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From DRUP to PAC and Back

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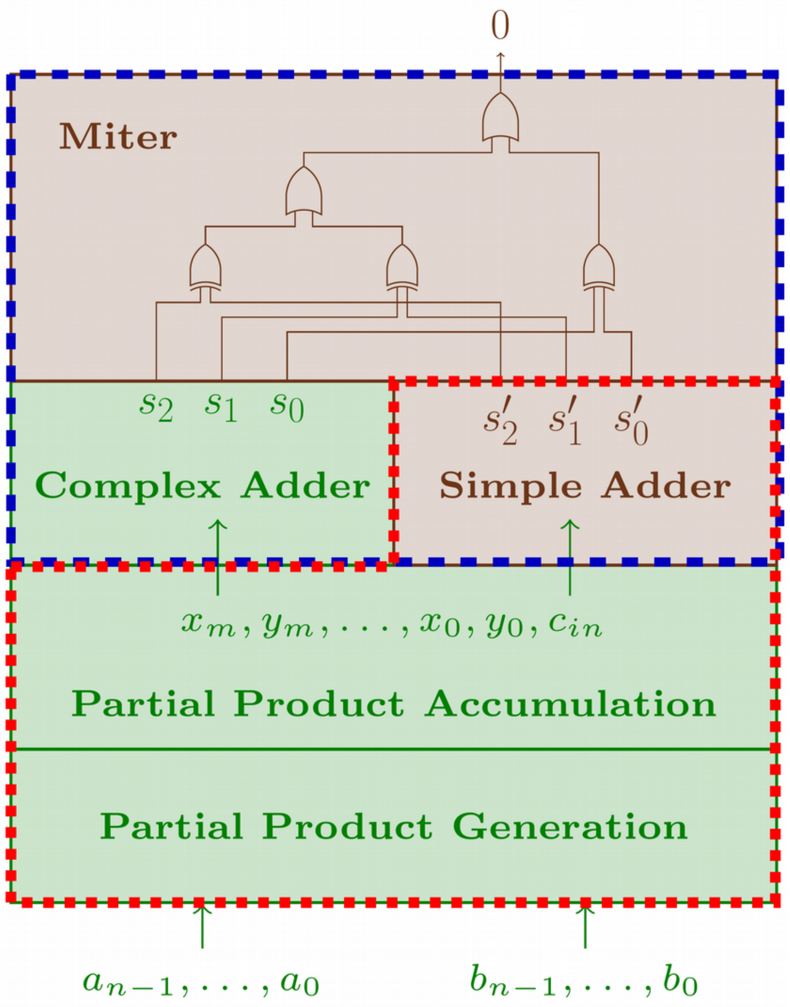
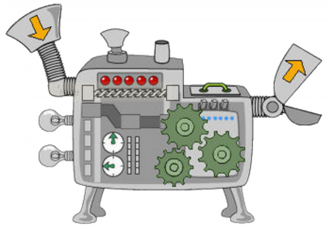
Motivation





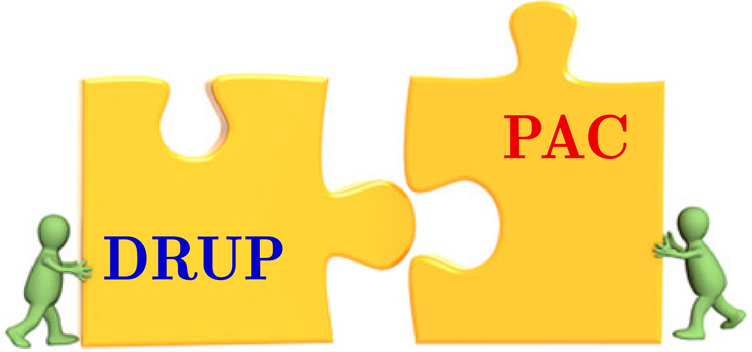
SAT

Computer Algebra

DRUP

Practical Algebraic Calculus

?



Computer Algebra & PAC

- Let $f \in \mathbb{Z}[X]$ and $P \subseteq \mathbb{Z}[X]$. We are interested whether the polynomial equation $f = 0$ is implied by the equations $p = 0$ with $p \in P$, i.e., decide $f \in \langle P \rangle$.
- All variables $x \in X$ represent logic gates and thus take only values in $\{0, 1\}$. This is enforced by *Boolean value constraints*. Let $B(X) = \{x(1-x) \mid x \in X\} \subseteq \mathbb{Z}[X]$ be the set of Boolean value constraints for X.
- PAC proofs are sequences of proof rules, which model the ideal properties:

$+ : p_i,\ p_j,\ p_i + p_j;$	p_i, p_j appearing earlier in the proof or are contained in constraint set P and $p_i + p_j$ being reduced by $B(X)$
$* : p_i,\ q,\ qp_i;$	p_i appearing earlier in proof or in P and $q \in \mathbb{Z}[X]$ being arbitrary and qp_i being reduced by $B(X)$

Example: Let $P = \{-x + 3z, 2xz\} \subseteq \mathbb{Z}[x,y,z]$ and let $f = -2x \in \mathbb{Z}[x,y,z]$. The proof shows $f \in \langle P \cup B(X) \rangle$:

```
* :   -x+3z,   2x, -2x+6xz;
* :     2xz,   -3, -6xz;
+ : -2x+6xz, -6xz, -2x;
```

SAT & DRUP

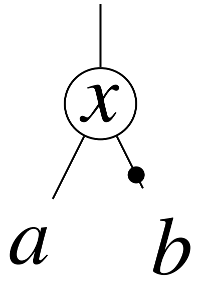
- The SAT problem seeks for an assignment such that a formula F in conjunctive normal form evaluates to **true**. If no satisfying assignment can be found it is *unsatisfiable*.
- The most basic clausal proof format is *reverse unit propagation* (RUP).
  Let $\overline{C}$ denote the negation of a clause C. We say C is a *RUP clause* if $F \wedge \overline{C}$ evaluates to **false**. A RUP proof is a sequence of RUP clauses containing the empty clause. DRUP extends RUP by adding deletion information.

Example: This is an unsatisfiable CNF in DIMACS format (left) with DRUP (middle) and TraceCheck (right) proofs.

```
p cnf 3 5         -2 0         1  1 -2 -3 0 0
 1 -2 -3  0     d  3 0         2  1  2  0 0
 1  2      0     d  1 -2 -3 0  3 -1 -2  0 0
-1 -2      0     d -1 -2 0     4 -1  2  0 0
-1  2      0         0         5  3  0  0
       3  0                    6 -2  0 3 1 5 0
                               7  0  4 2 6 0
```

From DRUP to PAC

1. Polynomial encoding of CNF



	Propostional Formula	Polynomial Relation
	$(x \leftrightarrow a \wedge \overline{b}) = \top$	$-x + a(1-b) = 0$
CNF	$(x \vee \overline{a} \vee b) = \top$	$(1-x)a(1-b) = 0$
	$(\overline{x} \vee a) = \top$	$x(1-a) = 0$
	$(\overline{x} \vee \overline{b}) = \top$	$xb = 0$

Using the fact that $x^2 - x = 0,\ b^2 - b = 0$ and $a^2 - a = 0$ we multiply the polynomial equation $-x + a(1-b)$ by different factors to derive the desired polynomials.

$$\begin{aligned}
0 = &\quad (-x + a(1-b))(-ba + a) &&= (1-x)a(1-b) \\
0 = &\quad (-x + a(1-b))(b-1) &&= x(1-a) \\
0 = &\quad (-x + a(1-b))(-a) &&= xb
\end{aligned}$$

From DRUP to PAC

2. Encoding of resolution steps

- Add bit-flipping polynomials (similar to “Polynomial Calculus with Resolution”):
  Clause $x \vee y \vee z$ can be translated to $(1-x)(1-y)(1-z) = 0$, which generates $2^3 - 1$ monomials.
  If we introduce $-f_x + 1 - x = 0$, $-f_y + 1 - y = 0$, $-f_z + 1 - z = 0$, the same equation can be depicted as $f_x f_y f_z = 0$.

- Encode resolution using the traces provided in TraceCheck:

```
1  1 -2 -3 0 0
2  1  2  0 0
3 -1 -2  0 0
4 -1  2  0 0
5  3  0  0
6 -2  0 3 1 5 0
7  0  4 2 6 0
```

Let $a = $ `1`, $b = $ `2` and $c = $ `3`.
We encode the first resolution step of rule 6 (resolving clause 3 and 1).
Thus from $a \vee \overline{b} \vee \overline{c}$ and $\overline{a} \vee \overline{b}$ we resolve the clause $\overline{b} \vee \overline{c}$.

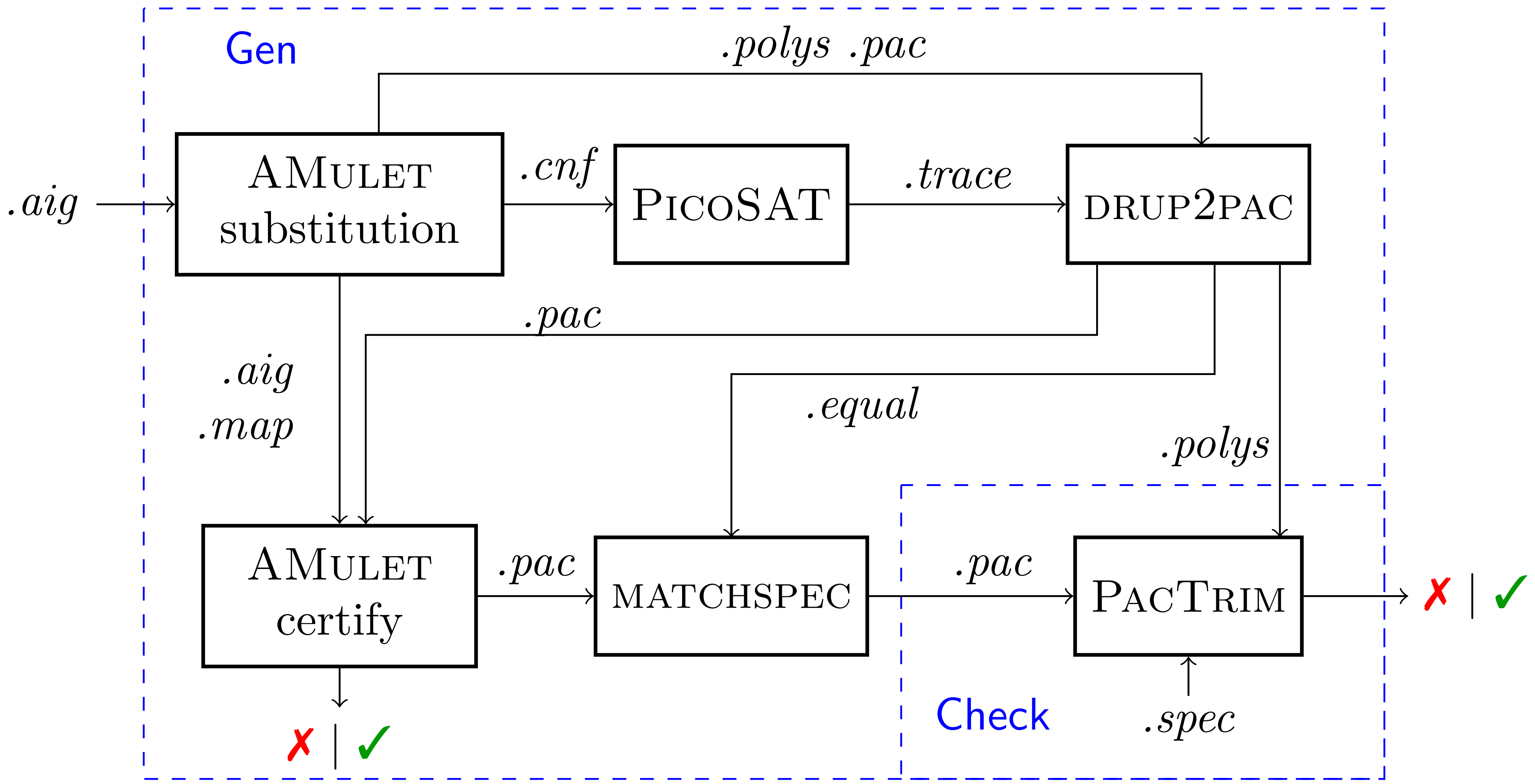
The corresponding PAC encoding is:

```
* :       b*a,     c,  c*b*a;
+ : -c*b*a+c*b, c*b*a,    c*b;
```

3. Merge PAC proofs

PAC proofs can be merged by combining constraint sets and proof rules.

From DRUP to PAC



From PAC to DRUP

1. **SMT encoding**

- The polynomial proof is translated into a bit-vector proof.
- To this end we encode the PAC proof as an SMT problem over the theory of quantifier free fixed size bit vectors.
- Each PAC rule is individually translated to SMT.
- Each variable in the PAC proof represents the input or output of a gate As a consequence we encode each variable in the PAC proof as a single bit and the coefficients are encoded as bit vectors.
- **Gap:** It is not checked that the specification is derived at the end.

From PAC to DRUP

1. Encode as SMT formula

Consider the following PAC rule

$$+ : 3x - z,\ 2y - 3x,\ 2y - z;$$

Checking the correctness of this rule can be encoded as:

```
(set-logic QF_BV)
(declare-fun x () (_ BitVec 1))
(declare-fun y () (_ BitVec 1))
(declare-fun z () (_ BitVec 1))
(assert
(let (($v0 (bvadd (bvand #b011 ((_ sign_extend 2) x))
      (bvand #b111 ((_ sign_extend 2) z)))))
(let (($w0 (bvadd (bvand #b010 ((_ sign_extend 2) y))
      (bvand #b101 ((_ sign_extend 2) x)))))
(let (($p0 (bvadd (bvand #b010 ((_ sign_extend 2) y))
      (bvand #b111 ((_ sign_extend 2) z)))))
(let (($e0 (= (bvadd $v0 $w0)  $p0)))

(not $e0))))))
(check-sat)
```

From PAC to DRUP

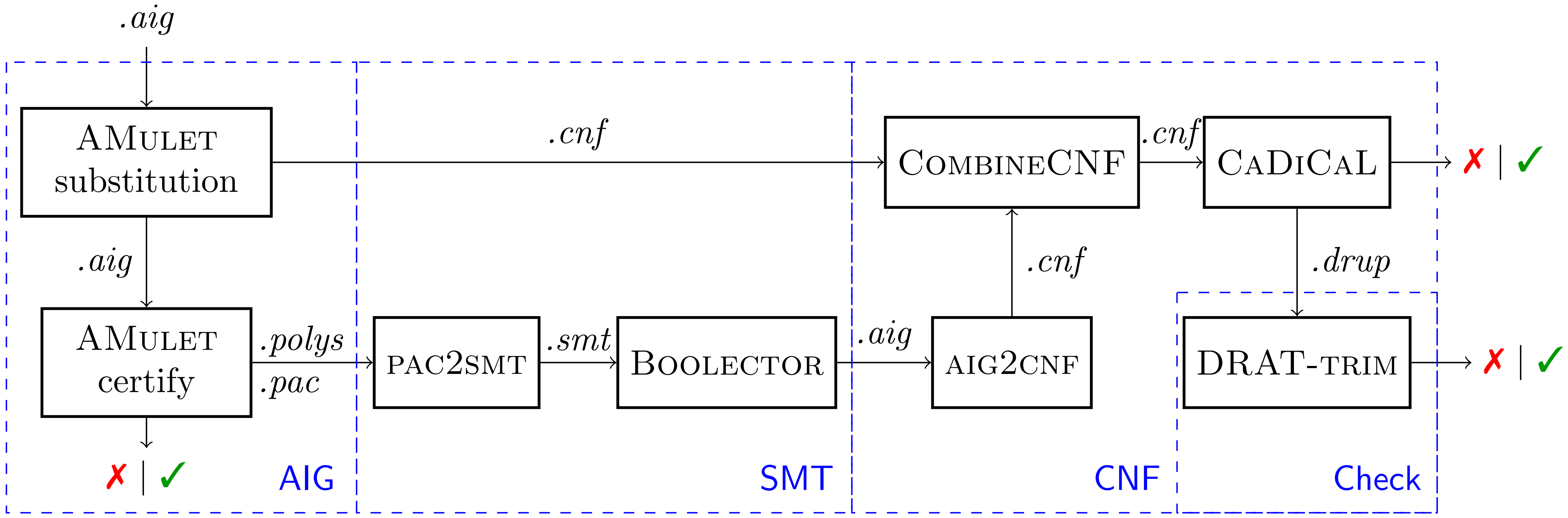
2. SMT to CNF

- SMT encoding is given to SMT solver Boolector.
- **Gap:** Internal rewriting steps are not covered.
- Boolector generates an And-Inverter-Graph that is translated into CNF.

3. Merge proofs

- Collect all clauses of both CNFs, except for output assumptions.
- The two output clauses $C_0 = l_0, C_1 = l_1$ are merged into the clause $l_0 \vee l_1$.
- Merged CNF is solved using SAT solver and DRUP proof is generated.

From PAC to DRUP



Experimental Results

architecture	n	Separate Proofs							PAC				DRUP					
		DRUP			PAC			total										
		gen	check	size	gen	check	size		gen	check	total	size	aig	smt	cnf	check	total	size
sp-ar-cl	16	0	0	1 299	0	0	7 962	0	2	2	3	185 588	0	7	300	264	570	19 317 884
sp-dt-lf	16	0	0	1 167	0	0	7 787	0	1	1	2	136 349	0	6	279	277	562	18 153 668
bp-ct-bk	16	0	0	1 029	0	0	7 205	0	1	1	2	128 720	0	7	TO	-	-	-
bp-wt-cl	16	0	0	2 902	0	0	7 946	0	30	11	41	614 742	0	7	TO	-	-	-
sp-ar-cl	32	0	0	14 927	0	1	33 834	1	133	31	164	1 597 897	0	56	TO	-	-	-
sp-dt-lf	32	0	0	3 138	0	1	33 451	1	2	3	5	321 720	0	52	TO	-	-	-
bp-ct-bk	32	0	0	2 276	0	1	27 312	1	1	2	3	217 128	0	49	TO	-	-	-
bp-wt-cl	32	1	1	46 502	0	1	30 561	2	3 133	242	3 375	5 536 176	0	55	TO	-	-	-

PPG: simple (sp), Booth (bp)

PPA: array (ar), Dadda tree (dt), compressor tree (ct), Wallace tree (wt)

FSA: carry look-ahead (cl), Ladner-Fischer (lf), Brent-Kung (bk)

TO = 3600 sec

Conclusion

From DRUP to PAC:

- requires algebraic reasoning
- include bit-flipping techniques to reduce size
- use TraceCheck format

From PAC to DRUP:

- encode PAC proof as an SMT problem
- translated into CNF using bit-blasting
- leaves gaps in the proof

Single DRUP proofs are three orders of magnitude larger than PAC proofs and contain gaps.

From DRUP to PAC and Back

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