

BIG BACKBONES

Improving Anytime CadiBack



Nils Frolleyks, Emily Yu, Armin Biere

Backbone

The backbone of a satisfiable formula is the set of literals that are true in all satisfying assignments.

$$\begin{aligned} & a \quad \wedge \quad \bar{a} \vee b \quad \wedge \quad \bar{a} \vee \bar{b} \vee c \quad \wedge \\ & d \vee e \vee f \quad \wedge \quad \bar{d} \vee e \vee f \quad \wedge \quad d \vee \bar{e} \vee f \quad \wedge \quad \bar{d} \vee \bar{e} \vee f \quad \wedge \\ & \text{PHP} \vee g \end{aligned}$$

CadiBack

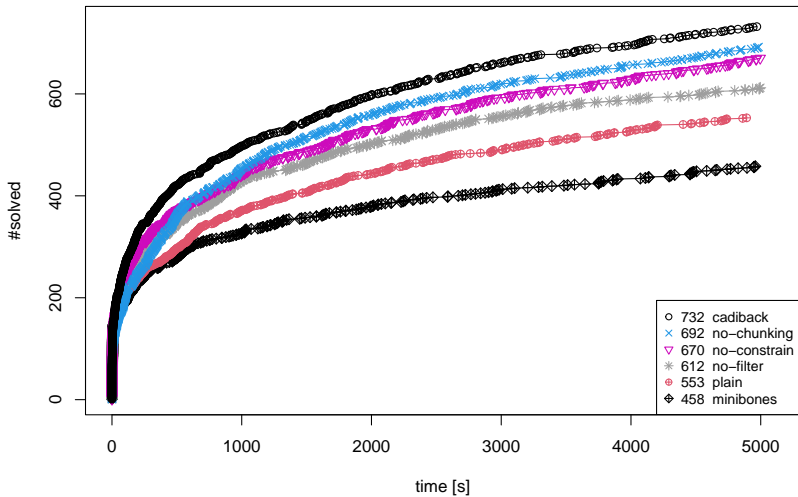
backbone (CNF φ)

```
1  ( $res, \sigma$ )  $\leftarrow$  SAT( $\varphi$ )
2   $\mathcal{B} \leftarrow \emptyset, \quad \Lambda \leftarrow \sigma$ 
3  while  $\Lambda \neq \emptyset$ 
4       $\ell \leftarrow$  pick literal from  $\Lambda$ 
5      ( $res, \sigma$ )  $\leftarrow$  SAT( $\varphi \mid \neg\ell$ )
6      if  $res$  then // satisfiable
7           $\Lambda \leftarrow \Lambda \setminus \{\ell\}$ 
8      else // unsatisfiable
9           $\mathcal{B} \leftarrow \mathcal{B} \cup \{\ell\} \quad \varphi = \varphi \wedge \ell$ 
10          $\Lambda \leftarrow \Lambda \setminus \{\ell\}$ 
11 return  $\mathcal{B}$ 
```

CadiBack

backbone (CNF φ)

```
1   $(res, \sigma) \leftarrow \text{SAT}(\varphi)$ 
2   $\mathcal{B} \leftarrow \emptyset, \quad \Lambda \leftarrow \sigma, \quad k \leftarrow 1$ 
3  while  $\Lambda \neq \emptyset$ 
4       $\Gamma \leftarrow$  pick  $k$  literals from  $\Lambda$  // chunk
5       $(res, \sigma) \leftarrow \text{SAT}(\varphi \mid \neg \ell \bigvee_{\ell \in \Gamma} \neg \ell)$ 
6      if  $res$  then // satisfiable
7           $\Lambda \leftarrow \Lambda \setminus \{\ell\} \wedge \Lambda \cap \sigma$ 
8      else // unsatisfiable
9           $\mathcal{B} \leftarrow \mathcal{B} \cup \{\ell\} \wedge \Gamma \quad \varphi = \varphi \wedge \ell$ 
10          $\Lambda \leftarrow \Lambda \setminus \{\ell\} \wedge \Gamma$ 
11          $k \leftarrow |\Lambda|$ 
12 return  $\mathcal{B}$ 
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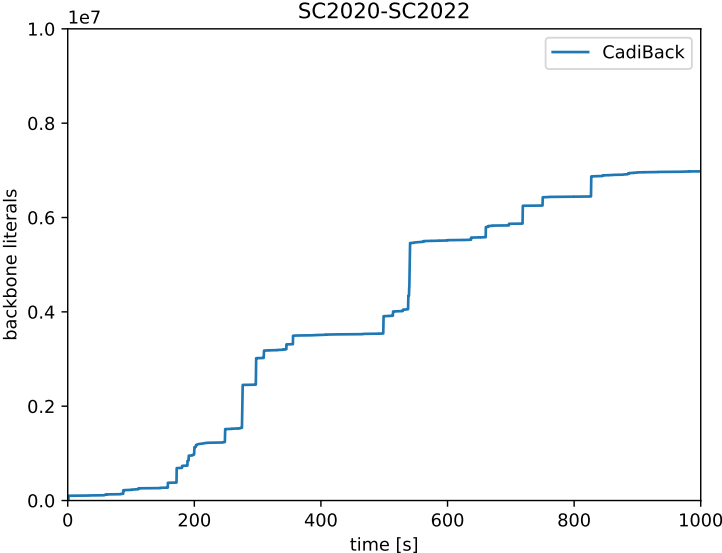


1798 satisfiable instances from SAT competitions 2004 – 2022

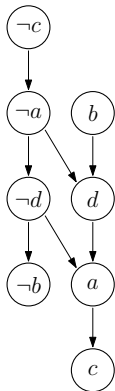
“How quickly do you find the majority of these backbones. I would like to use this as preprocessing for model counting.”

- Johannes Fichte
A bit after this paper was submitted

Backbone literals found over time



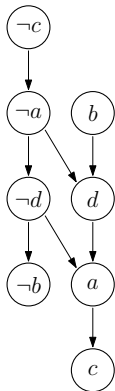
Focusing on the BIG



Binary Implication Graph

- $V = \mathcal{L}$
- $E = \{(\neg u, v), (\neg v, u) \mid (u, v) \in \mathcal{F}\}$
- Symmetric (Contraposition)

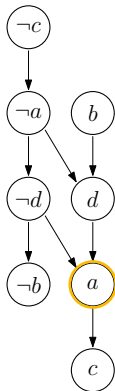
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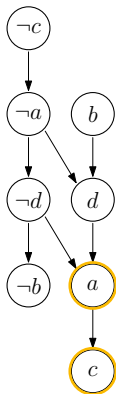
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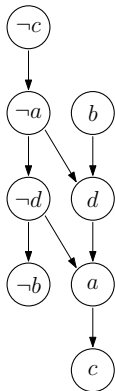
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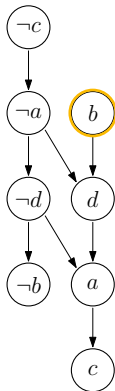
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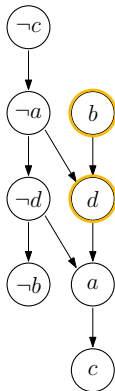
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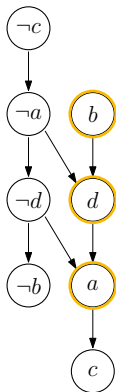
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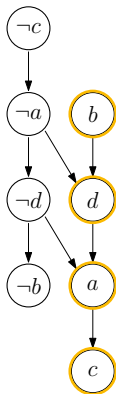
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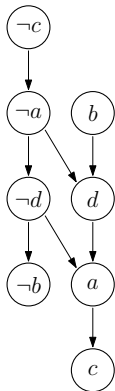
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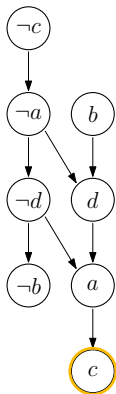
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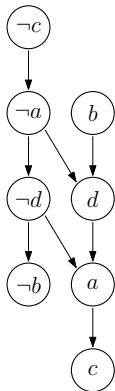
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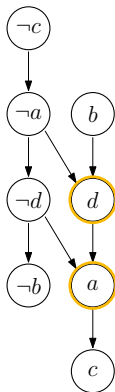
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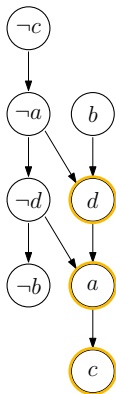

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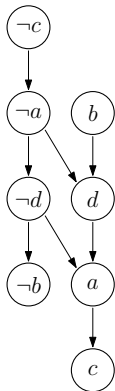
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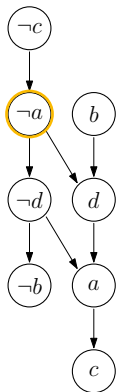
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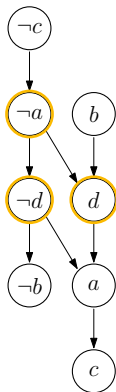
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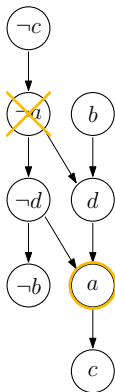
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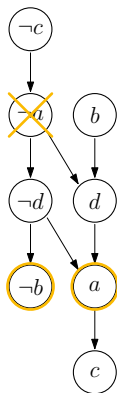
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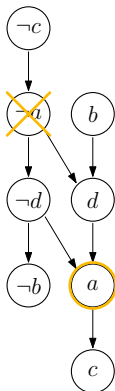
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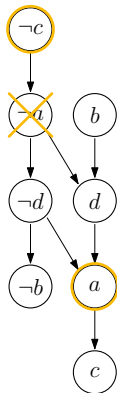
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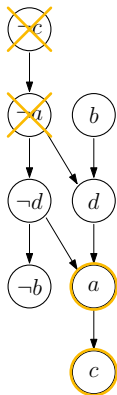
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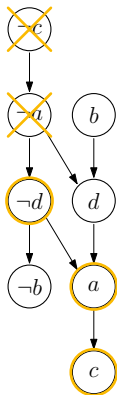
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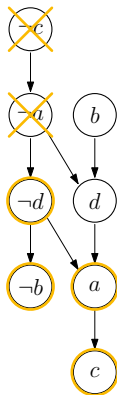
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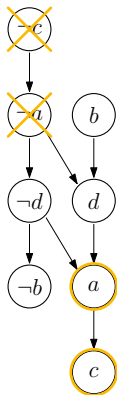
Focusing on the BIG



$$\Lambda = \begin{array}{cccc} a & b & c & d \\ \neg a & \neg b & \neg c & \neg d \end{array}$$

- 1 $\mathcal{B} \leftarrow \emptyset, \quad \Lambda \leftarrow \mathcal{L}$
- 2 **while** $\Lambda \neq \emptyset$
- 3 $\ell \leftarrow$ pick literal from Λ
- 4 $\sigma \leftarrow \{k \mid \mathcal{F}_{|\ell} \vdash_1 k\}$
- 5 **if** $\ell \notin \sigma$ **then** // satisfiable
- 6 $\Lambda \leftarrow \Lambda \setminus \{\ell\}$
- 7 **else** // unsatisfiable
- 8 $\mathcal{B} \leftarrow \mathcal{B} \cup \{\ell\} \quad \varphi = \varphi \wedge \ell$
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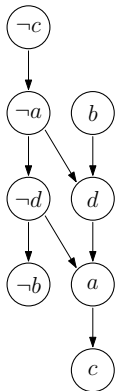
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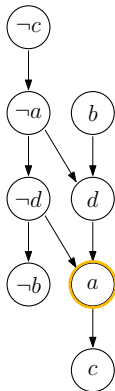
Keep Assignment Big Backbone



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14    $\Lambda \leftarrow \Lambda \setminus \Delta$ 
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Keep Assignment Big Backbone

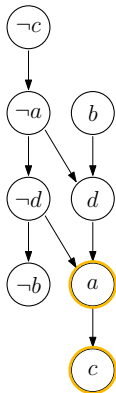


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Keep Assignment Big Backbone



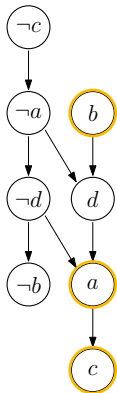
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Keep Assignment Big Backbone

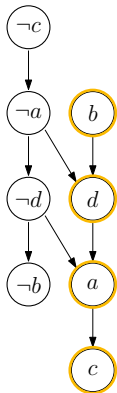


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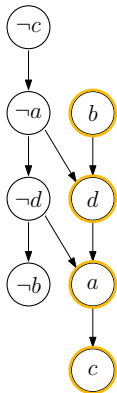


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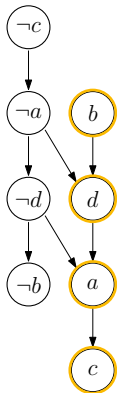
Keep Assignment Big Backbone



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Keep Assignment Big Backbone



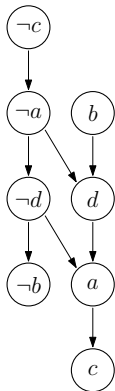
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Keep Assignment Big Backbone



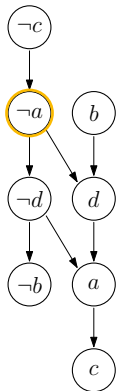
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Keep Assignment Big Backbone

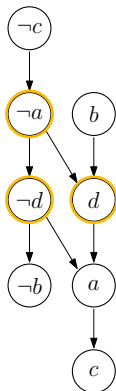


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Keep Assignment Big Backbone



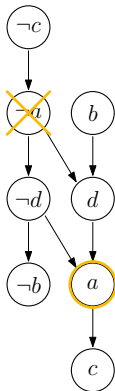
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Keep Assignment Big Backbone

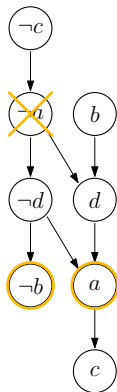


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Keep Assignment Big Backbone

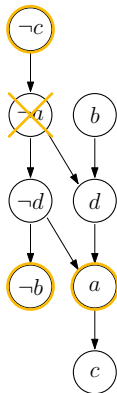


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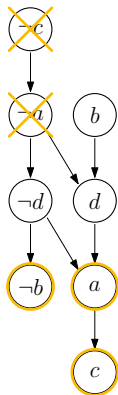
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Keep Assignment Big Backbone

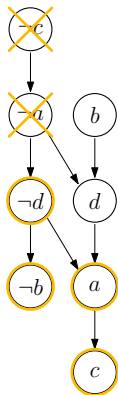


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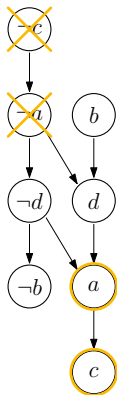


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Only Sound on 2-CNF

$(\neg a \vee \neg b \vee \neg c) \wedge (\neg a \vee \neg b \vee c)$

$\Lambda = a, b, \dots$

Models

$(\neg a \wedge b \wedge c), (a \wedge \neg b \vee \neg c), \dots$

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Models

$$(\neg a \wedge b \wedge c), (a \wedge \neg b \vee \neg c), \dots$$

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Only Sound on 2-CNF

$$(\neg a \vee \neg b \vee \neg c) \wedge (\neg a \vee \neg b \vee c)$$

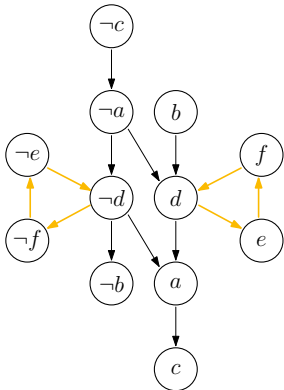
$$\Lambda = a, b, \dots$$

Models

$$(\neg a \wedge b \wedge c), (a \wedge \neg b \vee \neg c), \dots$$

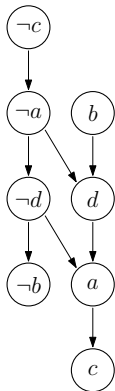
```
1  $\mathcal{B} \leftarrow \emptyset, \quad \Lambda \leftarrow \mathcal{L}$ 
2 while  $\Lambda \neq \emptyset$ 
3    $\sigma \leftarrow \mathcal{B}, \quad \Delta \leftarrow \emptyset$ 
4   for  $\ell \in \Lambda$  // next candidate
5     if  $\neg \ell \in \sigma$  continue
6      $\Delta \leftarrow \Delta \cup \{\ell\}$ 
7     if  $\ell \in \sigma$  continue
8      $\sigma' \leftarrow \{k \mid \mathcal{F}_{|\sigma \wedge \ell} \vdash_1 k\}$ 
9     if  $\ell \in \sigma'$  then
10        $\mathcal{B} \leftarrow \mathcal{B} \cup \{\neg \ell\}$ 
11        $\Delta \leftarrow \Delta \cup \mathcal{B} \cup \neg \mathcal{B}$ 
12        $\sigma \leftarrow \sigma \cup \mathcal{B}$ 
13     else  $\sigma \leftarrow \sigma'$ 
14    $\Lambda \leftarrow \Lambda \setminus \Delta$ 
15 return  $\mathcal{B}$ 
```

Equivalent Literal Substitution



```
1  $\mathcal{B} \leftarrow \emptyset, \quad \Lambda \leftarrow \mathcal{L}$ 
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15 return  $\mathcal{B}$ 
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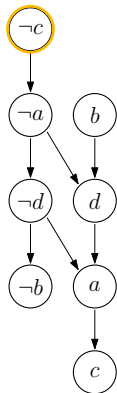
Roots Only



$$\Lambda = \neg c, b$$

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1  $\mathcal{B} \leftarrow \emptyset, \quad \Lambda \leftarrow \mathcal{L}$ 
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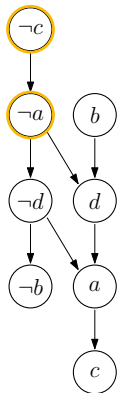
Search Order



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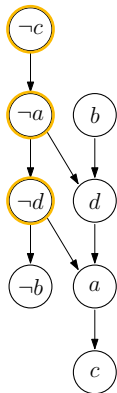
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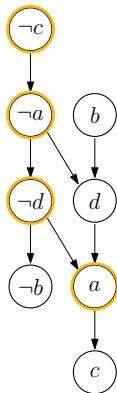

Search Order



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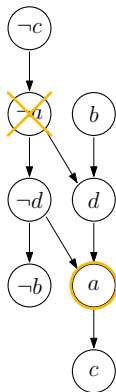
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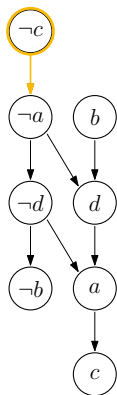
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15 return  $\mathcal{B}$ 
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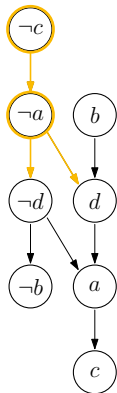
Unique Implication Point



$$\Lambda = \neg c, b$$

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1  $\mathcal{B} \leftarrow \emptyset, \quad \Lambda \leftarrow \mathcal{L}$ 
2 while  $\Lambda \neq \emptyset$ 
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5     if  $\neg \ell \in \sigma$  continue
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9     if  $\ell \in \sigma'$  then
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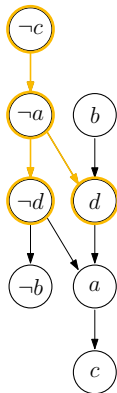
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6      $\Delta \leftarrow \Delta \cup \{l\}$ 
7     if  $l \in \sigma$  continue
8      $\sigma' \leftarrow \{k \mid \mathcal{F}_{|\sigma \wedge l} \vdash_1 k\}$ 
9     if  $\not\exists \ell \in \sigma'$  then
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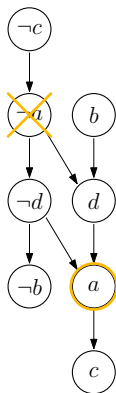
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15 return  $\mathcal{B}$ 
```

	backbone		KB3	
	BFS	DFS	BFS	DFS
Base	21 136	21 287	648	728
ELS	20 524	20 757	640	733
ELS+Roots	18 164	18 756	644	721
ELS+Roots+UIP			822	

- Total time in seconds on 1798 instances
- ELS takes 51 seconds

Anytime advantage with KB3

