on threshold automata (in theory)

Accelerated systems: partial order reduction and SMT

Partial orders and SMT beat counter abstraction



Partial orders and SMT beat counter abstraction (2)



Our new solution consists of the key ingredients:

Contexts: In every execution, evaluation of a guard changes at most once

e.g., $x \ge t + 1 - f$ is initially false and later turns to true. A *context* keeps track of all unlocked guards.

Representatives: As before, transform every execution to a representative by reordering and accelerating the rules with the same context.

the schedule $r_1^1 r_2^1 r_1^1 r_2^1 r_2^1$ becomes $r_1^2 r_2^3$.

Schemas: Representatives are generated by schemas.

e.g., r_1r_2 generates schedule $r_1^2r_2^3$ by picking acceleration factors 2 and 3.

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Contexts and representatives

Contexts



 Φ is the set of all threshold guards of TA, e.g., $\Phi = \{\varphi_1, \varphi_2\}$

A subset $\Omega \subseteq \Phi$ is a **context**, e.g., \emptyset , $\{\varphi_1\}$, and $\{\varphi_1, \varphi_2\}$ are contexts

Contexts and executions



Every execution defines a **monotonically increasing** sequence of contexts: e.g., for a configuration σ with n = 5, t = 1, f = 1 and $\kappa_1 = 1, \kappa_2 = 3$

Transitions $r_1^1, r_1^1, r_2^1, r_1^1, r_4^1$ applied to σ define the sequence of contexts $\emptyset \subset \{\varphi_1\} \subset \{\varphi_1, \varphi_2\}.$

Or, annotated, {} $r_1^1 \{\varphi_1\} r_1^1, r_2^1, r_1^1 \{\varphi_1, \varphi_2\} r_4^1 \{\varphi_1, \varphi_2\}$

Constructing short representatives



 $\varphi_1 \equiv x \ge t+1, \quad \varphi_2 \equiv x \ge n-t$

{} $r_1^1 \{\varphi_1\} r_1^1, r_2^1, r_1^1 \{\varphi_1, \varphi_2\} r_4^1 \{\varphi_1, \varphi_2\}$

the transitions with the same context are **sorted**, e.g., if $r_1 \leq^{lin} r_2 \leq^{lin} r_4$: {} $r_1^1 \{\varphi_1\} r_1^1, r_1^1, r_2^1 \{\varphi_1, \varphi_2\} r_4^1 \{\varphi_1, \varphi_2\}$

and the instances of the same rule are accelerated:

$$\{\} r_1^1 \{\varphi_1\} r_1^2, r_2^1 \{\varphi_1, \varphi_2\} r_4^1 \{\varphi_1, \varphi_2\}$$

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Formal result on representatives

By applying sorting and acceleration, we prove:

Proposition 9 [CAV'15]

Given a threshold automaton, a configuration σ , and schedule τ applicable to σ , there exists a schedule rep $[\sigma, \tau]$ with the following properties:

1 rep $[\sigma, \tau]$ is applicable to σ , and rep $[\sigma, \tau](\sigma) = \tau(\sigma)$,

2
$$|\operatorname{rep}[\sigma, \tau]| \leq 2 \cdot |\mathcal{R}| \cdot (|\Phi| + 1) + |\Phi|.$$

where

- **\square** \mathcal{R} is the set of rules (edges of TA),
- Φ is the set of all threshold guards used in \mathcal{R} .

(the new ingredient)

What can we do with the representatives?



To check reachability, we have to explore all the representatives.

For a monotonically increasing sequence of contexts, e.g., \emptyset , { φ_1 }, { φ_1 , φ_2 }

all representatives follow the same pattern:

{}
$$r_1 \{\varphi_1\} r_1, r_2 \{\varphi_1, \varphi_2\} r_1, r_2, r_3, r_4 \{\varphi_1, \varphi_2\}$$

A schema is a sequence of contexts and rule sequences:

$$\mathcal{S} = \{\Omega_0\}\rho_1\{\Omega_1\}\dots\{\Omega_{m-1}\}\rho_m\{\Omega_m\}$$

A schema generates paths (including the representatives):

```
e.g., {} r_1 {\varphi_1} r_1, r_3, r_4 {\varphi_1, \varphi_2}
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How to find a feasible path that reaches a bad state?

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How to find a feasible path that reaches a bad state?

Checking feasibility with SMT

It is easy to check with SMT, whether a schema generates a feasible path:

e.g., {} r_1 { φ_1 } r_2 { φ_1, φ_2 } r_4 { φ_1, φ_2 }

Complete parameterized reachability checking

Sound and complete algorithm for parameterized reachability in TA:

For each monotonically increasing sequence Ω of contexts: construct a schema S for Ω if there is a path π generated by S that reaches a bad state, then report π as a counterexample

Theorem 1 [CAV'15]

For a threshold automaton, there is a complete schema set of cardinality at most $|\Phi|!$, where the length of each schema does not exceed $(3 \cdot |\Phi| + 2) \cdot |\mathcal{R}|$.

Note:

This result also holds for the guards like n faulty < f

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Results

Now we can verify **safety** of the parameterized algorithms:

- Reliable broadcast (FRB, STRB, ABA)
- Non-blocking atomic commit with failure detectors (NBAC, NBACG)
- Condition-based consensus (CBC)
- One-step consensus (CF1S, C1CS, BOSCO)



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Liveness?

"...when looking for errors, most of your effort should be devoted to examining the safety part." Leslie Lamport. Specifying Systems (2002) "Liveness is whatever prevents an empty system from being correct." Orna Kupferman. Beyond Safety Workshop (2004) 61/64

Conclusions

Standard model checkers are not tuned to the computational models of fault-tolerant distributed algorithms

Computational primitives in FTDAs are simpler than the standard ones

This and parameterization helped us to develop efficient techniques



check FTDAs used in the cloud: variations of Paxos, RAFT, etc.?

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Thank you! [http://forsyte.at/software/bymc]

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