Satisfiability Modulo Theories and Z3

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SMT : Basic Architecture

- Equality + UF
- Arithmetic
- Bit-vector
- ...

Ca

Nikolaj is Sober ∨
Nikolaj is Somber ∨
(Nikolaj is Drunk ∧
Nikolaj is Happy)

Theory[Alcohol]:
Sober ⊗ Drunk

Theory[Moodswings]:
Somber ⊗ Happy

Plan

Mon An invitation to SMT with Z3

Tue Equalities and Theory Combination

Wed Theories: Arithmetic, Arrays, Data types

Thu Quantifiers and Theories

Fri Programming Z3: Interfacing and Solving

Part 1

I. Satisfiability Modulo Theories in a nutshell

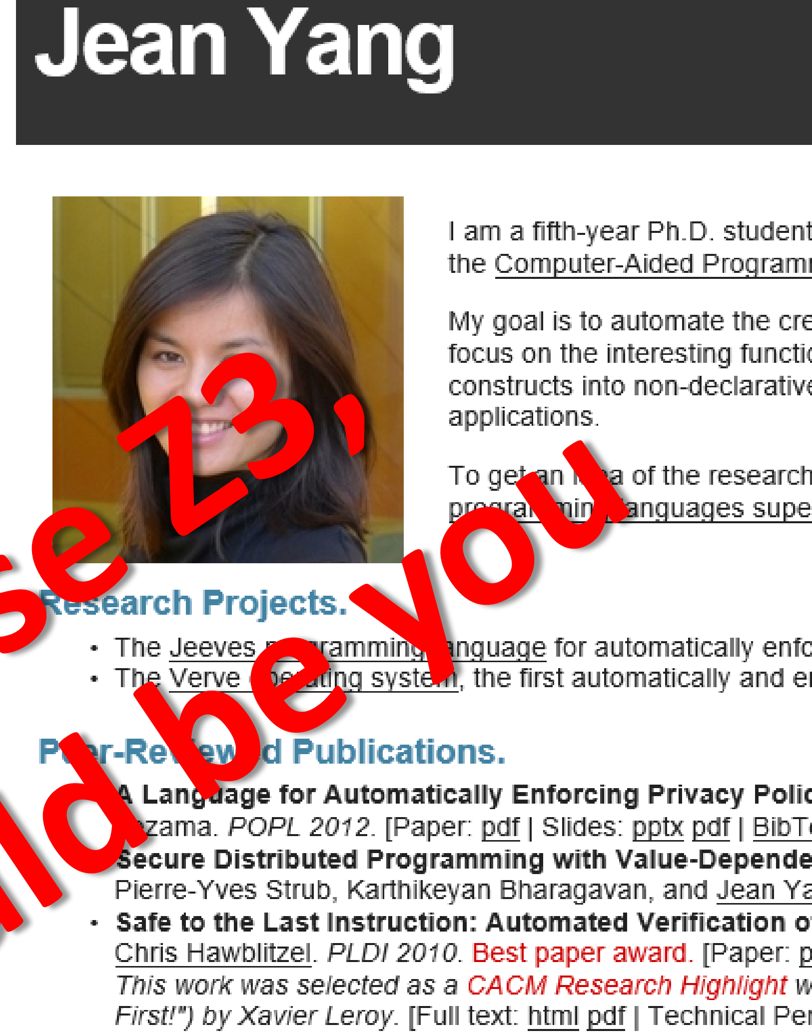
II. SMT *solving* in a nutshell

III. SMT by example

Takeaways:

- Modern SMT solvers are a often good fit for program analysis tools.
  - Handle domains found in programs directly.
- The selected examples are intended to show instances where sub-tasks are reduced to SMT/Z3.

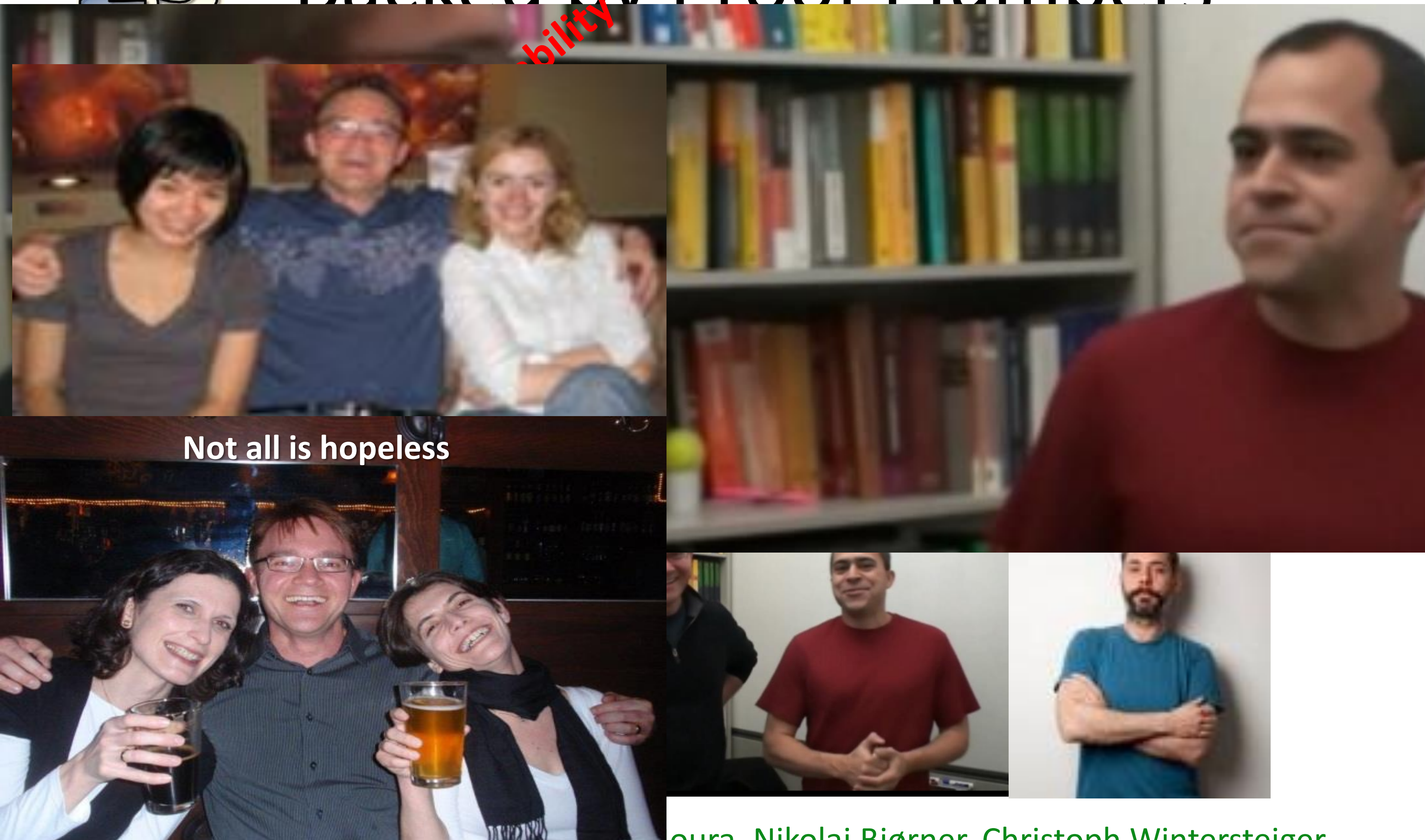

12



If you use Z3,

This could be you

Z3 – Backed by Proof Plumbers



oura, Nikolaj Bjørner, Christoph Wintersteiger

Background Reading: SMT

September 2011

isfiability Modulo Theories: Introduction & Applications

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RACT

int satisfaction problems arise in many diverse ar-
uding software and hardware verification, type infer-
atic program analysis, test-case generation, schedul-
anning and graph problems. These areas share a
n trait, they include a core component using logical
s for describing states and transformations between
The most well-known constraint satisfaction problem
ositional satisfiability*, SAT, where the goal is to de-
ether a formula over Boolean variables, formed using
connectives can be made *true* by choosing *true*/*false*
or its variables. Some problems are more naturally
ed using richer languages, such as arithmetic. A sup-
theory (of arithmetic) is then required to capture
ning of these formulas. Solvers for such formulations
monly called *Satisfiability Modulo Theories* (SMT)

SMT solvers have been the focus of increased recent attention thanks to technological advances and industrial applications. Yet, they draw on a combination of some of the most fundamental areas in computer science as well as discoveries from the past century of symbolic logic. They combine the problem of Boolean Satisfiability with domains, such as, those studied in convex optimization and term-manipulating symbolic systems. They involve the decision problem, completeness and incompleteness of logical theories, and finally complexity theory. In this article, we present an overview of the field of Satisfiability Modulo Theories, and some of its applications.

key driving factor [4]. An important ingredient is a common interchange format for benchmarks, called SMT-LIB [33], and the classification of benchmarks into various categories depending on which theories are required. Conversely, a growing number of applications are able to generate benchmarks in the SMT-LIB format to further inspire improving SMT solvers.

There is a relatively long tradition of using SMT solvers in select and specialized contexts. One prolific case is theorem proving systems such as ACL2 [26] and PVS [32]. These use decision procedures to discharge lemmas encountered during interactive proofs. SMT solvers have also been used for a long time in the context of program verification and *extended static checking* [21], where verification is focused on assertion checking. Recent progress in SMT solvers, however, has enabled their use in a set of diverse applications, including interactive theorem provers and extended static checkers, but also in the context of scheduling, planning, test-case generation, model-based testing and program development, static program analysis, program synthesis, and run-time analysis, among several others.

We begin by introducing a motivating application and a simple instance of it that we will use as a running example.

1.1 An SMT Application - Scheduling

Consider the classical *job shop scheduling* decision problem. In this problem, there are n jobs, each composed of m tasks of varying duration that have to be performed consecutively on m machines. The start of a new task can be delayed as long as needed in order to wait for a machine to become available, but tasks cannot be interrupted once

Some Microsoft Tools based on Z3

Program Verification

Auditing

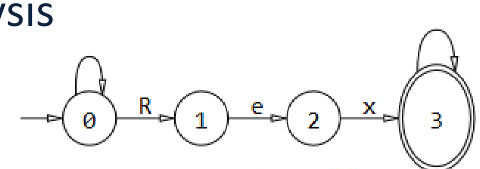
Type Safety

Over-Approximation

Under-Approximation

Testing

Analysis



Synthesis

SLAyer

SAGE

http://rise4fun.com/

rise4fun

1214986 programs analyzed

rise4fun

a community of software engineering tools

all tutorial automata concurrency design encoders infrastructure languages security synthesis testing verification

new!

f*
A verification tool for higher-order stateful programs

fast
A domain specific language for writing and analyzing tree manipulating programs

iz3
Efficient Interpolating Theorem Prover

microsoft

agl
Automatic Graph Layout

bek
A domain specific language for writing and analyzing common string functions

bex
A domain specific language for writing and analyzing string encoders and decoders

boogie
Intermediate Verification Language

chalice
A language and program verifier for reasoning about concurrent programs.

code contracts
Language agnostic modular program verification and repair with abstract interpretation.

counterdog
Theorem-prover for Counterfactual Datalog

dafny
A language and program verifier for functional correctness

dkal
Distributed Knowledge Authorization Language

esm
Empirical Software Engineering and Measurement Group

fast
A domain specific language for writing and analyzing tree manipulating programs

formula
Formal Modeling Using Logic Programming and Analysis

formula2
Formal Modeling Using Logic Programming and Analysis

try f#
Programming language combining functional, object-oriented and scripting programming.

f*
A verification tool for higher-order stateful programs

heapdbg
Runtime heap abstraction

iz3
Efficient Interpolating Theorem Prover

koka
A function-oriented language with effect inference

pex
Automatic test generation using Dynamic Symbolic Execution for .NET

quickcode
Programming-by-example technology for learning string transformation programs

concurrent revisions
Parallel and Concurrent Programming With Snapshots

rex
Regular Expression Exploration

seal
Side-Effects Analysis

slayer
Automatic formal verification for programs with heaps.

spec#
A formal language for API contracts

touchdevelop
Program your phone on your phone.

vcc
A Verifier for Concurrent C

visual c++
The Visual C++ compiler

z3
Efficient Theorem Prover

z34bio
SMT-based Analysis of Biological Computation

z3py
Python interface for the Z3 Theorem Prover

albert-ludwigs-universität freiburg

gravy
The Gradual Verifier

joogie
Infeasible Code Detection for Java

eth zurich - chair of software engineering

autoproof
a Program Verifier for Eiffel

boogaloo
the Boogie Interpreter

javanni
a Verifier for JavaScript

qfis
a Program Verifier for Integer Sequences

ku leuven

verifast
Verifier for C and Java Programs

multicore programming group, imperial college london

gpuverify-cuda
A verifier for CUDA/OpenCL kernels

gpuverify-opencl
A verifier for CUDA/OpenCL kernels

university of utah and imdea software institute

smack
Verifier for C/C++ Programs

SMT IN A NUTSHELL

Satisfiability Modulo Theories (SMT)

Is formula φ satisfiable modulo theory *T* ?

SMT solvers have specialized algorithms for *T*

Satisfiability Modulo Theories (SMT)

$$x + 2 = y \Rightarrow f(select(store(a, x, 3), y - 2)) = f(y - x + 1)$$

Array Theory

Arithmetic

Uninterpreted Functions

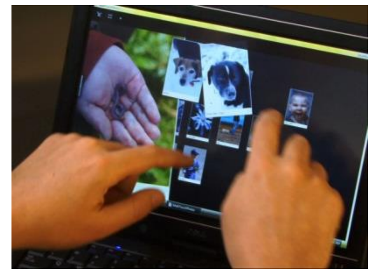
$$select(store(a, i, v), i) = v$$

$$i \neq j \Rightarrow select(store(a, i, v), j) = select(a, j)$$

SMT SOLVING IN A NUTSHELL

Job Shop Scheduling

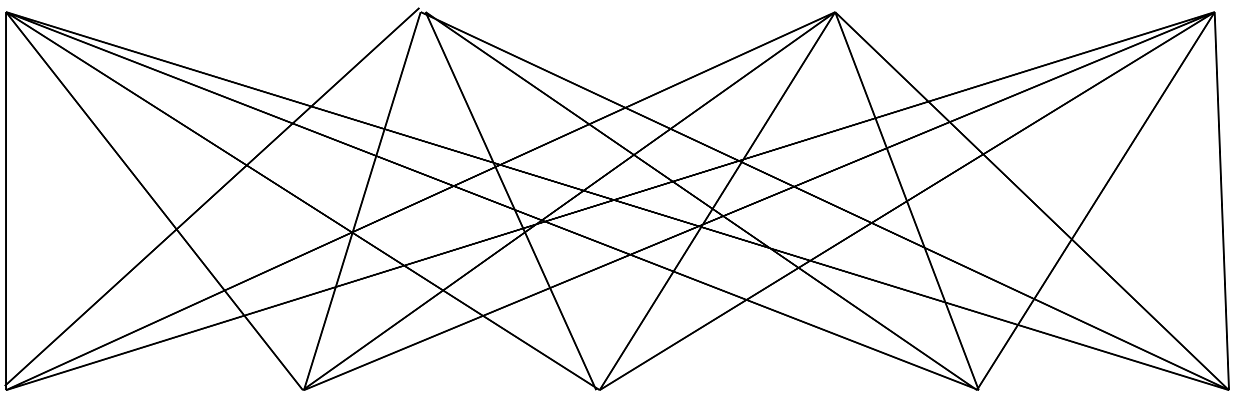
Job Shop Scheduling



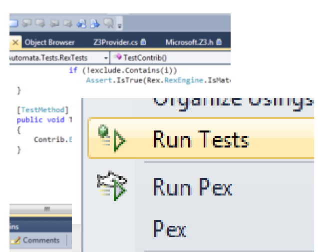
Machines

Tasks

Jobs



P = NP?

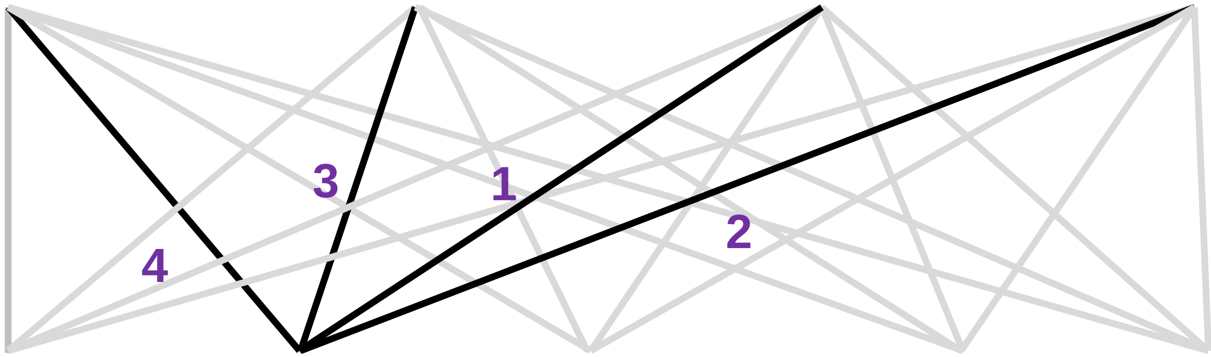


$\zeta(s) = 0 \Rightarrow s = \frac{1}{2} + ir$

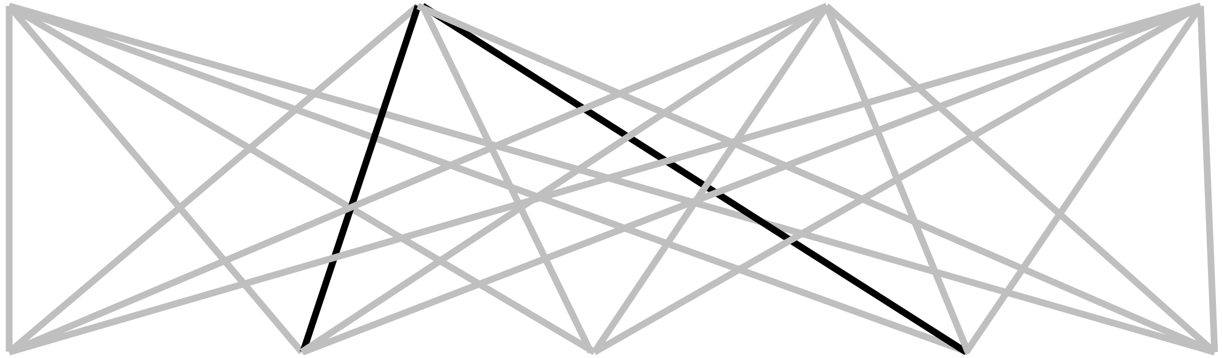
Job Shop Scheduling

Constraints:

Precedence: between two tasks of the same job



Resource: Machines execute at most one job at a time

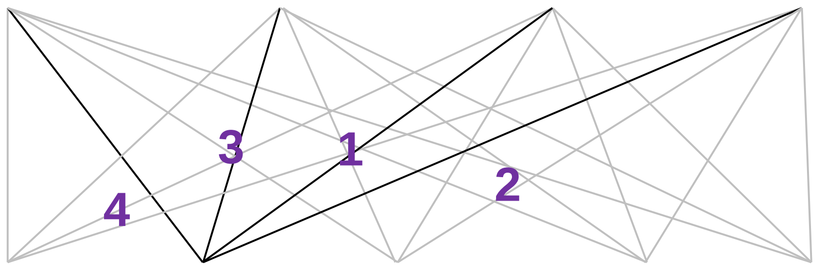


$$[start_{2,2}..end_{2,2}] \cap [start_{4,2}..end_{4,2}] = \emptyset$$

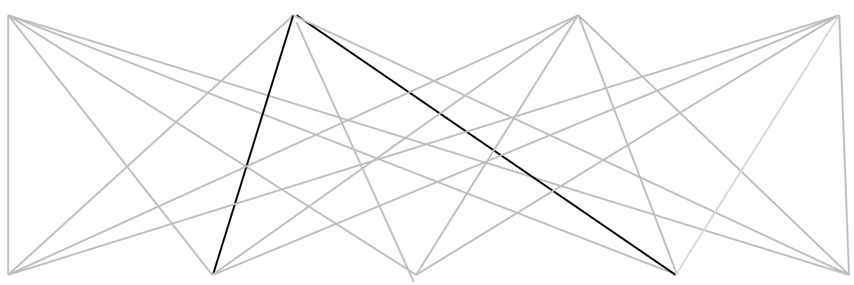
Job Shop Scheduling

Constraints:

Precedence:



Resource:



$$[start_{2,2}..end_{2,2}] \cap [start_{4,2}..end_{4,2}] = \emptyset$$

Encoding:

$t_{2,3}$ - start time of job 2 on mach 3

$d_{2,3}$ - duration of job 2 on mach 3

$$t_{2,3} + d_{2,3} \leq t_{2,4}$$

Not convex

$$t_{2,2} + d_{2,2} \leq t_{4,2}$$
$$\vee$$
$$t_{4,2} + \mathrm{d}_{4,2} \leq t_{2,2}$$

Job Shop Scheduling

$d_{i,j}$	Machine 1	Machine 2
Job 1	2	1
Job 2	3	1
Job 3	2	3

$max = 8$

Solution

$t_{1,1} = 5,\ t_{1,2} = 7,\ t_{2,1} = 2,$
$t_{2,2} = 6,\ t_{3,1} = 0,\ t_{3,2} = 3$

Encoding

$$(t_{1,1} \geq 0) \wedge (t_{1,2} \geq t_{1,1} + 2) \wedge (t_{1,2} + 1 \leq 8) \wedge$$
$$(t_{2,1} \geq 0) \wedge (t_{2,2} \geq t_{2,1} + 3) \wedge (t_{2,2} + 1 \leq 8) \wedge$$
$$(t_{3,1} \geq 0) \wedge (t_{3,2} \geq t_{3,1} + 2) \wedge (t_{3,2} + 3 \leq 8) \wedge$$
$$((t_{1,1} \geq t_{2,1} + 3) \vee (t_{2,1} \geq t_{1,1} + 2)) \wedge$$
$$((t_{1,1} \geq t_{3,1} + 2) \vee (t_{3,1} \geq t_{1,1} + 2)) \wedge$$
$$((t_{2,1} \geq t_{3,1} + 2) \vee (t_{3,1} \geq t_{2,1} + 3)) \wedge$$
$$((t_{1,2} \geq t_{2,2} + 1) \vee (t_{2,2} \geq t_{1,2} + 1)) \wedge$$
$$((t_{1,2} \geq t_{3,2} + 3) \vee (t_{3,2} \geq t_{1,2} + 1)) \wedge$$
$$((t_{2,2} \geq t_{3,2} + 3) \vee (t_{3,2} \geq t_{2,2} + 1))$$

Job Shop Scheduling

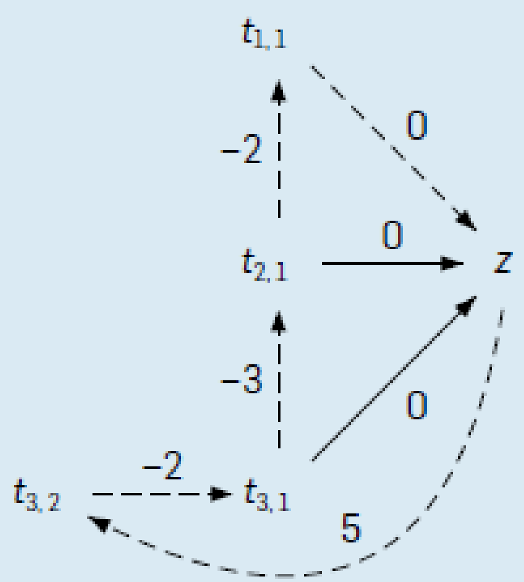
$$
\begin{aligned}
&(t_{1,1} \geq 0) \wedge (t_{1,2} \geq t_{1,1} + 2) \wedge (t_{1,2} + 1 \leq 8) \wedge \\
&(t_{2,1} \geq 0) \wedge (t_{2,2} \geq t_{2,1} + 3) \wedge (t_{2,2} + 1 \leq 8) \wedge \\
&(t_{3,1} \geq 0) \wedge (t_{3,2} \geq t_{3,1} + 2) \wedge (t_{3,2} + 3 \leq 8) \wedge \\
&((t_{1,1} \geq t_{2,1} + 3) \vee (t_{2,1} \geq t_{1,1} + 2)) \wedge \\
&((t_{1,1} \geq t_{3,1} + 2) \vee (t_{3,1} \geq t_{1,1} + 2)) \wedge \\
&((t_{2,1} \geq t_{3,1} + 2) \vee (t_{3,1} \geq t_{2,1} + 3)) \wedge \\
&((t_{1,2} \geq t_{2,2} + 1) \vee (t_{2,2} \geq t_{1,2} + 1)) \wedge \\
&((t_{1,2} \geq t_{3,2} + 3) \vee (t_{3,2} \geq t_{1,2} + 1)) \wedge \\
&((t_{2,2} \geq t_{3,2} + 3) \vee (t_{3,2} \geq t_{2,2} + 1))
\end{aligned}
$$

case split

Efficient solvers:
- Floyd-Warshal algorithm
- Ford-Fulkerson algorithm

case split

$$
\begin{array}{cccc}
z & - & t_{1,1} & \leq & 0 \\
z & - & t_{2,1} & \leq & 0 \\
z & - & t_{3,1} & \leq & 0 \\
t_{3,2} & - & z & \leq & 5 \\
t_{3,1} & - & t_{3,2} & \leq & -2 \\
t_{2,1} & - & t_{3,1} & \leq & -3 \\
t_{1,1} & - & t_{2,1} & \leq & -2
\end{array}
$$



$$z - z = 5 - 2 - 3 - 2 = -2 < 0$$

THEORIES

Theories

Uninterpreted functions

z3

Microsoft Research

Is this formula satisfiable? **Ask z3!**

```
(declare-sort () A)
(declare-fun f (A) A))
(declare-const a A)
(assert (= a (f (f a))))
(assert (= a (f (f (f a)))))
(check-sat)
(get-model)
(echo "Adding contradiction")
(assert (not (= a (f a))))
(check-sat)
```

ask z3

home tutorial video permal

Theories

Uninterpreted funct

[Arithmetic (linear)]

Microsoft Research

z3py

Explore the Z3 API using Python

```
t11, t12, t21, t22, t31, t32 = Ints('t11 t12 t21 t22 t31 t32')

s = Solver()

s.add(And([t11 >= 0, t12 >= t11 + 2, t12 + 1 <= 8]))
s.add(And([t21 >= 0, t22 >= t21 + 3, t22 + 1 <= 8]))
s.add(And([t31 >= 0, t32 >= t31 + 2, t32 + 3 <= 8]))

s.add(Or(t11 >= t21 + 3, t21 >= t11 + 2))
s.add(Or(t11 >= t31 + 2, t31 >= t11 + 2))
s.add(Or(t21 >= t31 + 2, t31 >= t21 + 3))
s.add(Or(t21 >= t22 + 1, t22 >= t12 + 1))
s.add(Or(t12 >= t32 + 3, t32 >= t12 + 1))
s.add(Or(t22 >= t32 + 3, t32 >= t22 + 1))

print ">>", s.check()
print ">>", s.model()
```

▶ tutorial home permalink

'▶' shortcut: Alt+B

```
>> sat
>> [t31 = 0, t21 = 4, t22 = 7, t32 = 2, t12 = 5, t11 = 2]
```

samples about Z3Py - Python interface for the Z3 Theorem P

Theories

Uninterpreted functions

Arithmetic (linear)

[Bit-vectors]

Microsoft Research

z3py

Explore the Z3 API using Python

```
x      = BitVec('x', 32)
powers = [ 2**i for i in range(32) ]
fast   = And(x != 0, x & (x - 1) == 0)
slow   = Or([ x == p for p in powers ])

prove(fast == slow)

print "buggy version..."

fast   = x & (x - 1) == 0

prove(fast == slow)
```

▶ tutorial home permalink

'▶' shortcut: Alt+B

```
proved
buggy version...
counterexample
[x = 0]
```

Theories

Uninterpreted functions

Arithmetic (linear)

Bit-vectors

[Algebraic data-types]

Microsoft Research

z3py

Explore the Z3 API using Python

```
List = Datatype('List')
List.declare('cons', ('car', IntSort()), ('cdr', List))
List.declare('nil')
List = List.create()
cons = List.cons
car  = List.car
cdr  = List.cdr
nil  = List.nil
l1 = cons(10, cons(20, nil))

print ">>", simplify(cdr(l1))

print ">>", simplify(car(l1))

print ">>", simplify(l1 == nil)


x, y = Ints('x y')
l1 = Const('l1',List)
l2 = Const('l2',List)
s = Solver()
```

▶ tutorial home permalink

'▶' shortcut: Alt+B

Theories

Uninterpreted functions

Arithmetic (linear)

Bit-vectors

Algebraic data-types

[Arrays]

```
; supported in Z3.
; This includes Combinatory Array Logic (de Moura &
;
(define-sort A () (Array Int Int))
(declare-fun x () Int)
(declare-fun y () Int)
(declare-fun z () Int)
(declare-fun a1 () A)
(declare-fun a2 () A)
(declare-fun a3 () A)
(push) ; illustrate select-store
(assert (= (select a1 x) x))
(assert (= (store a1 x y) a1))
(check-sat)
(get-model)
(assert (not (= x y)))
(check-sat)
(pop)
(define-fun all1_array () A ((as const A) 1))
(simplify (select all1_array x))
(define-sort IntSet () (Array Int Bool))
```

ask z3 | home | tutorial | video | permalink

```
sat
(model
  (define-fun y () Int
    1)
  (define-fun a1 () (Array Int Int)
    (_ as-array k!0))
  (define-fun x () Int
    1)
  (define-fun k!0 ((x!1 Int)) Int
    (ite (= x!1 1) 1
```

Theories

Uninterpreted functions

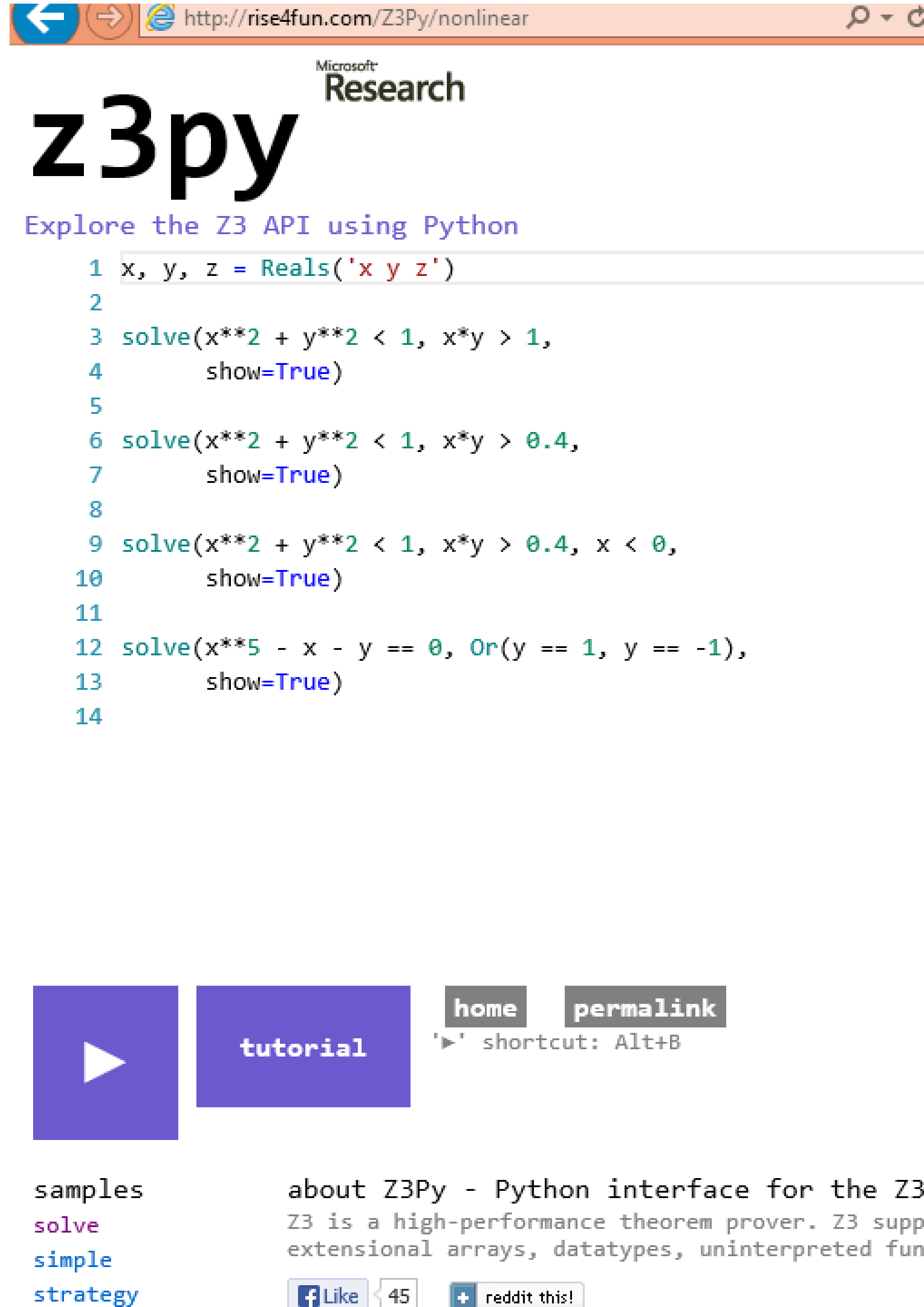
Arithmetic (linear)

Bit-vectors

Algebraic data-types

Arrays

Polynomial Arithmetic



QUANTIFIERS

Equality-Matching

$$p_{(\forall \ldots)}$$

$$\wedge \quad a = g(b, b)$$

$$\wedge \quad b = c$$

$$\wedge \quad f(a) \neq c$$

$$\wedge \quad p_{(\forall x \ldots)} \rightarrow f\big(g(c, b)\big) = b$$

$g(c, x)$ matches $g(b, b)$
with substitution $[x \mapsto b]$
modulo $b = c$

[de Moura, B. CADE 2007]

Quantifier Elimination

```
(define-fun stamp () Bool
  (forall((x Int))
   (=>
     (>= x 8)
     (exists ((u Int) (v Int))
       (and (>= u 0) (>= v 0) (= x (+ (* 3 u) (* 5 v))))))))

(simplify stamp)

(elim-quantifiers stamp)
```

Presburger Arithmetic,
Algebraic Data-types,
Quadratic polynomials

SMT integration to prune branches

[B. IJCAR 2010]

MBQI: Model based Quantifier Instantiation

```
(set-option :mbqi true)
(declare-fun f (Int Int) Int)
(declare-const a Int)
(declare-const b Int)

(assert (forall ((x Int)) (>= (f x x) (+ x a))))

(assert (< (f a b) a))
(assert (> a 0))
(check-sat)
(get-model)

(echo "evaluating (f (+ a 10) 20)...")
(eval (f (+ a 10) 20))
```

[de Moura, Ge. CAV 2008]
[Bonachnia, Lynch, de Moura CADE 2009]
[de Moura, B. IJCAR 2010]

Horn Clauses

mc(x) = x-10 if $x > 100$

mc(x) = mc(mc(x+11)) if $x \leq 100$

assert (mc(x) ≥ 91)

$$\forall X.\ X > 100 \rightarrow mc(X, X - 10)$$

$$\forall X, Y, R.\ X \leq 100 \wedge mc(X + 11, Y) \wedge mc(Y, R) \rightarrow mc(X, R)$$

$$\forall X, R.\ mc(X, R) \wedge X \leq 101 \rightarrow R = 91$$

***Solver finds solution for* mc**

[Hoder, B. SAT 2012]

MODELS, PROOFS, CORES & SIMPLIFICATION

Models

Click on a tool to load a sample then ask!

agl bek boogie code contracts concurrent revisions
dafny esm fine heapdbg poirot pex rex spec# vcc
z3

```
(define-sorts ((A (Array Int Int))))
(declare-funs ((x Int) (y Int) (z Int)))
(declare-funs ((a1 A) (a2 A) (a3 A)))
(assert (= (select a1 x) x))
(assert (= (store a1 x y) a1))
(check-sat)
(get-info model)
```

Logical Formula

ask z3 *Is this SMT formula satisfiable? **Click 'ask Z3'!** Read more or watch the video.*

```
sat
(("model" "
(define x 0)
(define a1 as-array[k!0])
(define y 0)
(define (k!0 (x1 Int))
(if (= x1 0) 0
1))"))
```

Sat/Model

Proofs

```
(set-logic QF_LIA)
(declare-funs ((x Int) (x1 Int)))
(declare-funs ((x3 Int) (x2 Int)))
(declare-funs ((x4 Int) (x5 Int)))
(declare-funs ((y Int) (z Int) (u Int)))
(assert (> x y))
(assert (= (- (* x 3) (* y 3)) (- z u)))
(assert (<= 0 z))
(assert (<= 0 u))
(assert (< z 3))
(assert (< u 3))
(check-sat)
(get-proof)
```

Logical Formula

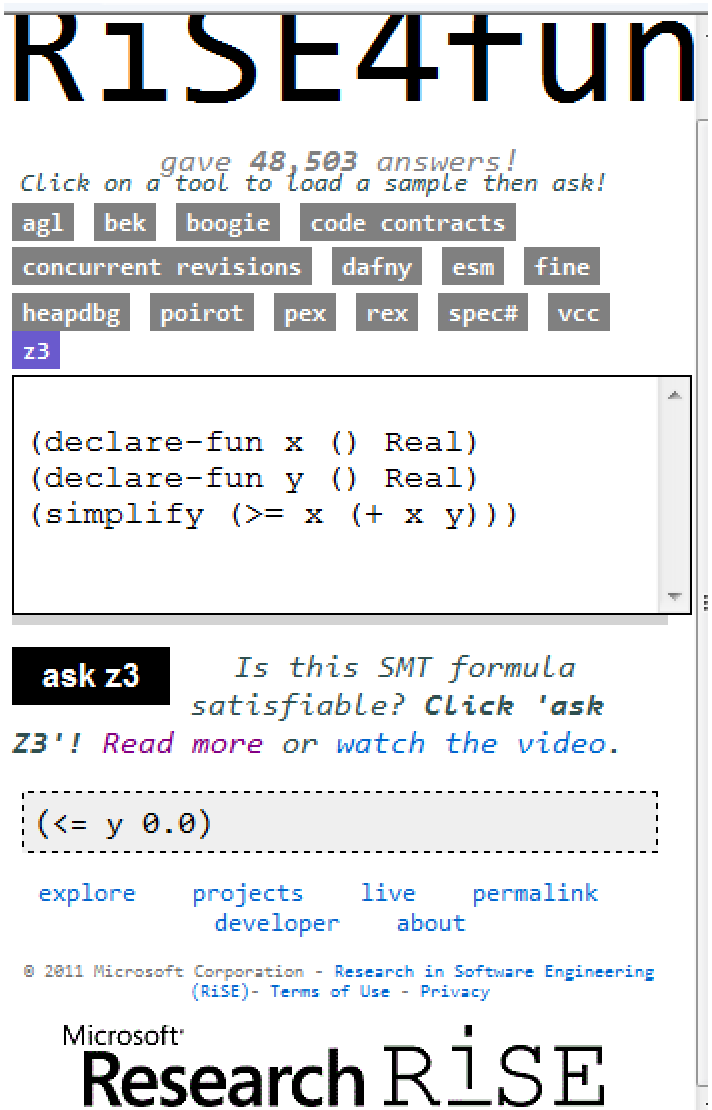
```
proof.smt2 PROOF_MODE=2

ted ((<= 0 u)] [rewrite (iff ((<= 0 u) (>= u 0))] (>= u 0)]
im

erted (= (- (* x 3) (* y 3)) (- z u))]
ns
onotonicity
[trans
  [monotonicity
    [rewrite (= (* x 3) (* 3 x))]
    [rewrite (= (* y 3) (* 3 y))]
    (= (- (* x 3) (* y 3)) (- (* 3 x) (* 3 y)))]
  [rewrite (= (- (* 3 x) (* 3 y)) (+ (* 3 x) (* -3 y)))]
  (= (- (* x 3) (* y 3)) (+ (* 3 x) (* -3 y)))]
[rewrite (= (- z u) (+ z (* -1 u)))]
(iff (= (- (* x 3) (* y 3)) (- z u))
     (= (+ (* 3 x) (* -3 y)) (+ z (* -1 u))))]
  [rewrite
    (iff (= (+ (* 3 x) (* -3 y)) (+ z (* -1 u)))
         (= (+ (* 3 x) (+ (* -3 y) (+ (* -1 z) u))) 0))]
  (iff (= (- (* x 3) (* y 3)) (- z u))
       (= (+ (* 3 x) (+ (* -3 y) (+ (* -1 z) u))) 0))]
 (= (+ (* 3 x) (+ (* -3 y) (+ (* -1 z) u))) 0)]
[rewrite
 (iff (= (+ (* 3 x) (+ (* -3 y) (+ (* -1 z) u))) 0)
      (not (or (not ((<= (+ (* 3 x) (+ (* -3 y) (+ (* -1 z) u))) 0))
               (not ((>= (+ (* 3 x) (+ (* -3 y) (+ (* -1 z) u))) 0)))))]
(not (or (not ((<= (+ (* 3 x) (+ (* -3 y) (+ (* -1 z) u))) 0))
         (not ((>= (+ (* 3 x) (+ (* -3 y) (+ (* -1 z) u))) 0))))]
 ((<= (+ (* 3 x) (+ (* -3 y) (+ (* -1 z) u))) 0)]
[mp
 [asserted ((> x y)]
 [rewrite (iff ((> x y) (not ((<= (+ x (* -1 y)) 0)))]
 (not ((<= (+ x (* -1 y)) 0))]
[mp [asserted ((< z 3)] [rewrite (iff ((< z 3) (not ((>= z 3)))] (not ((>= z 3))]
false]
```

Unsat/Proof

Simplification



Logical Formula

Simplify

Cores

```
(declare-preds ((p) (q) (r) (s)))
(set-option enable-cores)
(assert (or p q))
(assert (implies r s))
(assert (implies s (iff q r)))
(assert (or r p))
(assert (or r s))
(assert (not (and r q)))
(assert (not (and s p)))
(check-sat)
(get-unsat-core)
```

Logical Formula

ask z3 *Is this SMT formula satisfiable? **Click 'ask Z3'!** Read more or watch the video.*

```
unsat
((or p q)
(=> r s)
(or r p)
(or r s)
(not (and r q))
(not (and s p)))
```

Unsat. Core

TACTICS, SOLVERS

Tactics

```
(declare-const x (_ BitVec 16))
(declare-const y (_ BitVec 16))

(assert (= (bvor x y) (_ bv13 16)))
(assert (bvslt x y))

(check-sat-using (then simplify solve-eqs bit-blast sat))
(get-model)
```

Composition of tactics:

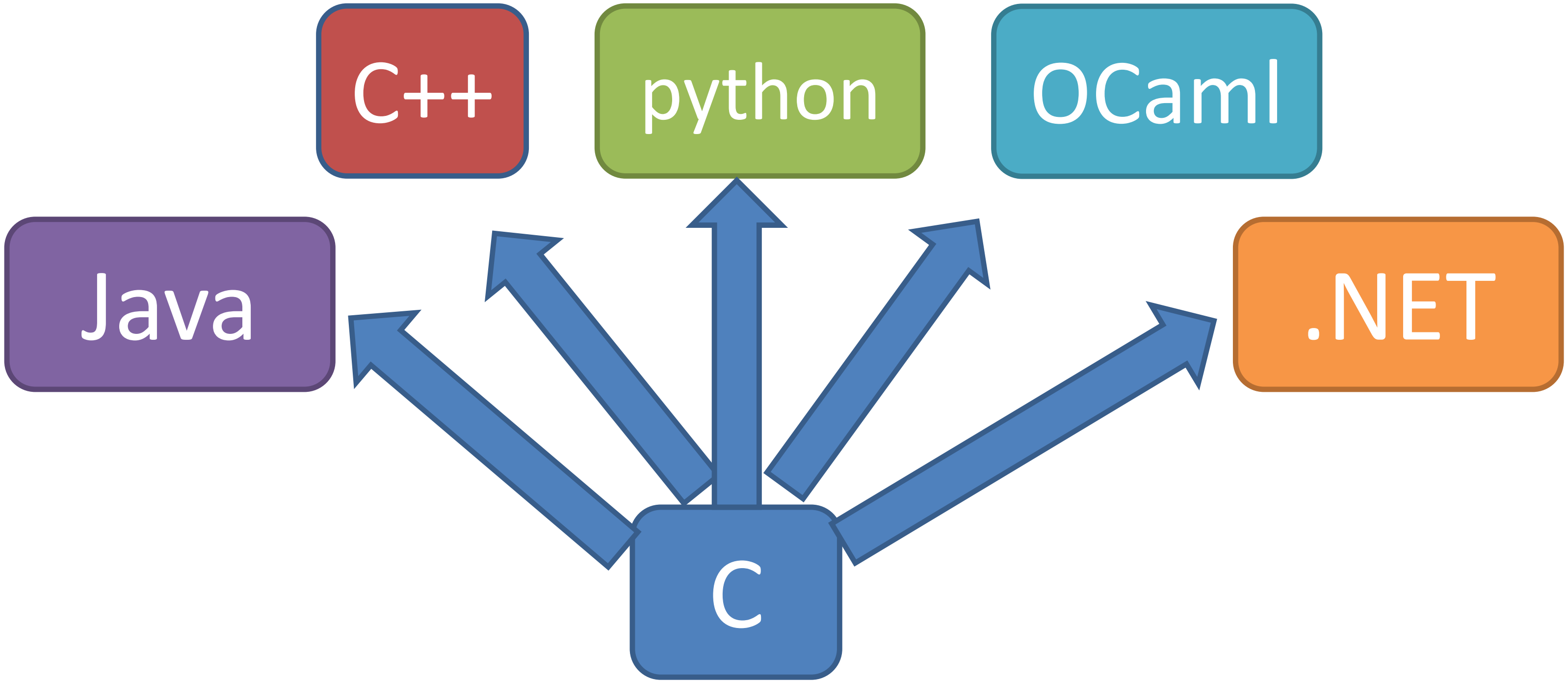
- (then t s)
- (par-then t s) applies t to the input goal and s to every subgoal produced by t in parallel.
- (or-else t s)
- (par-or t s) applies t and s in parallel until one of them succeed.
- (repeat t)
- (repeat t n)
- (try-for t ms)
- (using-params t params) Apply the given tactic using the given parameters.

Solvers

- Tactics take goals and reduce to sub-goals
- Solvers take tactics and serve as logical contexts.
    - push
    - add
    - check
    - model, core, proof
    - pop

```
bv_solver = Then(With('simplify', mul2concat=True),
                 'solve-eqs',
                 'bit-blast',
                 'aig',
                 'sat').solver()
x, y = BitVecs('x y', 16)
bv_solver.add(x*32 + y == 13, x & y < 10, y > -100)
print bv_solver.check()
m = bv_solver.model()
print m
print x*32 + y, "==", m.evaluate(x*32 + y)
print x & y, "==", m.evaluate(x & y)
```

APIS



SMT SOLVING

SMT : Basic Architecture

Case Analysis

- Equality + UF
- Arithmetic
- Bit-vectors
- …

SAT + Theory solvers

Basic Idea

$x \geq 0,\ y = x + 1,\ (y > 2 \vee y < 1)$

Abstract (aka "naming" atoms)

$p_1,\ p_2,\ (p_3 \vee p_4)$

$p_1 \equiv (x \geq 0),\ p_2 \equiv (y = x + 1),$

$p_3 \equiv (y > 2),\ p_4 \equiv (y < 1)$

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SAT
Solver

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SAT Solver

Assignment

$p_1,\ p_2,\ \neg p_3,\ p_4$

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SAT Solver

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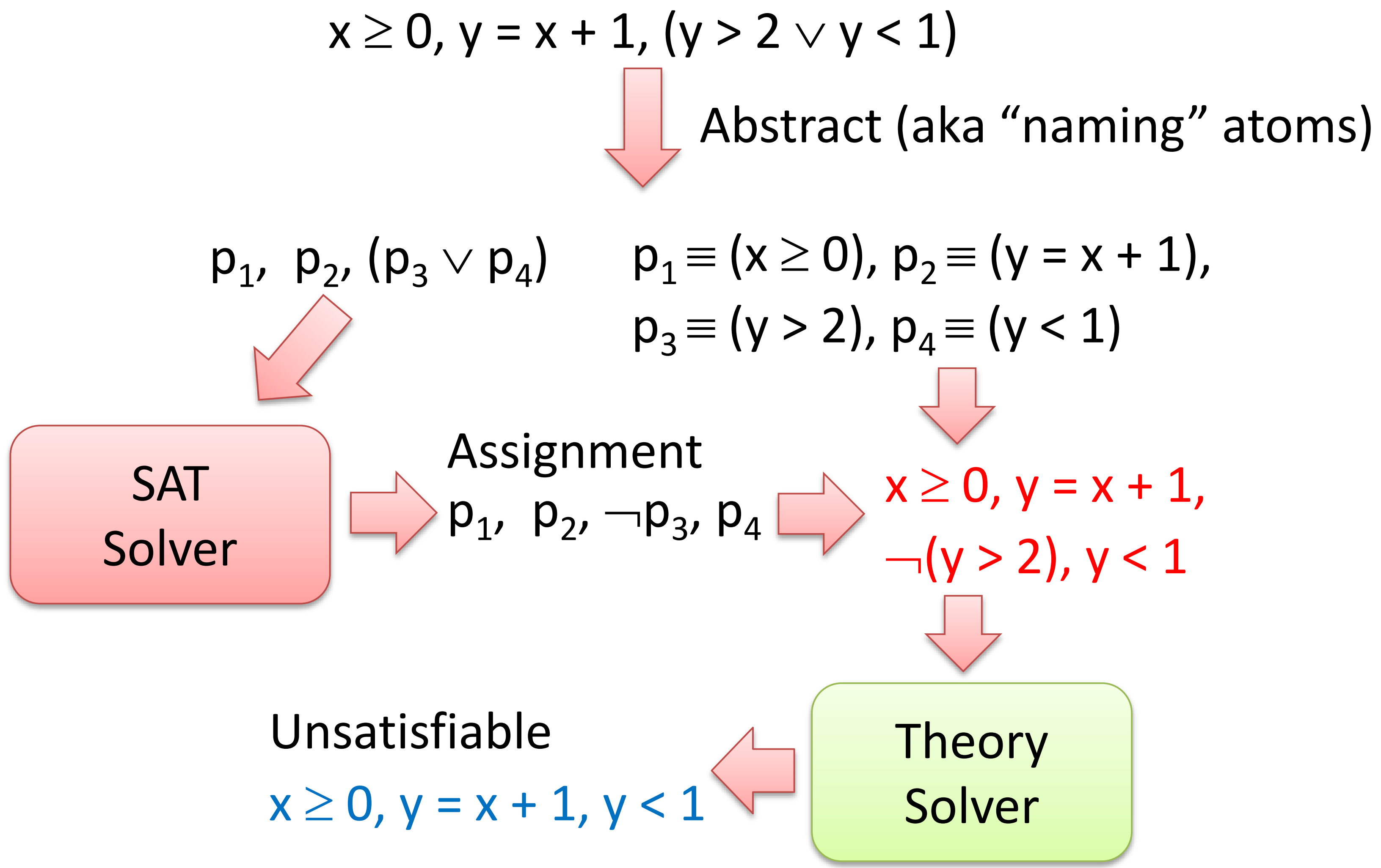
$x \geq 0,\ y = x + 1,$
$\neg(y > 2),\ y < 1$

SAT + Theory solvers

Basic Idea



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SAT Solver

Assignment

$p_1,\ p_2,\ \neg p_3,\ p_4$

$x \geq 0,\ y = x + 1,$

$\neg(y > 2),\ y < 1$

Theory Solver

Unsatisfiable

$x \geq 0,\ y = x + 1,\ y < 1$

New Lemma

$\neg p_1 \vee \neg p_2 \vee \neg p_4$

SAT + Theory solvers

New Lemma

$\neg p_1 \vee \neg p_2 \vee \neg p_4$

Unsatisfiable

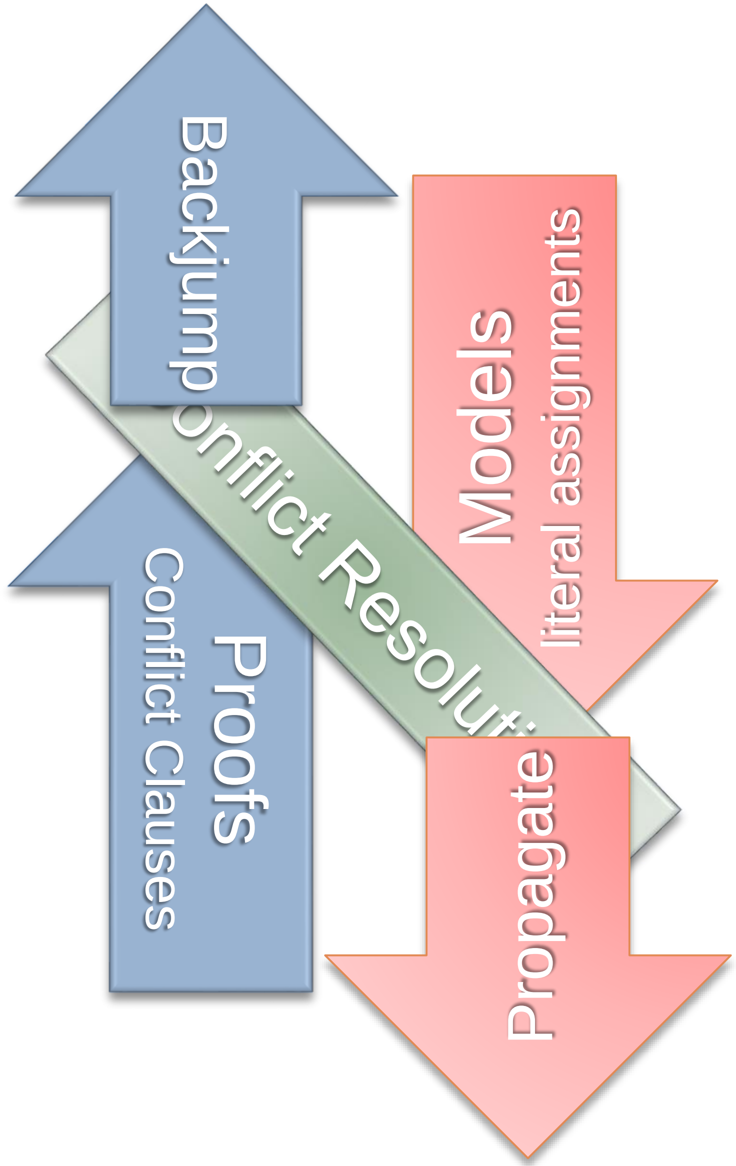
$x \geq 0,\ y = x + 1,\ y < 1$

Theory Solver

AKA
Theory conflict

SAT/SMT SOLVING USING DPLL(T)/CDCL

Mile High: Modern SAT/SMT search



Core Engine in Z3: Modern DPLL/CDCL

Initialize	$\epsilon \mid F$	F *is a set of clauses*	
Decide	$M \mid F \implies M, \ell \mid F$	ℓ *is unassigned*	Model
Propagate	$M \mid F, C \vee \ell \implies M, \ell^{C \vee \ell} \mid F, C \vee \ell$	C *is false under* M	Model
Sat	$M \mid F \implies M$	F *true under* M	Model
Conflict	$M \mid F, C \implies M \mid F, C \mid C$	C *is false under* M	Proof
Learn	$M \mid F \mid C \implies M \mid F, C \mid C$		Proof
Unsat	$M \mid F \mid \emptyset \implies Unsat$		Proof
Backjump	$MM' \mid F \mid C \vee \ell \implies M\ell^{C \vee \ell} \mid F$	$\bar{C} \subseteq M, \neg\ell \in M'$	Conflict Resolution
Resolve	$M \mid F \mid C' \vee \neg\ell \implies M \mid F \mid C' \vee C$	$\ell^{C \vee \ell} \in M$	Conflict Resolution
Forget	$M \mid F, C \implies M \mid F$	C is a learned clause	
Restart	$M \mid F \implies \epsilon \mid F$		

[Nieuwenhuis, Oliveras, Tinelli J.ACM 06] customized

DPLL(*T*) solver interaction

T- Propagate $M \mid F, C \vee \ell \implies M, \ell^{C \vee \ell} \mid F, C \vee \ell$ $\quad C\ is\ false\ under\ T + M$

T- Conflict $M \mid F \implies M \mid F \mid \neg M'$ $\quad M' \subseteq M\ and\ M'\ is\ false\ under\ T$

T- Propagate $a > b, b > c \mid F, a \leq c \vee b \leq d \implies$

$$a > b, b > c, b \leq d^{a \leq c \vee b \leq d} \mid F, a \leq c \vee b \leq d$$

T- Conflict $M \mid F \implies M \mid F,\ a \leq b \vee b \leq c \vee c < a$

$$where\ a > b, b > c, a \leq c \subseteq M$$

Summary

Z3 supports several theories

- Using a default combination
- Providing custom tactics for special combinations

Z3 is more than sat/unsat

- Models, proofs, unsat cores,
- simplification, quantifier elimination are tactics

Prototype with python/smt-lib2

- Implement using smt-lib2/programmatic API